MADHA

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ENGINEERING COLLEGE

Department of Mechanical Engineering

Lecture Notes

Subject Code : MA3351

Subject Name: TRANSFORM AND PARTIAL DIFFERENTIAL EQUATIONS

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21 : 09 : 2021

Applications of Partial differential equations

2 marks

1. Write down the three posible solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}.$$

i) $y(x,t) = (Ax+B)(Ct+D)$

ii) $y(x,t) = (Ae^{px} + Be^{-px})(Ce^{pat} + De^{-pat})$.

iii) $y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat)$.

2. What are the possible solution of the 1-D heat equation.

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$$

i) $u(x,t) = (Ax+B)C$

ii) $u(x,t) = (Ae^{px} + Be^{-px})(Ce^{\alpha^2 p^2 t})$.

iii) $u(x,t) = (A\cos px + B\sin px)\, Ce^{-\alpha^2 p^2 t}$.

3. Write down the possible solution of the 2D heat flow (or) Laplace equation. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

i) $u(x,y) = (Ax+B)(Cy+D)$

ii) $u(x,y) = (A\cos py + B\sin py)(Ce^{px} + De^{-px})$.

iii) $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$.

4. In the one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 stands for?

$\alpha^2 \rightarrow$ diffusivity constant

$\alpha^2 \rightarrow \frac{k}{\rho c}$.

where $k \rightarrow$ thermal conductivity.

$\rho \rightarrow$ density of the bar.

$c \rightarrow$ specific heat capacity.

5. In the 1-D wave equation: $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. What does a^2 stands for?

$a^2 = \frac{T}{m} \Rightarrow \frac{\text{Tension}}{\text{mass}}$.

6. Derive the solution of one dimensional heat flow equation in steady state conditions.

Let 1-D heat flow is

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \rightarrow (1)$$

In steady state, $\frac{\partial u}{\partial t} = 0$

$$(1) \Rightarrow \alpha^2 \frac{\partial^2 u}{\partial x^2} = 0$$

since $\alpha^2 \neq 0$ $\therefore$ $\frac{\partial^2 u}{\partial x^2} = 0$

i.e $\frac{d^2 u}{dx^2} = 0$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = 0$$

Integrate it

$$\frac{du}{dx} = a$$

$\therefore$ $du = a\,dx$

Again integrate it

$$\int du = a \int dx$$

$$u = ax + b$$

(i.e) $u(x) = ax + b$.

7. A rod 30 cm long has its end A and B kept at 20°C and 80°C respectively until steady conditions prevail. Find the steady state temperature in the rod.

Let the temperature function be

$$u(x) = ax + b \rightarrow (1).$$

With i) $u(0) = 20$ ii) $u(l) = 80$

Applying (i) & (1)

$$u(0) = a(0) + b = 20$$

$$\therefore \boxed{b = 20}$$

$$(1) \Rightarrow u(x) = ax + 20 \rightarrow (2)$$

Applying (ii) in (2)

$$u(l) = al + 20 = 80$$

$$al = 80 - 20 = 60$$

$$al = 60$$

$$\boxed{a = \frac{60}{l}}$$

$$u(x) = \frac{60}{l} x + 20$$

$$u(x) = \frac{60}{30} x + 20$$

$$\boxed{u(x) = 2x + 20}$$

Alternate method:

Given: $\theta_1 = 20$, $\theta_2 = 80$, $l = 30$

$$u(x) = \left(\frac{\theta_2 - \theta_1}{l}\right) x + \theta_1$$

$$u(x) = \left(\frac{80 - 20}{30}\right) x + 20$$

$$u(x) = \frac{60}{30} x + 20$$

$$\boxed{u(x) = 2x + 20}$$

8. Write down the initial conditions of the wave equation, if the string has an initial displacement but no initial velocity.

i) $y(0,t) = 0 \quad \forall t \geq 0.$

ii) $y(l,t) = 0 \quad \forall t \geq 0.$

iii) $\frac{\partial y}{\partial t}(x,0) = 0 \quad \forall x \text{ in } (0,l)$

iv) $y(x,0) = f(x)$

9. State the boundary condition of the 1-D wave equation with non-zero velocity:

i) $y(0,t) = 0 \quad \forall t \geq 0$

ii) $y(l,t) = 0 \quad \forall t \geq 0$

iii) $y(x,0) = 0 \quad \forall x \text{ in } (0,l)$

iv) $\frac{\partial y}{\partial t}(x,0) = f(x)$

10. What is the basic difference between the solution of 1-D wave equation and 1-D heat equation?

* 1-D wave equation is periodic with respect to time t.
* 1-D heat equation is non periodic with respect to time t.

11. What conditions are assumed in deriving the one dimensional wave equation:

1. The string is homogenous.
2. The string is perfectly elastic and so it does not offer any resistance to bending.

3. The tension T caused by stretching is so large so that the action of the gravitational force on the string can be neglected

4. The string platform small transverse motions in a vertical plane so that the deflection y and the slope are in absolute value and hence their powers (higher) may be neglected.

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Half range expansions:

Cosine series:

cosine series of f(x) in (0,l)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)dx.$$

$$a_n = \frac{2}{l}\int_0^l f(x) \cos \frac{n\pi x}{l} dx.$$

cosine series of f(x) in $(0,\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx.$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx.$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx dx.$$

Cor: Case(i): At $x = x_0$ is a point of continuity, Fourier series converges to $f(x_0)$.

Case(ii): Parseval's identity is

$$\frac{2}{l}\int_0^l [f(x)]^2\, dx + \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

(or)

$$\frac{2}{l}\int_0^{\pi} [f(x)]^2\, dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

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Unit-5

Z- Transforms

Difference equations:

If $z[y(n)] = y(z)$ then.

i) $z[y(n+1)] = z\, y(z) - z\, y(0)$.

ii) $z[y(n+2)] = z^2 y(z) - z^2 y(0) - z\, y(1)$.

iii) $z[y(n+3)] = z^3 y(z) - z^3 (y(0)] - z^2 y(1) - z\, y(2)$.

1. Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z- transforms:

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n.$$

Take z transforms on both sides. $\because z(a^n) = \frac{z}{z-a}$

$$z[y_{n+2}] + 6z[y_{n+1}] + 9z[y_n] = z[2^n].$$

$$[z^2 y(z) - z^2 y(0) - z y(1)] + 6[z y(z) - z y(0)] + 9 y(z) = \frac{z}{z-2}$$

$$y(z)[z^2 + 6z + 9] - z^2(0) - z(0) - 6(0) = \frac{z}{z-2}$$

$$y(z) = \frac{z}{(z-2)(z^2+6z+9)} = \frac{z}{(z-2)(z+3)^2}$$

$$z[y(n)] = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{y(n)}{z} = z^{-1}\left[\frac{1}{(z-2)(z+3)^2}\right] \rightarrow (1)$$

$$\frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2} \rightarrow (2)$$

$$\frac{1}{(z-2)(z+3)^2} = \frac{A(z+3)^2 + B(z+3)(z-2) + C(z-2)}{(z-2)(z+3)^2}$$

Put $z = -3$

$$1 = A(0) + B(0) + C(-3-2)$$

$$1 = -5C$$

$$\boxed{C = -\frac{1}{5}}$$

Put $z = 2$

$$1 = A(2+3)^2 + B(0) + C(0)$$

$$1 = A(25)$$

$$\boxed{A = \frac{1}{25}}$$

Put $z = 0$.

$$9A - 6B - 2C = 1$$

$$9\left(\frac{1}{25}\right) - 6B - 2\left(\frac{-1}{5}\right) = 1$$

$$\frac{9}{25} + \frac{2}{5} - 1 = 6B$$

$$\frac{9+10}{25} - 1 = 6B$$

$$\frac{9+10-25}{25} = 6B$$

$$\frac{-6}{25} = 6B$$

$$\boxed{B = -\frac{1}{25}}$$

Sub the value of A, B & C in (2).

$$\frac{1}{(z-2)(z+3)^2} = \frac{\frac{1}{25}}{(z-2)} - \frac{\frac{1}{25}}{(z+3)} - \frac{\frac{1}{5}}{(z+3)^2}.$$

Taking z^{-1} on both sides.

$$z^{-1}\left[\frac{1}{(z-2)(z+3)^2}\right] = \frac{1}{25} z^{-1}\left[\frac{1}{(z-2)}\right] - \frac{1}{25} z^{-1}\left[\frac{1}{z+3}\right] - \frac{1}{5} z^{-1}\left[\frac{1}{(z+3)^2}\right]$$

By eqn (1).

$$\frac{y(n)}{z} = \frac{1}{25} z^{-1}\left[\frac{1}{z-2}\right] - \frac{1}{25} z^{-1}\left[\frac{1}{z+3}\right] - \frac{1}{5} z^{-1}\left[\frac{1}{(z+3)^2}\right]$$

$$y(n) = \frac{1}{25} z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25} z^{-1}\left[\frac{z}{z+3}\right] - \frac{1}{5} z^{-1}\left[\frac{z}{(z+3)^2}\right].$$

$$y(n) = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n - \frac{1}{5}\cdot\frac{1}{3} z^{-1}\left[\frac{3z}{(z+3)^2}\right].$$

$$\left[\because z\left[a^n n\right] = \frac{az}{(z-a)^2}\right].$$

$$z\left[(-a)^n n\right] = \frac{-az}{(z+a)^2} \Rightarrow (-a)^n n = z^{-1}\left[\frac{az}{(z+a)^2}\right].$$

$$y(n) = \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n - \frac{1}{15}(-3)^n \cdot n.$$

Verification:

$$y(0) = \frac{1}{25} - \frac{1}{25} + 0 = 0$$

$$y(1) = \frac{2}{25} + \frac{3}{25} - \frac{3}{15}$$

$$= \frac{5}{25} = \frac{1}{5} = 1$$

2. Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$ by z-transforms:

Given that: $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$.

$y_0 = 0$, $y_1 = 1$.

Take z transform on both sides.

$$z[y_{n+2}] + 4z[y_{n+1}] + 3z[y_n] = z[3^n].$$

$$\left[z^2[y(z)] - z^2 y(0) - z y(1)\right] + 4\left[z y(z) - z y(0)\right] + 3y(z) = \frac{z}{z-3}$$

$$y(z)\left[z^2 + 4z + 3\right] - 0 - z - 0 = \frac{z}{z-3}$$

$$y(z)(z^2 + 4z + 3) = \frac{z}{z-3} + z = \frac{z + z(z-3)}{(z-3)}$$

$$y(z) = \frac{z + z^2 - 3z}{(z-3)(z^2+4z+3)}$$

$$y(z) = \frac{z^2 - 2z}{(z-3)(z+1)(z+3)}$$

$$z[y(n)] = \frac{z(z-2)}{(z+1)(z-3)(z+3)}$$

$$\frac{y(n)}{z} = z^{-1}\left[\frac{(z-2)}{(z+1)(z-3)(z+3)}\right] \rightarrow (1)$$

$$\frac{z-2}{(z+1)(z-3)(z+3)} = \frac{A}{(z+1)} + \frac{B}{(z-3)} + \frac{C}{(z+3)} \rightarrow (2).$$

$$\frac{z-2}{(z+1)(z-3)(z+3)} = \frac{A(z-3)(z+3) + B(z+1)(z+3) + C(z+1)(z+3)}{(z+1)(z-3)(z+3)}$$

Put $z = 3$

$3-2 = A(0) + B(3+1)(3+3) + C(0).$

$1 = B(4)(6)$

$1 = B(24)$

$$\boxed{B = \frac{1}{24}}$$

Put $z = -3$

$-3-2 = A(0) + B(0) + C(-3+1)(-3-3).$

$-5 = C(-2)(-6)$

$-5 = 12C$

$$\boxed{C = \frac{-5}{12}}$$

Put $z = -1$

$-1-2 = A(-1-3)(-1+3) + B(0) + C(0)$

$-3 = A(-4)(2)$

$-3 = -A(8)$

$$\boxed{A = \frac{3}{8}}$$

Sub the value of A, B & C in (a).

$$\frac{z-2}{(z+1)(z+3)(z-3)} = \frac{3/8}{z+1} + \frac{1/24}{z-3} + \frac{-5/12}{z+3}$$

Take z^{-1} on both sides.

By (1).

$$z^{-1}\left[\frac{z-2}{(z-1)(z+3)(z-3)}\right] = \frac{3}{8} z^{-1}\left[\frac{1}{z+1}\right] + \frac{1}{24} z^{-1}\left[\frac{1}{z-3}\right] - \frac{5}{12} z^{-1}\left[\frac{1}{z+3}\right]$$

$$\frac{y(n)}{z} = \frac{3}{8} z^{-1}\left[\frac{1}{z+1}\right] + \frac{1}{24} z^{-1}\left[\frac{1}{z-3}\right] - \frac{5}{12} z^{-1}\left[\frac{1}{z+3}\right]$$

$$y(n) = \frac{3}{8} z^{-1}\left[\frac{z}{z+1}\right] + \frac{1}{24} z^{-1}\left[\frac{z}{z-3}\right] - \frac{5}{12} z^{-1}\left[\frac{z}{z+3}\right]$$

$$y(n) = \frac{3}{8}(-1)^n + \frac{1}{24}(3)^n - \frac{5}{12}(-3)^n.$$

Verification:

$$y(0) = \frac{3}{8}(1) + \frac{1}{24} - \frac{5}{12}.$$

$$= \frac{9+1-10}{24} = \frac{10-10}{24} = 0.$$

$$y(1) = \frac{-3}{8} + \frac{3}{24} + \frac{15}{12}.$$

$$= -\frac{3}{8} + \frac{1}{8} + \frac{15}{12}$$

$$= \frac{-2}{8} + \frac{15}{12}$$

$$y(1) = \frac{4}{4} = 1$$

3. Solve: $y(n+3) - 3y(n+1) + 2y(n) = 0$ given that $y(0) = 4$, $y(1) = 0$, $y(2) = 8$.

Take z transforms on both sides.

$$z[y(n+3)] - 3z[y(n+1)] + 2z[y(n)] = 0$$

$$[z^3 y(z) - z^3 y(0) - z^2 y(1) - z y(2)] - 3[z y(z) - z y(0)] + 2y(z) = 0$$

$$y(z)\left[z^3-3z+2\right]-4z^3-0-8z+12z=0$$

$$y(z)\left[z^3-3z+2\right]=4z^3+8z-12z=4z^3-4z$$

$$y(z)=\frac{4z(z^2-1)}{z^3-3z+2}=\frac{4z(z+1)(z-1)}{(z+2)(z-1)^2}$$

$$y(z)=\frac{4z(z+1)}{(z+2)(z-1)}$$

$$y(z)=\frac{4z(z+1)}{(z-1)(z+2)}$$

$$z[y(n)]=\frac{4z(z+1)}{(z-1)(z+2)}$$

$$\frac{y(n)}{z}=z^{-1}\left[\frac{4(z+1)}{(z-1)(z+2)}\right]\rightarrow ①.$$

$$\frac{4(z+1)}{(z-1)(z+2)}=\frac{A}{(z-1)}+\frac{B}{(z+2)}\rightarrow ②.$$

$$\frac{A(z+1)}{(z-1)(z+2)}=\frac{A(z+2)+B(z-1)}{(z-1)(z+2)}$$

$$A(z+1)=A(z+2)+B(z-1)$$

put $z=-2$

$$4(-2+1)=A(0)+B(-2-1).$$

$$-4=-3B$$

$$\boxed{B=\frac{4}{3}}$$

put $z=1$

$$4(1+1)=A(1+2)+B(0)$$

$$8=3A$$

$$\boxed{A=\frac{8}{3}}$$

sub the value of A, B in ②

$$\frac{4(z+1)}{(z-1)(z+2)}=\frac{8/3}{(z-1)}+\frac{4/3}{(z+2)}$$

Take z^{-1} on both sides.

$$z^{-1}\left[\frac{4(z+1)}{(z-1)(z+2)}\right] = \frac{8}{3} z^{-1}\left[\frac{1}{z-1}\right] + \frac{4}{3} z^{-1}\left[\frac{1}{z+2}\right] \quad \text{By } ①.$$

$$\frac{y(n)}{2} = \frac{8}{3} z^{-1}\left[\frac{1}{z-1}\right] + \frac{4}{3} z^{-1}\left[\frac{1}{z+2}\right].$$

$$y(n) = \frac{8}{3}(1)^n + \frac{4}{3}(-2)^n.$$

Verification:

$$y(0) = \frac{8}{3} + \frac{4}{3} = \frac{12}{3} = \underline{\underline{4}}.$$

$$y(1) = \frac{8}{3} + \frac{4}{3}(-2) = \frac{8}{3} - \frac{8}{3} = \underline{\underline{0}}.$$

$$y(2) = \frac{8}{3} + \frac{4}{3} \times 4 = \frac{8}{3} + \frac{16}{3} = \frac{24}{3} = \underline{\underline{8}}$$

4. Solve : $y(n) + 3y(n-1) - 4y(n-2) = 0, \quad n \geq 2$

Given : $y(0) = 3, \; y(1) = -2$

Replace n by $n+2$ since $n \geq 2$

$$y(n+2) + 3y(n+2-1) - 4y(n+2-2) = 0$$

$$y(n+2) + 3y(n+1) - 4y(n) = 0$$

Take z, transform on both sides.

$$z[y(n+2)] + 3z[y(n+1)] - 4z[y(n)] = 0$$

$$[z^2 y(z) - z^2 y(0) - zy(1)] + 3[zy(z) - zy(0)] - 4y(z) = 0$$

$$y(z)[z^2 + 3z - 4] - 3z^2 + 2z - 9z = 0.$$

$$y(z)[z^3 + 3z - 4] = 3z^2 - 2z + 9z.$$

$$y(z) = \frac{3z^2 + 7z}{z^3 + 3z - 4}$$

$$z[y(n)] = \frac{z(3z+7)}{(z+4)(z-1)}$$

$$\frac{y(n)}{z} = z^{-1}\left[\frac{3z+7}{(z-1)(z+4)}\right] \longrightarrow ①$$

$$\frac{3z+7}{(z-1)(z+4)} = \frac{A}{(z-1)} + \frac{B}{(z+4)} \longrightarrow (2)$$

$$\frac{3z+7}{(z-1)(z+4)} = \frac{A(z+4)+B(z-1)}{(z-1)(z+4)}$$

$$3z+7 = A(z+4)+B(z-1)$$

Put $z=-4$

$$3z+7 = A(0)+B[-4-1]$$

$$-12+7 = -5B$$

$$-5 = -5B$$

$$\boxed{B=1}$$

Put $z=1$

$$3(1)+7 = A(1+4)+B(0).$$

$$10 = A(5)$$

$$\boxed{A=2}$$

Sub the values of A & B in (2).

$$\frac{3z+7}{(z-1)(z+4)} = \frac{2}{z-1} + \frac{1}{z+4}$$

Take z^{-1} on both sides.

$$z^{-1}\left[\frac{3z+7}{(z-1)(z+4)}\right] = 2z^{-1}\left[\frac{1}{z-1}\right] + z^{-1}\left[\frac{1}{z+4}\right]$$

By eqn (1).

$$\frac{y(n)}{z} = 2z^{-1}\left[\frac{1}{z-1}\right] + z^{-1}\left[\frac{1}{z+4}\right]$$

$$y(n) = 2z^{-1}\left[\frac{z}{z-1}\right] + z^{-1}\left[\frac{z}{z+4}\right]$$

$$y(n) = 2(1)^n + (-4)^n.$$

Verification:

$$y(0) = 2+1 = 3$$

$$y(1) = 2-4 = -2$$

5. Solve: $y_{n+2} - y_n = 2^n$, given that $y_0 = y_1 = 0$.

Take z-transform on both sides.

$$z[y_{n+2}] - z[y_n] = z[2^n].$$

$$[z^2 y(z) - z^2 y(0) - z y(1)] - y(z) = \frac{z}{z-2}$$

$$y(z)[z^2-1] - 0 - 0 = \frac{z}{z-2}$$

$$y(z) = \frac{z}{(z-2)(z^2-1)}.$$

$$z[y(n)] = \frac{z}{(z-2)(z+1)(z-1)}$$

$$\frac{y(n)}{z} = z^{-1}\left[\frac{1}{(z-1)(z+1)(z-2)}\right] \rightarrow (1)$$

$$\frac{1}{(z-1)(z+1)(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z+1)} + \frac{C}{(z-2)} \rightarrow (2)$$

$$\frac{1}{(z-1)(z+1)(z-2)} = \frac{A(z+1)(z-2) + B(z-1)(z-2) + C(z-1)(z+1)}{(z-1)(z+1)(z-2)}$$

put $z = -1$

$1 = A(0) + B[-1-1][-1-2] + C(0)$

$1 = B(-2)(-3)$

$\boxed{B = \frac{1}{6}}$

put $z = 2$

$1 = A(0) + B(0) + C(2-1)(2+1)$

$1 = C(3)$

$\boxed{C = \frac{1}{3}}$

put $z = 1$.

$1 = A(1+1)(1-2) + B(0) + C(0).$

$1 = A(2)(-1)$

$\boxed{A = \frac{-1}{2}}$

sub the value of A, B & C in ②.

$$\frac{1}{(z-1)(z+1)(z-2)} = \frac{-1/2}{(z-1)} + \frac{1/6}{(z+1)} + \frac{1/3}{(z-2)}$$

Take z^{-1} on both sides.

$$z^{-1}\left[\frac{1}{(z-1)(z+1)(z-2)}\right] = \frac{-1}{2} z^{-1}\left[\frac{1}{z-1}\right] + \frac{1}{6} z^{-1}\left[\frac{1}{z+1}\right] + \frac{1}{3} z^{-1}\left[\frac{1}{z-2}\right].$$

By eqn ①.

$$\frac{y(n)}{z} = \frac{-1}{2} z^{-1}\left[\frac{1}{z-1}\right] + \frac{1}{6} z^{-1}\left[\frac{1}{z+1}\right] + \frac{1}{3} z^{-1}\left[\frac{1}{z-2}\right].$$

$$y(n) = \frac{-1}{2} z^{-1}\left[\frac{z}{z-1}\right] + \frac{1}{6} z^{-1}\left[\frac{z}{z+1}\right] + \frac{1}{3} z^{-1}\left[\frac{z}{z-2}\right].$$

$$y(n) = -\frac{1}{2}(1)^n + \frac{1}{6}(-1)^n + \frac{1}{3}(2)^n$$

Verification:

$$y(0) = \frac{-1}{2}(1) + \frac{1}{6} + \frac{1}{3}$$

$$= \frac{-3+1+2}{6} = \frac{0}{6} = 0$$

$$y(1) = -\frac{1}{2}(1) + \frac{1}{6}(-1) + \frac{1}{3}(2)$$

$$= \frac{-1}{2} + \frac{1}{6} + \frac{2}{3}$$

$$= \frac{-3+1+2}{6} = \frac{0}{6} = 0$$

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Using convolution theorem to find $z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$

$$= z^{-1}\left[\frac{8z^2}{2\left(z-\frac{1}{2}\right)4\left(z+\frac{1}{4}\right)}\right]$$

$$= z^{-1}\left[\frac{z}{z-\frac{1}{2}} \cdot \frac{z}{z+\frac{1}{4}}\right]$$

$$= z^{-1}\left[\frac{z}{z-\frac{1}{2}}\right] * z^{-1}\left[\frac{z}{z+\frac{1}{4}}\right]$$

$$= \left(\frac{1}{2}\right)^n * \left(\frac{-1}{4}\right)^n$$

$$f(n) * g(n)$$

Since $f(n) * g(n) = \sum_{k=0}^{n} f(k)\, g(n-k)$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{-1}{4}\right)^{n-k}$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{-1}{4}\right)^n \left(\frac{-1}{4}\right)^{-k}$$

$$= \left(\frac{-1}{4}\right)^n \sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)^k}{\left(\frac{-1}{4}\right)^k}$$

$$= \left(\frac{-1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2} \times \frac{-4}{1}\right)^k$$

$$= \left(\frac{-1}{4}\right)^n \sum_{k=0}^{n} (-2)^k$$

$$= \left(-\frac{1}{4}\right)^n \left[1+(-2)+(-2)^2+\dots+(-2)^n\right].$$

$$1+x+x^2+\dots+x^n = \frac{1-x^{n+1}}{1-x}. \qquad [\because -2<1].$$

$$= \left(\frac{-1}{4}\right)^n \left[\frac{1-(-2)^{n+1}}{1-(-2)}\right]$$

$$= \left(\frac{-1}{4}\right)^n \left[\frac{1-(-2)^n(-2)^1}{1+2}\right]$$

$$= \left(\frac{-1}{4}\right)^n \left[\frac{1+2(-2)^n}{3}\right]$$

$$= \frac{1}{3}\left(\frac{-1}{4}\right)^n + \frac{2}{3}(-2)^n\left(\frac{-1}{4}\right)^n$$

$$= \frac{1}{3}\left(\frac{-1}{4}\right)^n + \frac{2}{3}\left(-2\times\frac{1}{4_2}\right)^n$$

$$= \frac{1}{3}\left(\frac{-1}{4}\right)^n + \frac{2}{3}\left(\frac{1}{2}\right)^n.$$

01/12/2021.

1. Solve: $(D^3 - 2D^2D')z = \sin(x+2y) + 3x^2y$. → Type II — Nov, Dec, 19.

A.E: $m^3 - 2m^2 = 0$

$m^2(m-2) = 0$

$m^2 = 0$, $m - 2 = 0$.
m_1, m_2

$m = 0$ (twice) ; $m_3 = 2$.

$C.F = f_1(y+m_1x) + x\, f_2(y+m_1x) + f_3(y+m_3x)$.

$= f_1(y+0x) + x f_2(y+0x) + f_3(y+2x)$.

$= f_1(y) + x f_2(y) + f_3(y+2x)$.

$$P.I_1 = \frac{\sin(\overset{a}{x}+\overset{b}{2y})}{D^3 - 2D^2D'}$$

$a = 1$, $b = 2$. $DDD' \to DD'.D$

Replace D^2 by $-a^2 = -1$, DD' by $-ab = -2$, D'^2 by $-b^2 = -4$

$$P.I_1 = \frac{\sin(x+2y)}{D^2D - 2DD'D}$$

$$= \frac{\sin(x+2y)}{(-1)D - 2(-2)D}$$

$$= \frac{\sin(x+2y)}{(-1)D - 2(-2)D}.$$

$$= \frac{\sin(x+2y)}{-D+4D}$$

$$= \frac{\sin(x+2y)}{3D}$$

$$= \frac{1}{3} \cdot \frac{1}{D} \left(\sin(x+2y) \right).$$

$$= \frac{1}{3} \int \sin(x+2y)\, dx.$$

$$= \frac{1}{3} \; -\cos(x+2y)$$

$$P.I_1 = -\frac{1}{3} \cos(x+2y).$$

$$P.I_2 = \frac{3x^2y}{D^3 - 2D^2D'} \quad \rightarrow \text{Type III}$$

pulled out higher power of D in the denominator.

$$PI_2 = \frac{3x^2y}{D^3\left[1 - \frac{2D^2D'}{D^3}\right]}$$

$$= \frac{1}{D^3}\left[1 - \frac{2D'}{D}\right]^{-1} (3x^2y).$$

$$\boxed{(1-x)^{-1} = 1+x+x^2+\ldots\ldots}$$

$$\frac{1}{D} = \int dx$$

$$D = \frac{\partial}{\partial x}$$

$$D' = \frac{\partial}{\partial y}$$

$$= \frac{1}{D^3}\left[1 + \frac{2D'}{D}\right](3x^2y)$$

$$= \frac{1}{D^3}\left[3x^2y + \frac{2D'}{D}(3x^2y)\right]. \quad 2 \cdot D' \cdot \frac{1}{D}$$

$$= \frac{1}{D^3}\left[3x^2y + \frac{2}{D}(3x^2 \cdot 1)\right] \qquad \frac{\partial}{\partial y}(3x^2y) \quad 3x^2(1) \qquad \frac{\partial}{\partial y}\; x \cdot \frac{\partial x}{x}$$

$$= \frac{1}{D^3}\left[3x^2y + 2\int 3x^2\, dx\right]$$

$$= \frac{1}{D^3}\left[3x^2y + 2 \cdot \frac{3x^3}{3}\right].$$

$$\frac{3x^3}{3} y +$$

$$= \frac{1}{D^2}\left[\frac{3x^3y}{3} + \frac{2x^4}{42}\right]$$

$$= \frac{1}{D}\left[\frac{x^4y}{4} + \frac{x^5}{10}\right]$$

WKT

$$P.I_2 = \frac{x^5y}{20} + \frac{x^6}{60}$$

The general solution is.

$Z = CF + P.I_1 + PI_2$.

$$Z = f_1(y) + x f_2(y) + f_3(y+2x) + \frac{1}{3}\cos(x+2y) + \frac{x^5y}{20} + \frac{x^6}{60}$$

2. Solve. $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$. → Type III   May, June 2016

A.E : $m^3 - 2m^2 = 0$

$m^2(m-2) = 0$

$m^2 = 0,\ m - 2 = 0$

$m = 0$ (twice), $m_3 = 2$

$C.F = f_1(y+mx) + x f_2(y+mx) + f_3(y+m_3x)$.

$= f_1(y+0x) + x f_2(y+0x) + f_3(y+2x)$

$= f_1(y) + x f_2(y) + f_3(y+2x)$

$$P.I_1 = \frac{2e^{2x}}{D^3 - 2D^2D'}$$

$$= \frac{2e^{2x+0y}}{D^3 - 2D^2D'} \qquad e^{ax+by}$$

Here $a = 2,\ b = 0$

Replace D by $a = 2$

D' by $b = 0$

$$P.I_1 = \frac{2e^{2x+0y}}{2^3 - 2(2)^2(0)}$$

$$= \frac{2e^{2x}}{8}$$

$$P.I_1 = \frac{e^{2x}}{4}$$

$P.I_2 \rightarrow$ same as the previous problem.

3. Solve: $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$

A.E: $m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$ (twice).

$C.F = f_1(y+mx) + x f_2(y+mx)$

$= f_1(y-x) + x f_2(y-x)$

$$P.I_1 = \frac{x^2y}{D^2 + 2DD' + D'^2}$$

$$= \frac{x^2y}{(D+D')^2}$$

$$P.I_1 = \frac{x^2y}{D^2\left[1+\frac{D'}{D}\right]^2}$$

$$PI_2 = \frac{e^{x-y}}{D^2 + 2DD' + D'^2}$$

Here $a = 1$

$$= \frac{1}{D^2}\left[1+\frac{D'}{D}\right]^{-2}(x^2y).$$

$(1+x)^{-2} = 1 - 2x + 3x^2 - \dots$

$$P.I_1 = \frac{1}{D^2}\left[1 - 2\frac{D'}{D}\right](x^2y).$$

$$= \frac{1}{D^2}\left[x^2y - \frac{2D'}{D}(x^2y)\right]$$

$$= \frac{1}{D^2}\left[x^2y - \frac{2}{D}(x^2 1)\right]$$

$$= \frac{1}{D^2}\left[x^2y - 2\int x^2 dx\right]$$

$$= \frac{1}{D^2}\left[x^2y - 2\frac{x^3}{3}\right]$$

$$= \frac{1}{D}\left[\frac{x^3y}{3} - \frac{2x^4}{12}\right]$$

$$P.I_1 = \frac{x^4y}{12} - \frac{x^5}{30}.$$

$$P.I_2 = \frac{e^{x-y}}{D^2+2DD'+D'^2}$$

Here $a=1$, $b=-1$

Replace D by $a=1$

D' by $b=-1$

$$P.I_2 = \frac{e^{x-y}}{1+2(1)(-1)+(-1)^2}$$

$$= \frac{e^{x-y}}{1-2+1}$$

$$P.I_2 = \frac{e^{x-y}}{0}.$$

Multiply the numerator by x & differentiate the denominator with respect to D.

$$P.I_2 = x.\frac{e^{x-y}}{2D+2D'}$$

$$= x \cdot \frac{e^{x-y}}{2(1)+2(-1)}$$

$$= x \cdot \frac{e^{x-y}}{2-2} = \frac{x e^{x-y}}{0}$$

$$= x^2 \cdot \frac{e^{x-y}}{2}$$

$$P.I_2 = \frac{x^2}{2} e^{x-y}$$

The general solution is

$$z = C.F + P.I_1 + P.I_2.$$

$$= f_1(y-x) + x f_2(y-x) + \frac{x^4 y}{12} - \frac{x^5}{30} + \frac{x^2}{2} e^{+x-y}$$

4. Solve: $(D^2 + 4DD' - 5D'^2) z = \sin(x-2y) + e^{2x-y}$.

A.E: $m^2 + 4m - 5 = 0$ $\qquad$ $P \to -5 \quad (-1 \times +5)$
 $\qquad\qquad\qquad\qquad\qquad\qquad$ $S \to 4 \quad (-1+5)$.

$(m+5)(m-1) = 0$.

$\therefore m_1 = 1, \; m_2 = -5$

$C.F = f_1(y + m_1 x) + f_2(y + m_2 x)$

$C.F = f_1(y+x) + f_2(y-5x)$.

$$P.I_1 = \frac{\sin(x-2y)}{D^2 + 4DD' - 5D'^2}$$

Here $a = 1$, $b = -2$

Replace D^2 by $-a^2 = -1$

DD' by $-ab = 2$

D'^2 by $-b^2 = -4$

$$P.I_1 = \frac{\sin(x-2y)}{-1+4(2)-5(-4)}$$

$$= \frac{\sin(x-2y)}{-1+8+20}$$

$$P.I_1 = \frac{1}{27}\sin(x-2y)$$

$$P.I_2 = \frac{e^{2x-y}}{D^2+4DD'-5D'^2}$$

Here $a=2$, $b=-1$

Replace D by $a=2$.

D' by $b=-1$

$$P.I_2 = \frac{e^{2x-y}}{2^2+4(2)(-1)-5(-1)^2}$$

$$P.I_2 = \frac{e^{2x-y}}{4-8-5}$$

$$P.I_2 = \frac{e^{2x-y}}{-9}$$

The general solution is

$$z = C.F + P.I_1 + P.I_2$$

5. Solve: $(D^2+DD'-6D'^2)z = y\cos x$.

Sol: A.E: $m^2+m-6=0$.

$P \rightarrow -6$ $(3\times(-2)$.

$S \rightarrow 1$ $(3, -2$

$(m+3)(m-2)=0$.

$m_2=-3$, $m_1=2$

$CF = f_1(y+m_1x)+f_2(y+m_2x)$

$= f_1(y+2x)+f_2(y-3x)$.

$$P.I = \frac{y\cos x}{D^2+DD'-6D'^2}$$

$$= \frac{y\cos x}{(D+3D')(D-2D')}$$

put $y = c+mx$

$y = c-2x$

$$= \frac{1}{D+3D'}\int (c-2x)\cos x\,dx.$$

$u \quad v \quad dv$

$$\int u\,dv = uv_1 - u'v_2 + u''v_3 - \dots$$

$$P.I = \frac{1}{(D+3D')}\left[(c-2x)\sin x - (0-2)(-\cos x)\right]$$

$$= \frac{1}{(D+3D')}\left[y\sin x - 2\cos x\right]$$

$c+3x$

$$= \int\left[(c+3x)\sin x - 2\cos x\right]dx.$$

put $y = c+mx$

$y = c+3x.$

$$= (c+3x)(-\cos x) - (0+3)(-\sin x) - 2\sin x.$$

$$= -y\cos x + 3\sin x - 2\sin x.$$

$$P.I = \sin x - y\cos x.$$

6. Solve: $(D^2 - 2DD')z = x^3y + e^{2x}.$

$A.E = m^2 - 2m = 0$

$m[m-2] = 0$

$m=0,\ m-2=0$

$m_1=0,\ m_2=2.$

$C.F = f_1(y+m_1x) + f_2(y+m_2x)$

$= f_1(y+0x) + f_2(y+2x).$

$$P.I = \frac{x^3y}{D^2-2DD'}$$

Pulled out the highest power of D in the denominator.

$$P.I_1 = \frac{x^3y}{D^2\left[1-\frac{2xD'}{Dx}\right]}$$

$$= \frac{1}{D^2}\left[1-\frac{2D'}{D}\right]^{-1}(x^3y).$$

$$(1-x)^{-1} = 1+x+x^2+\dots$$

$$P.I_1 = \frac{1}{D^2}\left[1+\frac{2D'}{D}\right](x^3y)$$

$$= \frac{1}{D^2}\left[x^3y+\frac{2D'}{D}(x^3y)\right].$$

$$= \frac{1}{D^2}\left[x^3y+\frac{2}{D}(x^3.1)\right]$$

$$= \frac{1}{D^2}\left[x^2y+2\int x^3dx\right]$$

$$= \frac{1}{D^2}\left[x^3y+\frac{2x^4}{4_2}\right].$$

$$= \frac{1}{D}\left[\frac{x^4y}{4}+\frac{x^5}{10}\right].$$

$$P.I_1 = \frac{x^5y}{20}+\frac{x^6}{60}.$$

$$P.I_2 = \frac{e^{2x}}{D^2-2DD'}$$

$$= \frac{e^{2x+0y}}{D^2-2DD'}.$$

Here $a=2$, $b=0$.

Replace D by $a=2$

D' by $b=0$.

$$P.I_2 = \frac{e^{2x+0y}}{2^2-2(2)(0)}.$$

$$= \frac{e^{2x}}{4}$$

The general solution is.

$z = C.F + P.I_1 + PI_2$.

7. Solve: $(D^2 - 5DD' + 6D'^2)z = y \sin x$.

A.E : $m^2 - 5m + 6 = 0$.

$P = 6 \Rightarrow -3 \times -2$

$S = -5 \Rightarrow -3 - 2$.

$(m-2)(m-3) = 0$.

$m_1 = 2, \ m_2 = 3$

$C.F = f_1(y + m_1 x) + f_2(y + m_2 x)$.

$= f_1(y + 2x) + f_2(y + 3x)$.

$$P.I = \frac{y \sin x}{D^2 - 5DD' + 6D'^2}$$

$$= \frac{y \sin x}{(D - 2D')(D - 3D')}$$

$c - 3x$

$y = c + mx$.

$y = c - 3x$.

$$= \frac{1}{(D - 2D')} \int (c - 3x) \sin x \, dx.$$

$\int uv\,dv = uv_1 - u'v_2 + u''v_3 - \ldots$

$$P.I = \frac{1}{D - 2D'} \left[(c - 3x)(-\cos x) - (0 - 3)(-\sin x) \right].$$

$$= \frac{1}{D - 2D'} \left[-y \cos x - 3 \sin x \right].$$

$$= -\int \left[(c - 2x) \cos x + 3 \sin x \right] dx.$$

put $y = c + mx$.

$y = c - 2x$.

$$= -\left[(c - 2x) \sin x - (-2)(-\cos x) + 3(-\cos x) \right]$$

$$= -\left[y \sin x - 2\cos x - 3\cos x \right].$$

$$= -\left[y\sin x - 5\cos x\right]$$

P.I. $= -y\sin x + 5\cos x$.

The general solution is.

$$z = C.F + P.I$$

$$= f_1(y+2x) + f_2(y+3x) - y\sin x + 5\cos x.$$

8. Solve: $(D^2 + 2DD' + D'^2)\, z = e^{x-y} + xy$.

$$m^2 + 2m + 1 = 0$$

$$(m+1)(m+1) = 0.$$

$$m = -1 \text{ (twice)}.$$

$$C.F = f_1(y+mx) + x\, f_2(y+mx).$$

$$= f_1(y-x) + x\, f_2(y-x)$$

$$P.I_1 = \frac{e^{+x-y}}{D^2 + 2DD' + D'^2}.$$

Here $a = 1$, $b = -1$

Replace D by $a = 1$

D' by $b = -1$

$$P.I_1 = \frac{e^{x-y}}{1 + 2(1)(-1) + (-1)^2}$$

$$= \frac{e^{x-y}}{1-2+1}$$

$$= \frac{e^{x-y}}{0}$$

Multiply the Nr of P.I by x & differentiate dr. w.r.to D.

$$P.I_1 = x \cdot \frac{e^{x-y}}{2D + 2D'}$$

$$= x \cdot \frac{e^{x-y}}{2(1)+2(-1)}$$

$$= x^2 \cdot \frac{e^{x-y}}{2}$$

$$P.I_1 = \frac{x^2 e^{x-y}}{2}$$

$$P.I_2 = \frac{xy}{D^2 + 2DD' + D'^2}$$

$$= \frac{xy}{(D+D')^2}$$

$$= \frac{xy}{D^2\left[1 + \frac{D'}{D}\right]^2}$$

$$= \frac{1}{D^2}\left[1 + \frac{D'}{D}\right]^{-2}(xy)$$

$$(1+x)^{-2} = 1 + 2x + 3x - \dots$$

$$P.I_2 = \frac{1}{D^2}\left[1 - 2\left(\frac{D'}{D}\right)\right](xy)$$

$$= \frac{1}{D^2}\left[xy - \frac{2D'}{D}(xy)\right]$$

$$= \frac{1}{D^2}\left[xy - \frac{2}{D}(x \cdot 1)\right]$$

$$= \frac{1}{D^2}\left[xy - 2\int x\,dx\right]$$

$$= \frac{1}{D^2}\left[xy - 2 \cdot \frac{x^2}{2}\right]$$

$$= \frac{1}{D}\left[\frac{x^2 y}{2} - \frac{x^3}{3}\right]$$

$$P.I_2 = \frac{x^3 y}{6} - \frac{x^4}{12}.$$

The general solution is.

$$z = C.F + P.I_1 + PI_2.$$

9. <u>Homework:</u>

$$(D^2 - 6DD' + 5D'^2)z = e^x \sin hy + xy.$$

<u>Soln:</u>

$$\sin hy = \frac{e^y - e^{-y}}{2}.$$

$$(D^2 - 6DD' + 5D'^2)z = e^x\left[\frac{e^y - e^{-y}}{2}\right] + xy.$$

$$= \left(\frac{e^x e^y - e^x e^{-y}}{2}\right) + xy$$

$$(D^2 - 6DD' + 5D'^2)z = xy + \frac{e^{x+y}}{2} - \frac{e^{x-y}}{2}$$

To find C.F

A.E : $m^2 - 6m + 5 = 0.$ $\qquad P \to 5 \quad (-1 \times -5)$

$(m-1)(m-5) = 0.$ $\qquad S \to -6 \quad (-1 - 5)$

$m_1 = 1, \; m_2 = 5.$

$$C.F = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$= f_1(y + x) + f_2(y + 5x).$$

$$P.I_1 = \frac{xy}{D^2 - 6DD' + 5D'^2} \qquad \text{(Type 3)}$$

pulled out the highest power in the denominator.

$$= \frac{xy}{D^2\left[1+\left(\frac{-6DD'+5D'^2}{D^2}\right)\right]}$$

$$= \frac{1}{D^2}\left[xy\right]\left[1+\left(\frac{-6D'}{D}+\frac{5D'^2}{D^2}\right)\right]^{-1}$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$$

$$P.I_1 = \frac{1}{D^2}\left[1-\left(\frac{-6D'}{D}+\frac{5D'^2}{D^2}\right)+\ldots\right]xy.$$

$$= \frac{1}{D^2}\left[1+\frac{6D'}{D}-\frac{5D'^2}{D^2}+\ldots\right]xy.$$

$$= \frac{1}{D^2}\left[xy+\frac{6D'}{D}(xy)-\frac{5D'^2}{D^2}xy+\ldots\right]xy.$$

$$D' = \frac{\partial}{\partial y}(y) = xy \Rightarrow (1).$$

$$D'^2 = \frac{\partial^2}{\partial^2 y^2}(y) = xy \Rightarrow (0).$$

$$P.I_1 = \frac{1}{D^2}\left[xy+\frac{6x}{D}-\frac{5(0)}{D^2}+\ldots\right]xy.$$

$$= \frac{1}{D^2}\left[xy+6\int x\,dx - 0+\ldots\right]xy.$$

$$= \frac{1}{D^2}\left[xy+\overset{3}{\cancel{6}}\frac{x^2}{\cancel{2}}\right]+\ldots$$

$$= \frac{1}{D^2}\left[xy+3x^2\right].$$

$$= \frac{1}{D}\int(xy+3x^2)$$

$$= \frac{1}{D}\left[\frac{x^2y}{2}+\frac{\cancel{3}x^3}{\cancel{3}}\right].$$

$$= \frac{1}{D}\left[\frac{x^2y}{2}+x^3\right]$$

$$P.I_1 = \frac{x^3 y}{6} + \frac{x^4}{4}$$

$$P.I_2 = \frac{e^{x+y}}{2 \times (D^2 - 6DD' + 5D'^2)} \quad \text{Type I:}$$

$$= \frac{1}{2} \cdot \frac{e^{x+y}}{D^2 - 6DD' + 5D'^2}$$

$a = 1, b = 1$

Replace D by $a \Rightarrow 1$

D' by $b \Rightarrow 1.$

$$P.I_2 = \frac{1}{2} \cdot \frac{e^{x+y}}{(1)^2 - 6(1)(1) + 5(1)^2}$$

$$= \frac{1}{2} \frac{e^{x+y}}{1 - 6 + 5}$$

$$= \frac{1}{2} \frac{e^{x+y}}{0}.$$

Multiply the numerator by x & differentiate the denominator with respect to D.

$$P.I_2 = \frac{x}{2} \frac{e^{x+y}}{2D - 6D'}$$

$$= \frac{x}{2} \frac{e^{x+y}}{2(1) - 6(1)}$$

$$= \frac{x}{2} \frac{e^{x+y}}{2-6}.$$

$$= \frac{x}{2} \cdot \frac{e^{x+y}}{-4}.$$

$$= -\frac{x}{2} \frac{e^{x+y}}{4}.$$

$$P.I_2 = \frac{-x \cdot e^{x+y}}{8}$$

$$P.I_3 = \frac{e^{x-y}}{D^2-6DD'+5D'^2} \cdot \frac{1}{2}$$ Type I:

Replace $D = a = 1$, $a = 1$

$D' = b = -1$. $b = -1$.

$$P.I_3 = \frac{e^{x+y}}{(1)^2-6(1)(-1)+5(-1)^2}\left(\frac{1}{2}\right)$$

$$= \frac{e^{x+y}}{1+6+5}\left(\frac{1}{2}\right)$$

$$= \frac{e^{x+y}}{12}\left(\frac{1}{2}\right)$$

$$P.I_3. = \frac{e^{x+y}}{24}.$$

The general solution is.

$$Z = C.F + P.I_1 + P.I_2 + P.I_3.$$

$$Z = f_1(y+x) + f_2(y+5x) + \frac{x^3y}{6} + \frac{x^4}{4} - \frac{x}{8} \cdot e^{x+y}.$$

$$+ \frac{1}{24} e^{x+y}.$$

06/12/2021.

1. A tightly stretched string of length l has its end fastened at $x = 0$, $x = l$, At $t = 0$

Initial velocity with zero

Sum. No ①.

Solution:

Let the 1-D wave equation is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \cdot \frac{\partial^2 y}{\partial x^2} \rightarrow ① \quad \text{where } a^2 = \frac{T}{m}.$$

The boundary conditions are.

(i) $y(0,t) = 0 \quad \forall t \geq 0$

(ii) $y(l,t) = 0 \quad \forall t \geq 0.$

(iii) $\frac{\partial y}{\partial t}(x,0) = 0 \quad \forall x \text{ in } (0,l).$

(iv) $y(x,0) = f(x) = kx(l-x).$

The correct solution is.

(i.e) $y(x,t) = (A\cos px + B\sin px)(C\cos pat + D\sin pat) \longrightarrow ②.$

Applying (i) in eqn ②.

② ⇒ $y(0,t) = (A\cdot 1 + B\cdot 0)(C\cos pat + D\sin pat) = 0.$

$$A[C\cos pat + D\sin pat] = 0.$$

Since $[C\cos pat + D\sin pat] \neq 0$
[∵ It is defined $\forall t$]

∴ $\boxed{A = 0}$

sub $A=0$ in ②,

② ⇒ $y(x,t) = B\sin px\,[C\cos pat + D\sin pat] \rightarrow ③.$

Applying (ii) in ③.

$y(l,t) = B\sin pl\,[C\cos pat + D\sin pat] = 0.$

Since $[C\cos pat + D\sin pat] \neq 0.$

and $B \neq 0$. [if $B=0$, it gives trivial solution].

$\sin pl = 0.$

$\sin pl = \sin 0$ $\quad$ [If $\sin\theta = \sin\alpha$ then $\theta = n\pi \pm \alpha$].

$\sin pl = \sin n\pi$

$pl = n\pi \pm 0$

$pl = n\pi$

$\boxed{p = \frac{n\pi}{l}}$, $n = 1, 2, 3 \ldots$

sub $p = \frac{n\pi}{l}$ in ③.

③ $\Rightarrow y(x,t) = B \sin\frac{n\pi x}{l}\left[C\cos\frac{n\pi at}{l} + D\sin\frac{n\pi at}{l}\right] \rightarrow$ ④.

Diff eqn ④ w.r.to 't' partially.

$$\frac{\partial y}{\partial t}(x,t) = B\sin\frac{n\pi x}{l}\left[\left[C\cdot\left(-\sin\frac{n\pi at}{l}\right)\cdot\frac{n\pi a}{l}\right] + D\left[\left(\cos\frac{n\pi at}{l}\right)\cdot\frac{n\pi a}{l}\right]\right] \rightarrow ⑤.$$

Applying (iii) in ⑤.

⑤ $\Rightarrow \frac{dy}{dt}(x,0) = B\sin\frac{n\pi x}{l}\left[C(0)\times\frac{n\pi a}{a} + D\cdot 1\cdot\frac{n\pi a}{l}\right] = 0.$

$$B\cdot D\,\frac{n\pi a}{l}\,\sin\frac{n\pi x}{l} = 0.$$

Since, $B \neq 0$, $\frac{n\pi a}{l} \neq 0$, $\sin\frac{n\pi x}{l} \neq 0$.

$\therefore \boxed{D=0}$

sub $D=0$ in eqn ④.

④ $\Rightarrow y(x,t) = BC\,\sin\frac{\pi x n}{l}\cdot\cos\frac{n\pi at}{l}$

The most general solution is:

$$y(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l} \rightarrow ⑥$$

where $B_n = BC$.

Applying (iv) in eqn ⑥.

⑥ $\Rightarrow y(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cdot 1 = f(x). \rightarrow$ ⑦

By Half range sine series in $(0,l)$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \rightarrow ⑧$$

where $b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$.

from ⑦ & ⑧

$$\boxed{B_n = b_n}$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l kx(l-x)\sin\frac{n\pi x}{l}\,dx \qquad \int uv\,dv = uV_1 - u'V_2 + u''V_3$$

$$= \frac{2k}{l}\int_0^l (lx-x^2)\sin\frac{n\pi x}{l}\,dx.$$

$$b_n = \frac{2k}{l}\left[(lx-x^2)\left(\frac{-\cos n\pi x/l}{\frac{n\pi}{l}}\right) - (l-2x)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (0-2)\left(\frac{\cos\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^l$$

$$B_n = b_n = \frac{2k}{l}\left[\left(0 - \frac{2\cos n\pi}{\frac{n^3\pi^3}{l^3}}\right) - \left(0 - \frac{2(1)}{\frac{n^3\pi^3}{l^3}}\right)\right]$$

$$= \frac{2k}{l}\cdot\frac{2}{\frac{n^3\pi^3}{l^3}}\left[1-\cos n\pi\right]$$

$$= \frac{4kl^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

$1-(-1)^1$; $1-(-1)$; $1-(-1)^2$; $(1-1) =$

$$B_n = b_n = \begin{cases} \frac{8kl^2}{n^3\pi^3} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even.} \end{cases} \rightarrow ⑨$$

use eqn ⑨ in ⑥.

$$y(x,t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{8kl^2}{n^3\pi^3}\sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}$$

April/ May 2019.

Initial Velocity with non zero.

(13.a) Let the 1-D wave equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial y^2}{\partial x^2} \rightarrow ①$$

where $a^2 = \frac{T}{m}$.

The boundary conditions are

(i) $y(0,t) = 0 \quad \forall t \geq 0$

(ii) $y(l,t) = 0 \quad \forall t \geq 0$.

(iii) $y(x,0) = 0, \quad \forall x \text{ in } (0,l)$.

(iv) $v = \frac{\partial y}{\partial t}(x,0) = 3x(l-x) = f(x)$.

The correct solution is

$y(x,t) = (A \cos px + B \sin px)(C \cos pat + D \sin pat) \rightarrow ②$.

Applying (i) in eqn ②.

② ⇒ $y(0,t) = (A \cos 0 + B \sin 0)(C \cos pat + D \sin pat)$

⇒ $A(\cos pat + D \sin pat) = 0$

Since $(C \cos pat + D \sin pat) \neq 0$ [∵ It is defined for all t].

∴ $\boxed{A=0}$

A=0 in eqn ②.

② ⇒ $y(x,t) = (B \sin px)(C \cos pat + D \sin pat) \rightarrow ③$.

Applying (ii) in eqn ③.

③ ⇒ $y(l,t) = B \sin pl (C \cos pat + D \sin pat) = 0$.

Since $(C \cos pat + D \sin pat) \neq 0$

and $B \neq 0$ ∵ [If B=0, it gives trivial solution]

∴ $\sin pl = 0$.

$\sin pl = \sin 0$

θ α

If $\sin\theta = \sin\alpha$
$\theta = n\pi \pm \alpha$

$Pl = n\pi \pm 0$

$$\boxed{P = \frac{n\pi}{l}}$$

sub $p = \frac{n\pi}{l}$ in eqn ③

$$y(x,t) = B \sin \frac{n\pi x}{l} \left(C \cos \frac{n\pi at}{l} + D \sin \frac{n\pi at}{l} \right] \rightarrow ④.$$

Applying (iii) in eqn ④.

$$④ \Rightarrow y(x,0) = B \sin \frac{n\pi x}{l} \left[C \cdot 1 + D\, 0 \right] = 0.$$

$$= BC \sin \frac{n\pi x}{l} = 0$$

$$\sin \frac{n\pi\pi}{l} \neq 0, \quad B \neq 0.$$

$$\therefore \boxed{C=0}.$$

sub $C=0$ in eqn ④

$$y(x,t) = B \sin \frac{n\pi x}{l} \left[0 + D \sin \frac{n\pi at}{l} \right].$$

$$y(x,t) = BD \sin \frac{n\pi x}{l} \cdot \sin \frac{n\pi at}{l}.$$

The most general solution.

$$y(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} \sin \frac{n\pi at}{l} \rightarrow ⑤.$$

where $B_n = BD$.

diff ⑤ w.r.to "t" partially.

$$\frac{\partial y}{\partial t}(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} \cos \frac{n\pi at}{l} \cdot \frac{n\pi a}{l} \rightarrow ⑥.$$

Applying (iv) in ⑥.

$$\frac{\partial y}{\partial t}(x,0) = \sum_{n=1}^{\infty} B_n \frac{n\pi a}{l} \sin \frac{n\pi x}{l} \cdot 1 = 3x(l-x) = f(x) \rightarrow ⑦.$$

By Half range sine series in $(0,l)$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}. \rightarrow ⑧$$

where $b_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l}\, dx.$

from ⑦ & ⑧

$$B_n \frac{n\pi a}{l} = b_n.$$

$$B_n = b_n \cdot \frac{l}{n\pi a}. \rightarrow (9).$$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l} dx.$$

$$= \frac{2}{l}\int_0^l 3x(l-x) \sin \frac{n\pi x}{l} dx.$$

$$= \frac{6}{l}\int_0^l (lx - x^2) \sin \frac{n\pi x}{l} dx.$$

(u = $lx - x^2$, dv = $\sin \frac{n\pi x}{l}$)

$$\int u\,dv = uV_1 - u'V_2 + u''V_3 - \dots$$

$$b_n = \frac{6}{l}\left[(lx-x^2)\left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (l-2x)\left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + (0-2)\left[\frac{\cos \frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right]\right]_0^l.$$

$$b_n = \frac{6}{l}\left[\left(0 - \frac{2\cos n\pi}{\frac{n^3\pi^3}{l^3}}\right) - \left(0 - \frac{2(1)}{\frac{n^3\pi^3}{l^3}}\right)\right].$$

$$b_n = \frac{6}{l} \cdot \frac{2}{\frac{n^3\pi^3}{l^{3}}} [1 - \cos n\pi].$$

$$= \frac{12 l^2}{n^3\pi^3}[1-(-1)^n].$$

$$b_n = \begin{cases} \frac{24 l^2}{n^3\pi^3}, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even.} \end{cases} \rightarrow (10).$$

Sub eqn (10) in (9).

$$B_n = \begin{cases} \frac{24l^2}{n^3\pi^3} \frac{l}{n\pi a}, & \text{if } n \text{ is odd.} \\ 0, & \text{if } n \text{ is even.} \end{cases}$$

$$B_n = \begin{cases} \frac{24l^3}{n^4\pi^4 a}, & \text{if } n \text{ is odd.} \\ 0, & \text{if } n \text{ is even.} \end{cases} \rightarrow (11)$$

sub (11) in eqn (5).

$$y(x,t) = \sum_{n=1,3,5..}^{\infty} \frac{24l^3}{n^4\pi^4 a} \sin\frac{n\pi x}{l} \sin\frac{n\pi a t}{l}.$$

Convolution theorem.

Fourier transform	Z- transform
If $F[f(x)] = f(s)$ & $F[g(x)] = G(s)$ then. $F[f(x) * g(x)] = F(s) \cdot G(s)$	If $z[f(n)] = f(z)$ & $z[g(n)] = G(z)$ then. $z[f(n) * g(n)] = F(z) \cdot G(z)$

10/12/21.

Find the Fourier series expansion upto the first 3 harmonic from the following data.

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Here, N=6.

$a_0 = \frac{2\Sigma y}{N} = \frac{2(8.7)}{6} = 2.9$

$a_1 = \frac{2\Sigma y \cos x}{N} = \frac{2(-1.10)}{6} = -0.37$

$a_2 = \frac{2\Sigma y \cos 2x}{N} = \frac{2(-0.3)}{6} = -0.1$

$a_3 = \frac{2\Sigma y \cos 3x}{N} = \frac{2(0.1)}{6} = 0.03$

$b_1 = \frac{2\Sigma y \sin x}{N} = \frac{2(0.53)}{6} = 0.17$

$b_2 = \frac{2\Sigma y \sin 2x}{N} = \frac{2(-0.17)}{6} = -0.06$

$b_3 = 0$

The required harmonic is

$$y = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin x) + (a_3 \cos 3x + b_3 \sin 3x)$$

$$y = 1.45 + [0.37 \cos x + 0.17 \sin x] + [0.1 \cos 2x - 0.06 \sin 2x] + 0.03 \cos x.$$

x	y	$y \cos x$	$y \cos 2x$	$y \cos 3x$	$y \sin x$	$y \sin 2x$	$y \sin 3x$
0	1.0	1.0	1.0	1.0	0	0	0
$\frac{\pi}{3}$ (60°)	1.4	0.7	-0.7	-1.4	1.21	1.21	0
$\frac{2\pi}{3}$ (120°)	1.9	-0.95	-0.95	1.9	1.64	-1.64	0
π (180°)	1.7	-1.7	1.7	-1.7	0	0	0
$\frac{4\pi}{3}$ (240°)	1.5	-0.75	-0.75	1.5	-1.29	1.29	0
$\frac{5\pi}{3}$ (300°)	1.2	0.6	-0.6	-1.2	-1.03	-1.03	0
	$\Sigma y = 8.7$	$\Sigma y \cos x = -1.10$	$\Sigma y \cos 2x = -0.3$	$\Sigma y \cos 3x = 0.1$	$\Sigma y \sin x = 0.53$	$\Sigma y \sin 2x = -0.17$	$\Sigma y \sin 3x = 0$

Determine the first two harmonics of the Fourier series for the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

x	y	y cos x	y cos 2x	y sin x	y sin 2x
0	1.98	1.98	1.98	0	0
$\frac{\pi}{3}$	1.30	0.65	−0.65	1.13	1.125 (1.13)
$\frac{2\pi}{3}$	1.05	−0.53	−0.53	0.91	−0.91
π	1.30	−1.30	1.30	0	0
$\frac{4\pi}{3}$	−0.88	0.44	0.44	0.76	−0.76
$\frac{5\pi}{3}$	−0.25	−0.13	0.13	0.22	0.22
	$\Sigma y = 4.5$	$\Sigma y\cos x = 1.11$	$\Sigma y\cos 2x = 2.67$	$\Sigma y\sin x = 3.02$	$\Sigma y \sin 2x = -0.32$

N=6.

$a_0 = \frac{2\Sigma y}{N} = \frac{2(4.5)}{6} = 1.5$

$a_1 = \frac{2\Sigma y\cos x}{N} = \frac{2(1.11)}{6} = 0.37$

$a_2 = \frac{2\Sigma y\cos 2x}{N} = \frac{2(2.67)}{6} = 0.89$

$b_1 = \frac{2\Sigma y\sin x}{N} = \frac{2(3.02)}{6} = 1.006 \simeq 1.01$

$b_2 = \frac{2\Sigma y\sin 2x}{N} = \frac{2(-0.32)}{6} = -0.106 \simeq -0.11$

$y = \frac{a_0}{2} + (a_1\cos x + b_1\sin x) + (a_2\cos 2x + b_2 \sin 2x)$

$y = 0.75\left[0.37\cos x + 1.01\sin x\right] + \left[0.89\cos 2x - 0.11\sin 2x\right]$

When the problem comes in period function.

Replace, $\boxed{\theta = \frac{2\pi x}{T}}$

3. Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier series expansion from the following data.

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

Here, $2l = 6$

$\boxed{l = 3}$.

x	y	$y\cos\frac{\pi x}{3}$	$y\cos\frac{2\pi x}{3}$	$y\sin\frac{\pi x}{3}$	$y\sin\frac{2\pi x}{3}$
0	9	9	9.	0.	0
1	18	9	−9	15.59.	15.6
2	24	−12	−12	20.78	20.8
3	28	−28.	28	0	0
4	26	−13	−13	−22.5	22.5
5	20	10.	−10	−17.32	−17.3
	$\Sigma y = 125$	$\Sigma y\cos\frac{\pi x}{3} = -25$	$\Sigma y\cos\frac{2\pi x}{3} = -7$	$\Sigma y\sin\frac{\pi x}{3} = -3.4$	$\Sigma y\sin\frac{2\pi x}{3} = 0$

$N = 6$.

$a_0 = \frac{2\Sigma y}{N} = \frac{2(125)}{6} = 41.7$

$b_2 = \frac{2\Sigma y\sin\frac{2\pi x}{3}}{N} = 0$.

$a_1 = \frac{2\Sigma y\cos\frac{\pi x}{3}}{N} = \frac{2(-25)}{6} = -8.3$

$a_2 = \frac{2\Sigma y\cos\frac{2\pi x}{3}}{N} = \frac{2(-7)}{6} = -2.3$

$b_1 = \frac{2\Sigma y\sin\frac{\pi x}{3}}{N} = \frac{2(-3.4)}{6} = -1.13$

$$y = \frac{a_0}{2} + \left[a_1 \cos\frac{\pi x}{3} + b_1 \sin\frac{\pi x}{3}\right] + \left[a_2 \cos\frac{2\pi x}{3} + b_2 \sin\frac{2\pi x}{3}\right].$$

$$= 20.85 + \left[-8.3 \cos\frac{\pi x}{3} - 1.13 \sin\frac{\pi x}{3}\right] + \left[-2.3 \cos\frac{2\pi x}{3}\right].$$

4. Obtain the constant term and first three harmonic in the fourier cosine series of $y = f(x)$ in $(0,6)$ from the following table.

x	0	1	2	3	4	5
y	4	8	15	7	6	2.

$2l = 6$.

~~$l = 3$~~

$l = 6$

x	y	$y\cos\frac{\pi x}{6}$	$y\cos\frac{2\pi x}{6}$	$y\cos\frac{3\pi x}{6}$
0	4	4.	4	4
1	8	6.93	4	0.
2	15	7.5	-7.5	-15
3	7	0	-7	0
4	6	-3	-3	6
5	2	-1.73	1	0
	$\Sigma y = 42$	$\Sigma y\cos\frac{\pi x}{6} = 13.7$	$\Sigma y\cos\frac{2\pi x}{6} = -8.5$	$\Sigma y\cos\frac{3\pi x}{6} = -5$

$$a_0 = \frac{2\Sigma y}{N} = \frac{2(42)}{6} = 14.$$

$$a_1 = \frac{2}{6}(13.7) = \frac{2\Sigma y\cos\frac{\pi x}{6}}{N} = 4.56$$

$$a_2 = \frac{2\Sigma y\cos\frac{2\pi x}{6}}{N} = \frac{2(-8.5)}{6} = -2.83$$

$$a_3 = \frac{2\Sigma y\cos\frac{3\pi x}{6}}{N} = \frac{2(-5)}{6} = -1.67$$

The required harmonic is:

$$y = \frac{a_0}{2} + a_1\cos\frac{\pi x}{6} + a_2\cos\frac{2\pi x}{6} + a_3\cos\frac{3\pi x}{6}.$$

$$= 7 + 4.57\cos\frac{\pi x}{6} - 2.83\cos\frac{2\pi x}{6} - 1.67\cos\frac{3\pi x}{6}.$$

Unit-1: PDE

1. Find the complete solution of $p = 2qx$.

This is in the form of $f(p, x) = f(q, y)$

$$\frac{p}{x} = 2q = k.$$

$$\frac{p}{x} = k, \quad 2q = k.$$

$$p = kx, \quad q = \frac{k}{2}$$

Since $z = \int p\,dx + q\,dy.$

$$z = \int kx\,dx + \frac{k}{2}\,dy.$$

$$\boxed{z = \frac{kx^2}{2} + \frac{k}{2}y + c}$$

2. Solve: $(D^2 - 6DD' + 9D'^2)z = 0$

$$A.E = m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$m = 3 \text{ (twice)}$$

$$z = f_1(y + mx) + x f_2(y + mx)$$

$$z = f_1(y + 3x) + x f_2(y + 3x)$$

3. Form the PDE by eliminating the arbitary constant.

$f(x^2+y^2, z-xy)=0$.

$x^2+y^2 = f(z-xy) \rightarrow ①$

diff w.r to x partially

$2x = f'(z-xy)\left[\frac{\partial z}{\partial x} - y\right]$.

$p = \frac{\partial z}{\partial x}$

$2x = f'(z-xy)(p-y) \rightarrow ②$.

$q = \frac{\partial z}{\partial y}$.

diff ① w.r to y partially.

$2y = f'(z-xy)\left[\frac{\partial z}{\partial y} - x\right]$.

$2y = f'(z-xy)(q-x) \rightarrow ③$.

$\frac{②}{③} \Rightarrow \frac{2x}{2y} = \frac{f'(z-xy)(p-y)}{f'(z-xy)(q-x)}$

$\frac{x}{y} = \frac{(p-y)}{(q-x)}$

$x(q-x) = y(p-y)$

$qx - x^2 = py - y^2$.

~~$-qx - py = y^2 - x^2$~~.

$y^2 - x^2 = py - qx$.

$yp - xq = y^2 - x^2$.

4. Find the complete soln of PDE $p^3 - q^3 = 0$.

This is in the form $f(p,q)=0$.

$p^3 - q^3 = 0 \rightarrow ①$.

Let $z = ax + by + c \rightarrow ②$ be the solution of ①

put $p=a$ & $q=b$ in ①.

$a^3 - b^3 = 0$

$a^3 = b^3$.

$a=b$

② ⇒ $z = ax + ay + c$ is the C.I. (complete Integral).

~~$z \neq ax + a$~~

5. Form the PDE by eliminating the arbitary function f,
$z = e^{ay} f(x+by)$.

$$z = e^{ay} f(x+by) \rightarrow ①.$$

diff ① w.r.to x partially.

$$\frac{\partial z}{\partial x} = e^{ay} f'(x+by) \cdot 1$$

$$p = e^{ay} f'(x+by) \rightarrow ②.$$

Diff ① w.r.to y partially.

$$\frac{\partial z}{\partial y} = e^{ay} \cdot f'(x+by) \cdot b + f(x+by) \cdot a e^{ay}$$

$$q = p \cdot b + a \cdot z \quad (\text{by ② , by ①})$$

$$\boxed{q = bp + az}$$

6. Solve $(D^3 - D^2 D' - 8DD'^2 + 12 D'^3) z = 0$.

$m_3 = -3, \quad m_2 = 2, \quad m_1 = 2 \qquad [m_1 = m_2 = 2], \; m_3 = -3$

$C.F = f_1(y + m_1 x) + f_2(y + m_2 x) + f_3(y + m_3 x)$.

$= f_1(y + 2x) + f_2(y + 2x) + f_3(y - 3x)$.

$C.F = f_1(y + 2x) + x f_2(y + 2x) + f_3(y - 3x)$.

2	1	-1	-8	12
	0	2	2	-12
	1	1	-6	0

$m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$m = -3, \; m = 2, \; m = 2$.

7. Find the complete integral of PDE; $z = px + qy + \sqrt{pq}$.

$z = px + pq + \sqrt{pq} \rightarrow (1)$.

put $p = a$, $q = b$ in (1).

$z = ax + by + \sqrt{ab}$ is the complete integral.

8. Solve: $(D^3 - 3DD'^2 + 2D'^3)z = 0$

A.E: $m^3 - 3m + 2 = 0$

$(m-1)(m-2) = 0$.

$m = 1$, $m = 2$.

1	1	0	-3	2
	1	1	-2	0
	1	1	-2	0

$m^2 + m - 2 = 0$

$(m-1)(m+2) = 0$.

$m = 1$, $m = -2$, $m = 1$

$m = 1$ (twice) $m = -2$.

$C.F = f_1(y + m_1 x) + x f_2(y + m_2 x) + f_3(y + m_3 x)$

$= f_1(y+x) + x f_2(y+x) + f_3(y-2x)$

9. Form the P.DE by eliminating the arbitary function f, $z = f\left(\frac{y}{x}\right)$.

$z = f\left(\frac{y}{x}\right) \rightarrow (1)$.

diff eqn (1) w.r.to x partially.

$\frac{1}{x} = x^{-1}$

$\frac{\partial z}{\partial x} = f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)$.

$\frac{d}{dx}(x^n) = n x^{n-1}$

$p = f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right) \rightarrow (2)$.

$-1\, x^{-1-1}$

$-1\, x^{-2}$

$\frac{-1}{x^2}$.

diff eqn (1) w.r.to y partially:

$\frac{\partial z}{\partial y} = f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)$.

$$q = f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) \rightarrow ③.$$

$$\frac{②}{③} \Rightarrow \frac{p}{q} = \frac{f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)}{f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)}$$

$$\frac{p}{q} = \frac{-y}{x}.$$

$$\boxed{px = -qy}$$

10. Find the complete solution of PDE $\sqrt{p} + \sqrt{q} = 1$.

Repeated

$$\sqrt{p} + \sqrt{q} = 1 \rightarrow ①.$$

$z = ax + by + c$ be the solution of ① $\rightarrow ②$.

Put $p = a$, $q = b$ in ①.

$$\sqrt{a} + \sqrt{b} = 1$$

$$\sqrt{b} = 1 - \sqrt{a}.$$

$$\therefore b = (1 - \sqrt{a})^2 \rightarrow ③.$$

sub b in ②.

$$z = ax + (1 - \sqrt{a})^2 y + c$$

11. Find the PDE of all spheres whose centre lies on the x axis.

Let the equation of the sphere,

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2 \rightarrow ①$$

Since centre lies on x axis.

i.e $(a, 0, 0)$.

diff (2) w.r. to x partially.

$$2(x-a)'.1 + 2z \cdot \frac{\partial z}{\partial x} = 0$$

$$2(x-a) + 2zp = 0$$

$$2(x-a) = -2zp$$

$$(x-a) = -zp \rightarrow (3)$$

diff (2) w.r. to y partially.

$$2y + 2z \cdot \frac{\partial z}{\partial y} = 0$$

$$2zq + 2y = 0$$

$$2zq = -2y$$

$$-zq = +y \rightarrow (4)$$

sub (3) & (4) in (2):

$$(-zp)^2 + (-zq)^2 + z^2 = r^2$$

$$z^2p^2 + z^2 + q^2 + z^2 = r^2$$

$$z^2[p^2 + q^2 + 1] = r^2$$

Find the PDE of all spheres whose centre lies on the z axis.

Let the equation of the sphere be

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2 \rightarrow (1).$$

Since the centre lies on z-axis

(i.e) $[0,0,c]$.

$$x^2+y^2+(z-c)^2=r^2 \rightarrow (2).$$

diff (2) w.r. to 'x' partially.

$$2x+2(z-c)\frac{\partial z}{\partial x}=0.$$

$$2(z-c)p=-2x$$

$$(z-c)p=-x, \quad (z-c)=\frac{-x}{p} \rightarrow (3).$$

diff (2) w.r. to 'y' partially.

$$2y+2(z-c)\frac{\partial z}{\partial y}=0$$

$$2(z-c)q=-2y$$

$$(z-c)q=-y$$

$$z-c=\frac{-y}{q} \rightarrow (4).$$

from eqn (3) & (4).

$$\frac{-x}{p}=\frac{-y}{q}$$

$$\boxed{qx=py}$$

12. Find the complete integral of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

$$\frac{z}{pq} = \frac{px + qy + \sqrt{pq}\cdot pq}{pq}$$

$$z = px + qy + (pq)^{3/2} \rightarrow (1).$$

Put $p = a$, $q = b$ in (1)

$$z = ax + by + (ab)^{3/2}$$

It is the complete integral.

Find the PDE of all spheres whose centre lies on the x & y axis:

Let the equation of the sphere.

$$(x-a)^2+(y-b)^2+(z-c)^2 = r^2 \rightarrow (1).$$

Since centre lies on x & y axis.

i.e $(x-a)^2+(y-b)^2+z^2 = r^2 \rightarrow (2).$

Diff (2) w.r. to x partially.

$$2(x-a).1+2z\frac{\partial z}{\partial x}=0$$

$$2(x-a)1+2z.\frac{\partial z}{\partial x}=0 \qquad 2(x-a)=-2zp$$

$$x-a=-zp \rightarrow (3).$$

diff (2) w.r. to y partially:

$$2(y-b).1+2z.\frac{\partial z}{\partial y}=0.$$

$$2(y-b)=-2zq$$

$$y-b=-zq \rightarrow (4).$$

sub (3) & (4) in eqn (2).

$$(-zp)^2+(-zq)^2+z^2=r^2$$

$$z^2p^2+z^2q^2+z^2=r^2$$

$$z^2(p^2+q^2+1)=r^2.$$

13. Find the PDE by eliminating the arbitary function, 'f' from the relation, $z=f(x^2-y^2)$.

$$z=f(x^2-y^2) \rightarrow (1).$$

diff (1) w.r. to x.

$$\frac{\partial z}{\partial x}=f'(x^2-y^2).2x.$$

$$p=f'(x^2-y^2).2x \rightarrow (2).$$

diff ② w.r. to y.

$$q = \frac{\partial z}{\partial y} = f'(x^2-y^2)(-2y) \longrightarrow ③$$

$$\frac{②}{③} \Rightarrow \frac{p}{q} = \frac{f'(x^2-y^2)\,2x}{f'(x^2-y^2)(-2y)}$$

$$\frac{p}{q} = \frac{x}{-y}$$

$$\boxed{py = -qx}$$

14. Find the PDE by eliminating the arbitrary constants 'a' and 'b'.

$$z = (x-a)^2 + (y-b)^2 + 1 \longrightarrow ①$$

diff eqn ① w.r. to x.

$$\frac{\partial z}{\partial x} = 2(x-a)\cdot 1$$

$$p = 2(x-a)$$

$$\frac{p}{2} = (x-a) \longrightarrow ②$$

diff eqn ① w.r. to y.

$$\frac{\partial z}{\partial y} = 2(y-b)\cdot 1$$

$$q = (y-b)\,2$$

$$\frac{q}{2} = (y-b) \longrightarrow ③$$

sub ② & ③ in ①.

$$① \Rightarrow z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2 + 1$$

$$z = \frac{p^2}{4} + \frac{q^2}{4} + 1$$

$$z = \frac{p^2+q^2+4}{4}$$

$$\boxed{4z = p^2 + q^2 + 4}$$

15. Find the complete integral of $p+q = x+y$.

$$P+q = x-y. \rightarrow ①$$

$$p - x = y - q = k.$$

$$P - x = k, \quad y - q = k$$

$$P = k + x \quad q = y - k.$$

Since $z = \int Pd\,x + q\,dy.$

$$z = \int [(k+x)\,dx + (y-k)\,dy]$$

$$z = kx + \frac{x^2}{2} + \frac{y^2}{2} - ky + C.$$

$$\boxed{z = \frac{x^2}{2} + \frac{y^2}{2} + kx - ky + c}$$

3rd unit → Applications of PDE

1. Classify the two dimensional steady state heat conduction equation.

The 2-D heat flow eqn in steady state is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Here, $A \cdot \frac{\partial^2 u}{\partial x^2} + B \cdot \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} = 0.$

$$A = 1, \quad B = 0, \quad C = 1$$

$$\Delta = B^2 - 4AC.$$

$$= 0 - 4(1)(1)$$

$$\Delta = -4 < 0$$

Zero → Parabolic

greater than 0 → Hyperbolic

∴ It is elliptic.

6. Give the mathematical formulation of the problem of 1-D heat conduction in a rod of length l with insulated ends & with initial temp $f(x)$.

Let the 1-D heat flow eqn is

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

In steady state, $\frac{\partial u}{\partial t} = 0$.

i.e, $0 = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

Since $\alpha^2 \neq 0$.

$$\therefore \frac{\partial^2 u}{\partial x^2} = 0$$

ie $\frac{d^2 u}{dx^2} = 0$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = 0.$$

Integrate it.

$$\frac{du}{dx} = 0 + a.$$

$$du = a\,dx.$$

Integrate again

$$\int du = a \int dx$$

$$\boxed{u = ax + b}.$$

7. Classify the PDE: $u_{xx} + u_{xy} = f(x,y)$.

$$u_{xx} + u_{xy} - f(x,y) = 0.$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - f(x,y) = 0.$$

Since $A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} = 0.$

Here $A=1, B=1, C=0.$

$\Delta = B^2 - 4(AC).$

$= 1^2 - 4(1)(0)$

$\Delta = 1 > 0.$

$\therefore$ It is Hyperbolic.

4. Write all the possible solutions of 1-D heat equation:

Repeated

Let the 1-D heat equation is.

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$$

The possible solution is.

(i) $u(x,t) = (Ax+B)\, C$

(ii) $u(x,t) = (Ae^{px} + Be^{-px})(Ce^{\alpha^2 p^2 t})$

(iii) $u(x,t) = (A\cos px + B\sin px)\, Ce^{-\alpha^2 p^2 t}.$

5. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u.$

where $u(x,0) = 6e^{-3x}$

No need. (8 marks)

6. Classify the PDE: $u_{xy} = u_x u_y + xy.$

$u_{xy} = u_x u_y + xy \rightarrow ①.$

$u_{xy} - u_x \cdot u_y - xy = 0$

$$\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} - xy = 0.$$

$$A \cdot \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2}.$$

Here, $A = 0, B = 1, C = 0$.

$$\Delta^2 = B^2 - 4AC$$

$$= 1^2 - 4(0)$$

$$\Delta = 1 - 0$$

$$\Delta = 1 > 0$$

$\therefore$ It is Hyperbolic.

7. Classify the PDE: $U_{xx} + U_{xy} + U_{yy} = 0$.

$$U_{xx} + U_{xy} + U_{yy} = 0$$

$A = 1, B = 1, C = 1$

$$\Delta = B^2 = 4AC$$

$$= 1^2 - 4(1)(1)$$

$$= 1 - 4$$

$$\Delta = -3 < 0$$

$\therefore$ It is elliptic.

8. State the assumptions in deriving one dimensional wave equation.

1. The string is homogenous.
2. The string is perfectly elastic and so it does not offer any resitance to bending.
3. The tension T caused by stretching is so large so that the action of the gravitational force on the string can be neglected.
4. The string platform small tranverse motions in a vertical plane so that the deflection y and the slope are in absolute value and hence their powers (higher) may be neglected.

9. What is the basic differcen between the soln of 1-D wave eqn & 1-D heat eqn:

* 1-D wave equation is periodic, w.r. to time 't'.
* 1-D heat equation is non-periodic w.r. to time 't'.

10. State any 2 solutions of the Laplace equation. $u_{xx}+u_{yy}=0$ involving exponential terms in x or y?

(i) $u(x,y) = (A\cos py + B\sin py)(Ce^{px} + De^{-px})$.

(ii) $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$.

11. Solve: $yu_x + xu_y = 0$ using separation of variables method:

Let $u = X(x)\, Y(y) \rightarrow (1)$.

$u_x = X'Y$ & $u_y = XY' \rightarrow (3)$

$\hookrightarrow (2)$.

(2) & (3) use in (1)

$yX'Y + xXY' = 0$

~~$y^2 x' + x^2 y' = 0$.~~ $yX'Y = -xXY'$

~~$y^2 x' = -x^2 y'$~~ $\frac{X'}{xX} = \frac{-Y'}{yY}$.

Integrate it,

$\frac{1}{x}\int \frac{X'}{X} = \frac{-1}{y}\int \frac{Y'}{Y}$

$\frac{1}{x}\log X = \frac{-1}{y}\log Y + \log c$

$\frac{1}{x}\log x - \frac{1}{y}\log y = \log c$.

Unit-2: Fourier series:

1. State the Dirichlet's conditions:

(i) $f(x)$ is periodic, single valued everywhere.

(ii) $f(x)$ must have a finite number of discontinuity in the given interval.

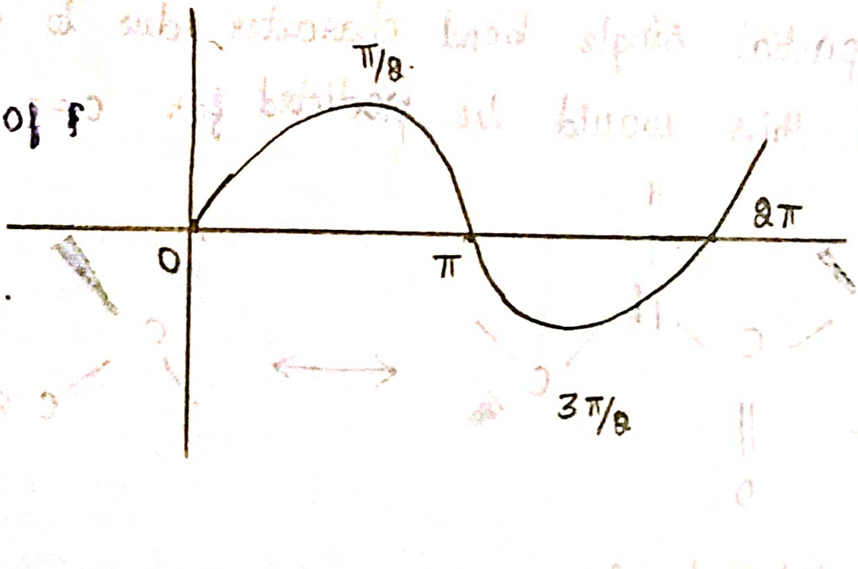
(iii) $f(x)$ has finite number of maxima & minima.

2. Sketch the even extension of the function $f(x) = \sin x$, $0 < x < \pi$.

Even extension of f

$f(x) = \sin x$

as follows.



Department of Mechanical Engineering

Lecture Notes

Subject Code : ME3351

Subject Name: ENGINEERING MECHANICS

Sem/Year : 03/II

Regulation : 2021

ME3351 ENGINEERING MECHANICS

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COURSE OBJECTIVES:

1 To Learn the use scalar and vector analytical techniques for analysing forces in statically determinate structures
2 To introduce the equilibrium of rigid bodies, vector methods and free body diagram
3 To study and understand the distributed forces, surface, loading on beam and intensity.
4 To learn the principles of friction, forces and to determine the apply the concepts of frictional forces at the contact surfaces of various engineering systems.
5 To develop basic dynamics concepts – force, momentum, work and energy;

UNIT I STATICS OF PARTICLES 9

Fundamental Concepts and Principles, Systems of Units, Method of Problem Solutions, Statics of Particles -Forces in a Plane, Resultant of Forces, Resolution of a Force into Components, Rectangular Components of a Force, Unit Vectors. Equilibrium of a Particle- Newton's First Law of Motion, Space and Free-Body Diagrams, Forces in Space, Equilibrium of a Particle in Space.

UNIT II EQUILIBRIUM OF RIGID BODIES 9

Principle of Transmissibility, Equivalent Forces, Vector Product of Two Vectors, Moment of a Force about a Point, Varignon's Theorem, Rectangular Components of the Moment of a Force, Scalar Product of Two Vectors, Mixed Triple Product of Three Vectors, Moment of a Force about an Axis, Couple - Moment of a Couple, Equivalent Couples, Addition of Couples, Resolution of a Given Force into a Force -Couple system, Further Reduction of a System of Forces, Equilibrium in Two and Three Dimensions - Reactions at Supports and Connections.

UNIT III DISTRIBUTED FORCES 9

Centroids of lines and areas – symmetrical and unsymmetrical shapes, Determination of Centroids by Integration, Theorems of Pappus-Guldinus, Distributed Loads on Beams, Centre of Gravity of a Three- Dimensional Body, Centroid of a Volume, Composite Bodies, Determination of Centroids of Volumes by Integration. Moments of Inertia of Areas and Mass - Determination of the Moment of Inertia of an Area by Integration, Polar Moment of Inertia, Radius of Gyration of an Area, Parallel-Axis Theorem, Moments of Inertia of Composite Areas, Moments of Inertia of a Mass - Moments of Inertia of Thin Plates, Determination of the Moment of Inertia of a Three-Dimensional Body by Integration.

UNIT IV FRICTION 9

The Laws of Dry Friction, Coefficients of Friction, Angles of Friction, Wedge friction, Wheel Friction, Rolling Resistance, Ladder friction.

UNIT V DYNAMICS OF PARTICLES 9

Kinematics - Rectilinear Motion and Curvilinear Motion of Particles. Kinetics- Newton's Second Law of Motion -Equations of Motions, Dynamic Equilibrium, Energy and Momentum Methods - Work of a Force, Kinetic Energy of a Particle, Principle of Work and Energy, Principle of Impulse and Momentum, Impact of bodies.

UNIT I STATICS OF PARTICLES

INTRODUCTION

Engineering Mechanics is all about mechanical interaction between bodies which means we will learn how different bodies apply forces on one another and how they then balance to keep each other in equilibrium. The branch of physical science that deals with the state of rest or the state of motion is termed as Mechanics. The state of rest and state of motion of the bodies under the action of different forces has engaged the attention of theorists, mathematicians and scientists. Starting from the analysis of rigid bodies under gravitational force and simple applied forces the mechanics has grown to the analysis of robotics, aircrafts, spacecrafts under dynamic forces, atmospheric forces, temperature forces etc.
Engineering mechanics is the application of mechanics to solve problems involving common engineering elements. The engineering mechanics is mainly classified into two branches. They are
1. Statics

2. Dynamics

Statics - Statics deals with the forces on a body at rest.
Dynamics - Dynamics deals with the forces acting on a body when the body is in motion.
Dynamics further subdivided in to two sub branches.
They are:

- **Kinematics:** Deals the motion of a body without considering the forces causing the motion.
- **Kinetics:** Deals with the relation between the forces acting on the body and the resulting motion

IMPORTANCE OF MECHANICS TO ENGINEERING

UNITS AND DIMENSIONS

Length (L), Mass (M) and Time (S) are the fundamental units in mechanics. The units of all other quantities may be expressed in terms of these basic units.
The three commonly used systems in engineering are
Metre—Kilogramme—Second (MKS) system,
Centimetre—Gramme—Second (CGS) system, and
Foot—Pound—Second (FPS) system.
The units of length, mass and time used in the system are used to name the systems. Using these basic units, the units for other quantities can be found.
Fundamental quantities

- The quantities that are independent of other quantities are called fundamental quantities.
- The units that are used to measure these fundamental quantities are called fundamental units.

BASE QUANTITY	NAME	SYMBOL
	SI BASED UNIT	
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Thermodynamic Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Derived quantities

- The quantities that are derived using the fundamental quantities are called derived quantities.
- The units that are used to measure these derived quantities are called derived units.

Area	Square meter	**m^2**
Volume	Cubic meter	**m^3**
Speed, Velocity	Meter per second	**m/s**
Acceleration	Meter per second squared	**m/s^2**
Wave Number	Reciprocal meter	**m^{-1}**
Mass Density	Kilogram per cubic meter	**kg/m^3**
Specific Volume	Cubic meter per kilogram	**m^3/kg**
Current Density	Ampere per square meter	**A/m^2**
Magnetic Field Strength	Ampere per meter	**A/m**
Amount-of-substance Concentration	Mole per cubic meter	**mol/m^3**
Luminance	Candela per square meter	**cd/m^2**
Mass Fraction	Kilogram per kilogram, which may be represented by the number 1	**kg/kg = 1**

Table 3 Derived SI units with special names

Physical quantity	SI unit	Symbol
Frequency	hertz	Hz
Energy	joule	J
Force	newton	N
Power	watt	W
Pressure	pascal	Pa
Electric charge or quantity of electricity	coulomb	C
Electric potential difference and emf	volt	V
Electric resistance	ohm	Omega / Ω
Electric conductance	siemen	S
Electric capacitance	farad	F
Magnetic flux	weber	Wb
Inductance	henry	H
Magnetic flux density	tesla	T
Illumination	lux	Lx
Luminous flux	lumen	Lm

International System of Units

SI base units - The SI is founded on seven SI base units for seven base quantities assumed to be mutually independent, as given in Table below:

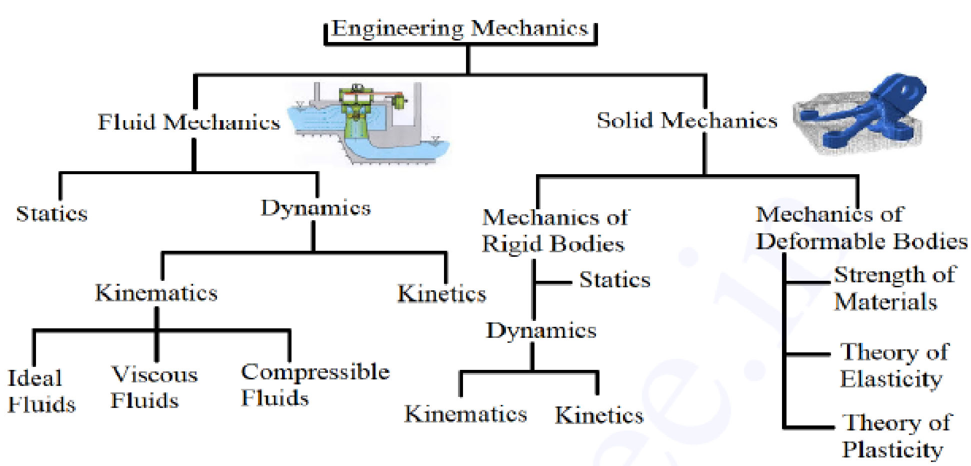


Fig. 1 Classification of Engineering Mechanics

LAWS OF MECHANICS

- Newton's First Law of Motion
- Newton's Second Law of Motion
- Newton's Third Law of Motion
- Newton's Law of Gravitation
- Parallelogram law of forces
- Principles of Transmissibility

Newton's first law of Motion

It states that each and every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

Everybody continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it.

Newton's Second Law

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it. Thus according to this law,

Force ∝ Rate of change of momentum. But momentum = Mass × Velocity

As mass do not change, Force ∝ Mass × Rate of change of velocity

Force ∝ Mass × Acceleration

$F \propto m \times a$

Newton's Third Law

It states that for every action there is an equal and opposite reaction. Consider the two bodies in contact with each other. Let one body applies a force *F on another. According to this* law, the second body develops a reactive force *R which is equal in magnitude to force F and* acts in the line same as *F but in the opposite direction.*

Newton's Law of Gravitation

The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. According to this law, the force of attraction between the bodies of mass m_1 and mass m_2 at a distance d .

$$F = G\, m_1\, m_2/d^2$$

Where,

G is the constant of proportionality and is known as constant of gravitation.

It states that two bodies will be attracted towards each other along their connecting line with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between the centres.

Parallelogram law of forces

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram ram passing through that point.

Principles of Transmissibility
The general principle states that the effect of force acting on a rigid body does not change if the force is moved along its line of action to another point on the body.
Example: Let F be the force acting on a rigid body at point A as shown in figure given below
According to the law of transmissibility of force, this force has the same effect on the state of body as the force F applied at point B

Triangular law of forces

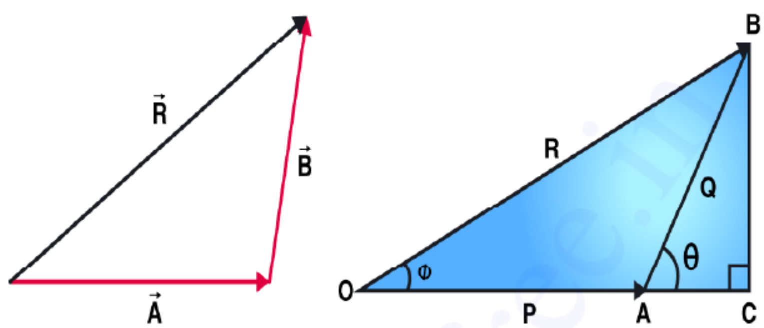
If two forces acting at a point are represented by the two sides of a triangle taken in order then their resultant force is represented by the third side taken in opposite order.

therefore, $R = \sqrt{P^2 + 2PQcos\Theta + Q^2}$

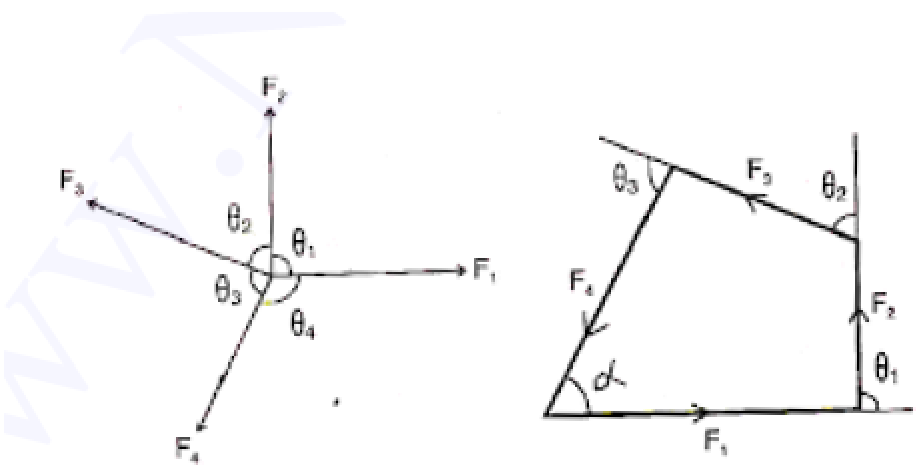
Above equation is the magnitude of the resultant vector.

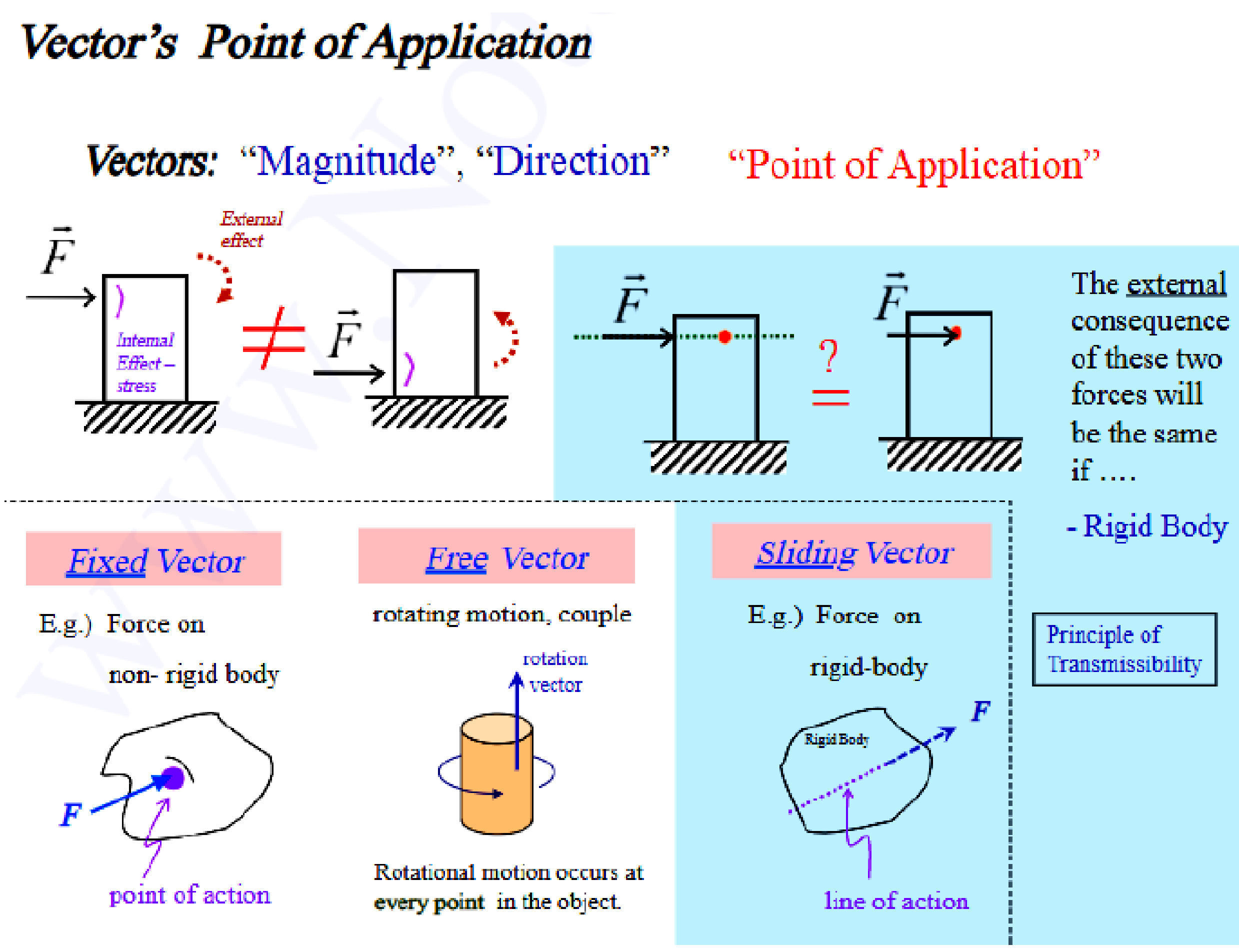
therefore, $\phi = tan^{-1}(\frac{Qsin\Theta}{P+Qcos\Theta})$

Above equation is the direction of the resultant vector.



Polygon law of Forces
If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order then the resultant of all three forces may be represented in magnitude and direction by the closing side of the polygon taken in opposite order.





FORCES

A force is a measure of the action of one body or media on another (push or pull)

Force has:

- Magnitude
- Direction
- Point of application

Types of Forces:

- ❖ *External Forces* – It represents the action of other bodies on the rigid body
- ❖ *Internal Forces* – The forces which hold together the particles forming the rigid body

Effects of a force

A force may produce the following effects in a body, on which it acts :

- ➢ It may change the motion of a body. *i.e*. if a body is at rest, the force may set it in motion.
- ➢ And if the body is already in motion, the force may accelerate it.
- ➢ It may retard the motion of a body.
- ➢ It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.

Characteristics of a force

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

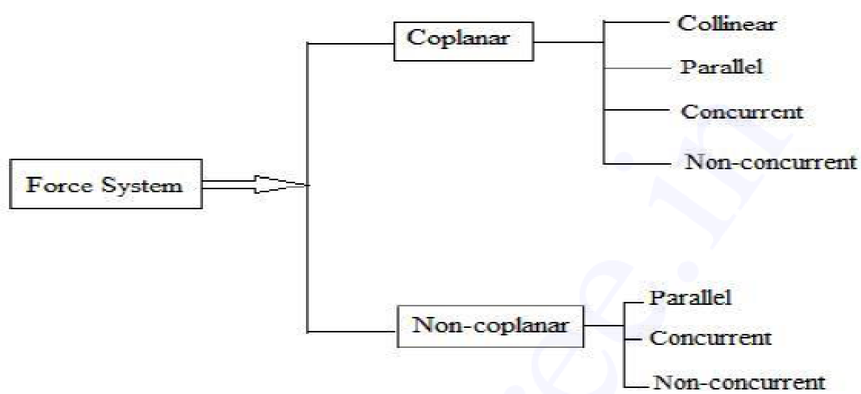
- Magnitude of the force (*i.e*., 100 N, 50 N, 20 kN, 5 kN, etc.)

- The direction of the line, along which the force acts (*i.e*., along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.

Nature of the force (*i.e*., whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.

- The point at which (or through which) the force acts on the body.

SYSTEM OF FORCES



Resolution of a force

Splitting up of a force into components along the fixed reference axis is called resolution o forces. The effect by single force and component forces remains the same.

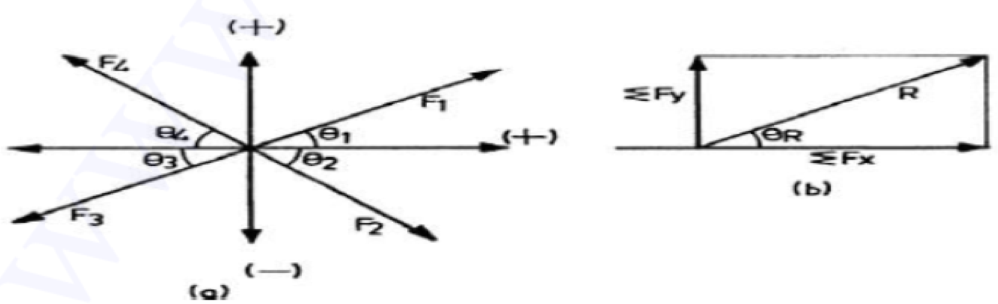


Fig. 17 Resolution of a force

Algebraic sum of horizontal components

$\Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 - F_3 \cos\theta_3 - F_4 \cos\theta_4$

Algebraic sum of vertical components

$\Sigma F_y = F_1\sin\theta_1 - F_2\sin\theta_2 - F_3\sin\theta_3 + F_4\sin\theta_4$

Resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

Angle α mode by the resultant with x axis is given by

$\tan\alpha = \Sigma F_y / \Sigma F_x$

A vertical force has no horizontal component

A horizontal force has no vertical component

Q1. Forces *R*, *S*, *T*, *U* are collinear. Forces *R* and *T* act from left to right. Forces *S* and *U* act from right to left. Magnitudes of the forces *R*, *S*, *T*, *U* are 40 N, 45 N, 50 N and 55 N respectively. Find the resultant of *R*, *S*, *T*, *U*.

Given data:

R=40 N

S=45 N

T=50 N

U=55N

Resultant= -R-U+S+T=-40-55+45+50=0

Q2. Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

Solution

Given:

First force ($F1$) = 100 N; Second force ($F2$) = 150 N and angle between $F1$ and $F2$ (θ) = 45°.

We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= (100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45° \text{ N}$$

$$= 10\,000 + 22\,500 + (30\,000 \times 0.707) \text{ N}$$

$$= 232 \text{ N}$$

EQUILIBRIUM OF A PARTICLE IN SPACE

In three dimension of space if the forces acting on the particle are resolved into their respective i, j, k components the equilibrium equation is written as,

$\Sigma Fxi + \Sigma Fj + \Sigma Fxk = 0$

The equation for equilibrium of a particle in space is,

$\Sigma Fx = 0$; $\Sigma Fy = 0$; $\Sigma Fz = 0$;

Resultant of Concurrent Force Systems in Space

Components of the resultant

$R_x = \Sigma F_x$, $R_y = \Sigma F_y$ and $R_z = \Sigma F_z$

Magnitude of the resultant

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Equilibrium of Concurrent Space Forces

The resultant of all forces is zero

$\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$

The sum of moment is zero

$\Sigma M_x = 0$, $\Sigma My = 0$ and $\Sigma M_z = 0$

Rectangular Components in Space

$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

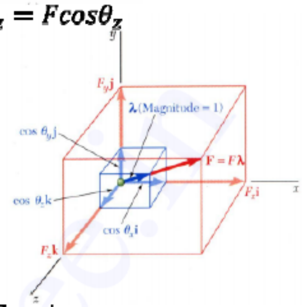
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\cos\theta_x\boldsymbol{i} + F\cos\theta_y\boldsymbol{j} + F\cos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\left(\cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}\right)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

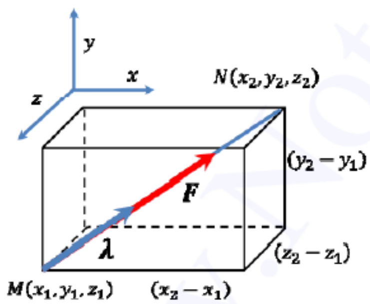
Where $\boldsymbol{\lambda} = \cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}$



$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

EQUIVALENT FORCE SYSTEMS

Two forces are said to be equivalent if they have the same magnitude and direction (i.e. they are equal) and produce the same moment about any point O (i.e. same line of action).

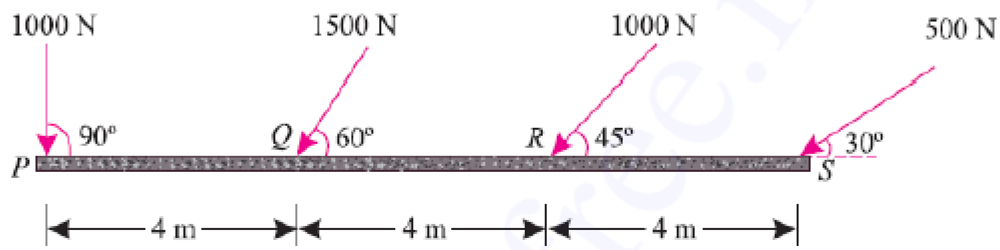
The basic idea - Two force systems are equivalent if they result in the same resultant force and the same resultant moment.

$$\begin{cases} \sum \underline{\mathbf{F}} \ \text{for system 1} = \sum \underline{\mathbf{F}} \ \text{for system 2} \\ \sum \underline{\mathbf{M}}_O \ \text{for system 1} = \sum \underline{\mathbf{M}}_O \ \text{for system 2} \end{cases} \Leftrightarrow \ \text{The two force systems are equivalent}$$

Moving a force along its line of action - Moving a force along its line of action results in a new force system which is equivalent to the original force system

2. A horizontal line PQRS is 12 m long, where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force.

Solution



Magnitude of the resultant force

Resolving all the forces horizontally,

$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30°$ N

$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866)$ N

$= 1890$ N ...(*i*)

and now resolving all the forces vertically,

$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30°$ N

$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5)$ N

$= 3256$ N ...(*ii*)

We know that magnitude of the resultant force,

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765$ N

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS*.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722$$

Note:
Since both the values of Σ*H* and Σ*V* are +ve, therefore resultant lies between 0° and 90°.

Position of the resultant force
Let *x* = Distance between *P* and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force

FREE BODY DIAGRAM

Free body diagram is a diagram which shows all the forces acting at a rigid body involving **1) self weight, 2) Normal reactions, 3) frictional force, 4) Applied force, 5) External moment applied**. In a rigid body mechanics, the concept of free body diagram is very useful to solve the problems.

Free body diagram for rigid bodies:

In order to draw the FBD for each member of a rigid body follow the instructions below:

- Isolate the object from its surroundings,
- Draw the outline of the object; consider all dimensions and angles,
- Include all forces and couple moments that the surroundings exert on the body. Forces include *loadings*, *support reactions* and *weights*. (See the support reaction section for detailed explanation)
- Known forces and moments should be labeled with their proper *magnitudes* and *directions*.

- Magnitudes and direction angles of unknown forces and moments should be represented with *letters*.

FBD is a sketch of the outlined shape of the body, which represents it being isolated from its surroundings.

- It is necessary to show all the forces and couple moments that the surroundings exert on the body so that these effects can be accounted for when equations of equilibrium are applied.
-

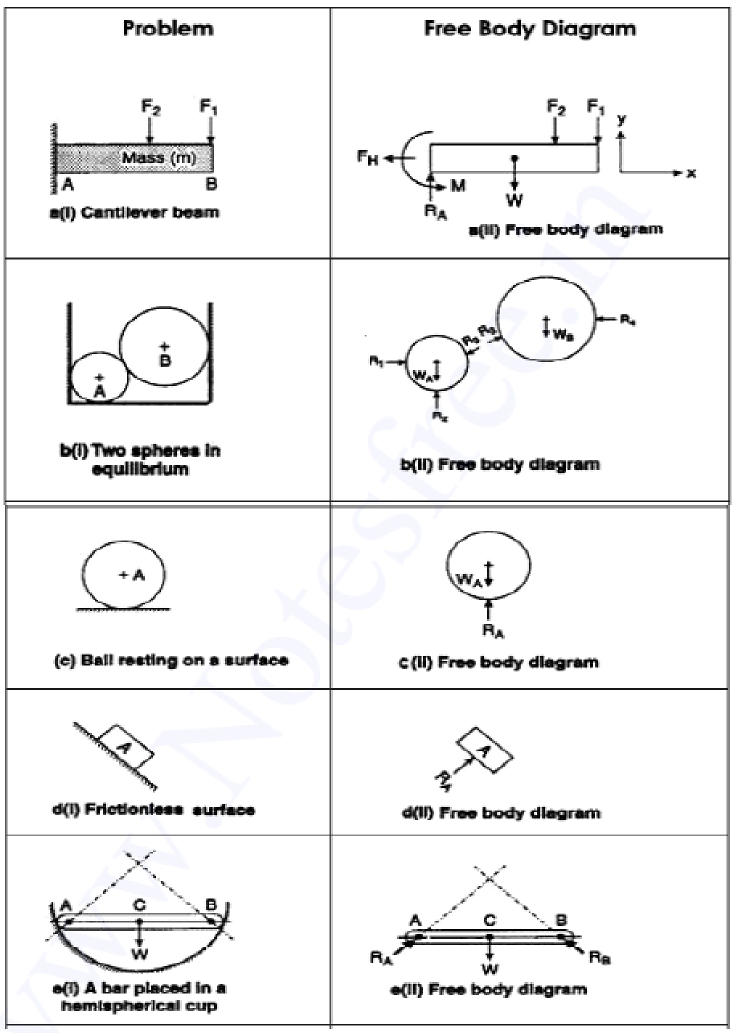
Free Body Diagram As a general rule, if
a support prevents translation of a
body in a given direction, then a force is developed on the body in the opposite
direction. Similarly, if rotation is prevented,
a couple moment is exerted on the body.

- The problem becomes much simple if each body is considered in isolation i.e, separate from the surrounding body or bodies. Such a body which has been so separated or isolated from the surrounding bodies is called as Free Body.
- The sketch showing all the forces and moments acting on the body is called as the free body diagram.

It is a diagram of the body in which the bodies under consideration are freed from all contact surfaces and all the forces acting on it are clearly indicated.

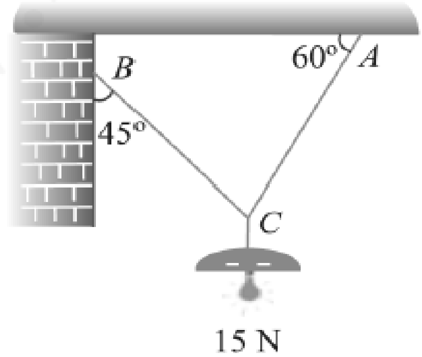
Procedure for Drawing a FBD:

1. Draw outlined shape - Isolate rigid body from its surroundings
2. Show all the forces - Show all the external forces and couple moments. These typically include
   - ❖ Applied Loads
   - ❖ Support reactions
   - ❖ The weight of the body
3. Identify each force
   - ❖ Known forces should be labeled with proper magnitude and direction
   - ❖ Letters are used to represent magnitude and directions of unknown forces.



Worked out examples

An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.



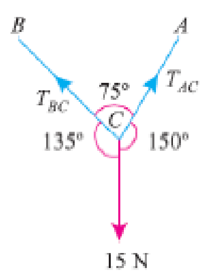
Given:

Weight at C = 15 N

Let TAC = Force in the string AC, and

TBC = Force in the string BC.

The system of forces is shown in Figure.



From the geometry of the figure, we find that angle between TAC and 15 N is 150° and angle between TBC and 15 N is 135°.

$$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

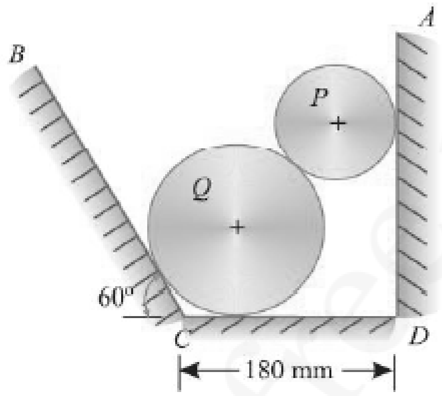
$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98\,\text{N}$$

and

$$T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \times 0.5}{0.9659} = 7.76\,\text{N}$$

Worked out examples

Two cylinders P and Q rest in a channel as shown in Figure. The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N. If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60°, determine the reactions at all the four points of contact.



Solution

Given:

Diameter of cylinder P = 100 mm

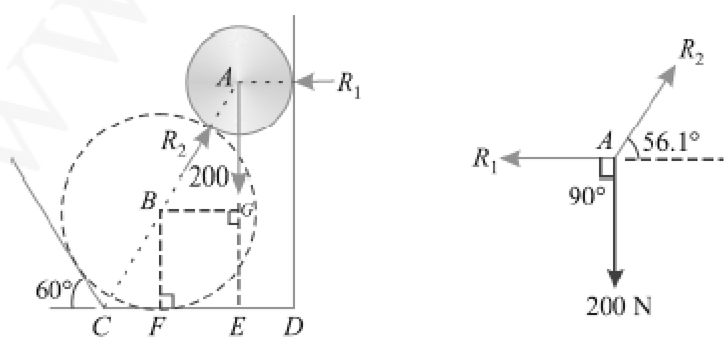
Weight of cylinder P = 200 N

Diameter of cylinder Q = 180 mm

Weight of cylinder Q = 500 N and

Width of channel = 180 mm.

First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A i.e., the centre of the cylinder P as shown in Figure (a) below. The system of forces at A is shown in Figure (b) below.



(*a*) Free body diagram (*b*) Force diagram

1. Weight of the cylinder (200 N) acting downwards.

2. Reaction (R_1) of the cylinder P at the vertical side.

3. Reaction (R_2) of the cylinder P at the point of contact with the cylinder Q.

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder } P = \frac{100}{2} = 50 \text{ mm}$$

Similarly $$BF = \text{Radius of cylinder } Q = \frac{180}{2} = 90 \text{ mm}$$

and $$\angle BCF = 60^\circ$$

$$\therefore \quad CF = BF \cot 60^\circ = 90 \times 0.577 = 52 \text{ mm}$$

$$\therefore \quad FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and $$AB = 50 + 90 = 140 \text{ mm}$$

$$\therefore \quad \cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or $$\angle ABG = 56.1^\circ$$

Applying Lami's equation at A,

$$\frac{R_1}{\sin (90^\circ + 56.1^\circ)} = \frac{R_2}{\sin 90^\circ} = \frac{200}{\sin (180^\circ - 56.1^\circ)}$$

$$\frac{R_1}{\cos 56.1^\circ} = \frac{R_2}{1} = \frac{200}{\sin 56.1^\circ}$$

$$R_1 = \frac{200 \cos 56.1^\circ}{\sin 56.1^\circ} = \frac{200 \times 0.5571}{0.830} = 134.2 \text{ N} \quad \textbf{Ans.}$$

$$R_2 = \frac{200}{\sin 56.1^\circ} = \frac{200}{0.8300} = 240.8 \text{ N} \quad \textbf{Ans.}$$

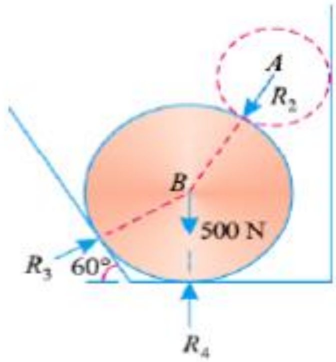
Now consider the equilibrium of the cylinder Q. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in Figure (*a*). The system of forces is shown in Figure (*b*).

1. Weight of the cylinder Q (500 N) acting downwards.

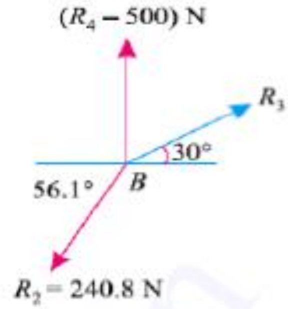
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.

3. Reaction R_3 of the cylinder Q on the inclined surface.

4. Reaction R_4 of the cylinder Q on the base of the channel.



(a) Free body diagram



(b) Force diagram

A little consideration will show that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

∴ Net downward force = $(R_4 - 500)$ N

Applying Lami's equation at B,

$$\frac{R_3}{\sin(90^\circ + 56.1^\circ)} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin(180^\circ + 30^\circ - 56.1^\circ)}$$

$$\frac{R_3}{\cos 56.1^\circ} = \frac{240.8}{\sin 60^\circ} = \frac{R_4 - 500}{\sin 26.1^\circ}$$

$$\therefore \quad R_3 = \frac{240.8 \times \cos 56.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 0.5577}{0.866} = 155 \text{ N} \quad \textbf{Ans.}$$

$$R_4 - 500 = \frac{240.8 \times \sin 26.1^\circ}{\sin 60^\circ} = \frac{240.8 \times 399}{0.866} = 122.3 \text{ N}$$

$$\therefore \quad R_4 = 122.3 + 500 = 622.3 \text{ N} \quad \textbf{Ans.}$$

TYPES OF SUPPORTS AND THEIR REACTIONS

In architectural structures, supports refer to the part of the structure which may help other parts to resist loads.

- Roller Supports
- Hinged Supports
- Fixed Supports

Roller Supports:

- Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped at any angle.

- The resulting reaction force is always a single force that is perpendicular to, and away from, the surface.
- Roller supports are commonly located at one end of long bridges.
- This allows the bridge structure to expand and contract with temperature changes.
- The expansion forces could fracture the supports at the banks if the bridge structure was "locked" in place.
- Roller supports can also take the form of rubber bearings, rockers, or a set of gears which are designed to allow a limited amount of lateral movement.
- A roller support cannot provide resistance to lateral forces. Imagine a structure on roller skates.
- It would remain in place as long as the structure must only support itself and perhaps a perfectly vertical load.
- As soon as a lateral load of any kind pushes on the structure it will roll away in response to the force.
- The lateral load could be a shove, a gust of wind or an earthquake.
- Since most structures are subjected to lateral loads it follows that a building must have other types of support in addition to roller supports.

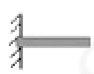
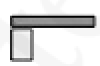
Hinged Supports:

- A hinged support can resist both vertical and horizontal forces but not a moment.
- They will allow the structural member to rotate, but not to translate in any direction.

- Many connections are assumed to be pinned connections even though they might resist a small amount of moment in reality.
- It is also true that a pinned connection could allow rotation in only one direction; providing resistance to rotation in any other direction.
- It is also used in doors to produce only rotation in a door.
- Hinge support reduces sensitivity to earthquake.

Fixed Support:

- Fixed support can resist vertical and horizontal forces as well as moment since they restrain both rotation and translation.
- They are also known as rigid support. For the stability of a structure there should be one fixed support.
- All three equations of equilibrium can be satisfied.
- A flagpole set into a concrete base is a good example of this kind of support. The representation of fixed supports always includes two forces (horizontal and vertical) and a moment.

S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads

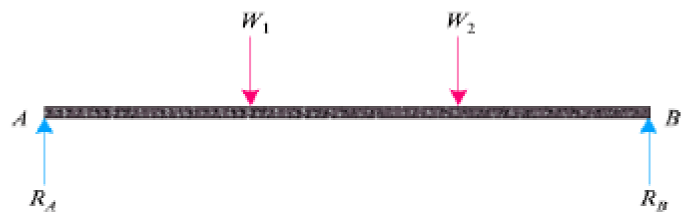
TYPES OF LOADING

Though there are many types of loading, yet the following are important from the subject point of view:

- Concentrated or point load,
- Uniformly distributed load,
- Uniformly varying load.

CONCENTRATED OR POINT LOAD

A load, acting at a point on a beam is known as a concentrated or a point load as shown in Figure.



UNIFORMLY DISTRIBUTED LOAD

A load, which is spread over a beam, in such a manner that each unit length is loaded to the same extent, is known as *uniformly distributed load (briefly written as U.D.L.) as shown in Figure*

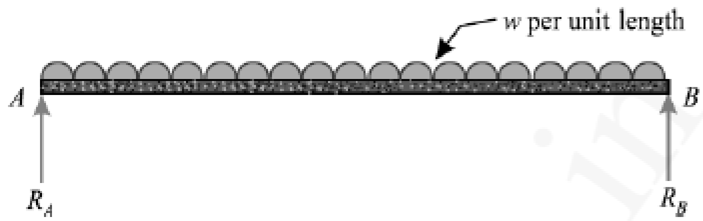


Fig. 7 uniformly distributed load

UNIFORMLY VARYING LOAD

- A load, which is spread over a beam, in such a manner that its extent varies uniformly on each unit length (say from *w_1 per unit length at one support to w_2 per unit length at the other support) is* known as *uniformly varying load as shown in Figure.*
- Sometimes, the load varies from zero at one support to w at the other. Such a load is also called triangular load.

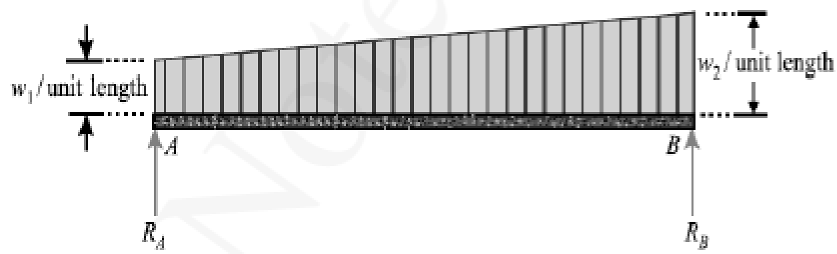


Fig. 8 uniformly varying load

MOMENTS AND COUPLES

A pair of two equal and unlike parallel forces (*i.e.* forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple. As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple.

Mathematically:

Moment of a couple = $P \times a$

Where,

P = Magnitude of the force, and

a = Arm of the couple.

Classification of Couples

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:

1) Clockwise couple, and
2) Anticlockwise couple

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a **clockwise couple** as shown in Figure (a). Such a couple is also called positive couple.

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an **anticlockwise couple** as shown in Figure (b). Such a couple is also called a negative couple.

Characteristics of a couple

A couple (whether clockwise or anticlockwise) has the following characteristics:

- The algebraic sum of the forces, constituting the couple, is zero.
- The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
- A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
- Any no. of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

VARIGNON's THEOREM

Moment of a force about any point is equal to the sum of the moments of the components of that force about the same point. To prove this theorem, consider the force **R** acting in the plane of the body shown in Figure.1. The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point O is

$\mathbf{M_0} = \mathbf{r} \times \mathbf{R}$

Because $\mathbf{R} = \mathbf{P}+\mathbf{Q}$, we may write

$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P}+\mathbf{Q})$

Using the distributive law for cross products, we have

$\mathbf{M_0} = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$

This says that the moment of R about O equals the sum of the moments about O of its components P and Q.

This proves the theorem. Varignon's theorem need not be restricted to the case of two components, but it applies equally well to three or more where we take the clockwise moment sense to be positive.

Theorem of Varignon's

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to some centre.

Introduction

In our day-to-day work, we see that whenever we apply a force on a body, it exerts a reaction, *e.g.*, when a ceiling fan is hung from a girder, it is subjected to the following two forces:

1. Weight of the fan, acting downwards, and
2. Reaction on the girder, acting upwards.

A little consideration will show, that as the fan is in equilibrium therefore, the above two forces must be equal and opposite. Similarly, if we consider the equilibrium of a girder supported on the walls, we see that the total weight of the fan and girder is acting through the supports of the girder on the walls. It is thus obvious, that walls must exert equal and upward reactions at the supports to maintain the equilibrium. The upward reactions, offered by the walls, are known as support reactions. As a matter of fact, the support reaction depends upon the type of loading and the support.

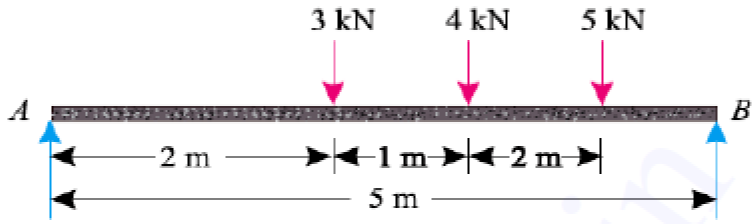
TYPES OF END SUPPORTS OF BEAMS

Though there are many types of supports, for beams and frames, yet the following three types of supports are important from the subject point of view:

1. Simply supported beams,
2. Roller supported beams, and
3. Hinged beams

Worked out examples

A simply supported beam AB of span 5 m is loaded as shown in Figure. Find the reactions at A and B.



Solution:

Given: Span (l) = 5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve analytically only. We know that anticlockwise moment due to R_B about A

$= R_B \times l = R_B \times 5$

$= 5\ R_B$ kN-m ...(i)

And sum of the clockwise moments about A,

$= (3 \times 2) + (4 \times 3) + (5 \times 4)$

$= 38$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii),

$5\ R_B = 38$

$$R_B = \frac{38}{5} = 7.6 \text{ kN}$$

$$R_A = (3 + 4 + 5) - 7.6 = 4.4 \text{ kN}$$

Equivalent Force Couple System

Every set of forces and moments has an **equivalent force couple** system. This is a single force and pure moment (couple) acting at a single point that is **statically equivalent** to the original set of forces and moments.

Any set of forces on a body can be replaced by a single force and a single couple acting tha is statically equivalent to the original set of forces and moments. This set of an equivalen force and a couple is known as the equivalent force couple system.

To find the equivalent force couple system, you simply need to follow the steps below.

1. First, choose a point to take the equivalent force couple system about. Any point will work, but the point you choose will affect the final values you find for the equivalent force couple system. Traditionally this point will either be the center of mass of the body or some connection point for the body.
2. Next resolve all the forces not acting though that point to a force and a couple acting at the point you chose.
3. To find the "force" part of the equivalent force couple system add together all the force vectors. This will give you the magnitude and the direction of the force in the equivalent force couple system.
4. To find the "couple" part of the equivalent force couple system, add together any moment vectors (this could be moments originally acting on the body or moments from the resolution of the forces into forces and couples). This will give you the magnitude and direction of the pure moment (couple) in the equivalent force couple system.

Unit-III Distributed forces

INTRODUCTION

An important part of the job of a skilled construction tradesperson involves making measurements based on instructions such as blueprints and then building based on those measurements. Before you begin construction, one of the challenges may be to take those measurements and to make calculations such as perimeter, area and volume. For example, to make a window frame, a glazier must calculate the perimeter around the glass in order to know how much trim will be needed. A reinforcing rod worker would need to calculate the total area of concrete coverage in order to determine the number of reinforcing rods to use.

This skill sheet reviews the steps in finding the perimeter, area and volume of simple two and three dimensional geometric figures, including:

1. Two dimensional figures
2. Finding the perimeter
3. Finding the area
4. Three dimensional figures
5. Finding the surface area
6. Finding the volume

TWO DIMENSIONAL GEOMETRIC FIGURES

A simple, closed, two dimensional (flat) figures with three or more straight sides is called a polygon. Triangles, squares, rectangles, and parallelograms (figures with 2 pair of opposite sides parallel) are all examples of polygons. A circle is also a flat, closed figure but it is a curve, consisting of points that are all the same distance from the centre.

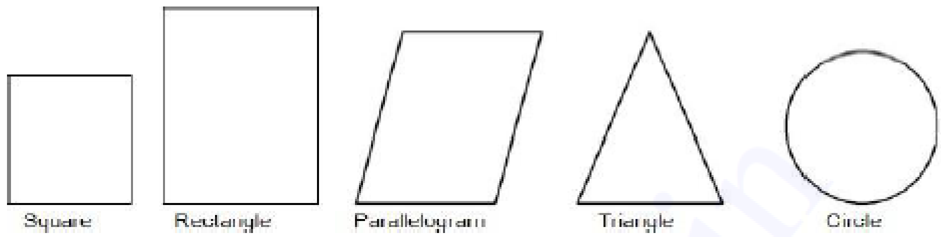


FIGURE 1: Some Simple Geometric Shapes

These figures can be measured in different ways.

a. Whenever we use measurements to make calculations with geometric figures, all measurements must be in the same linear units.
b. The units might be meters or centimeters, but they can't be a mix of meters and centimeters.

FINDING THE PERIMETER

The **perimeter** (P) of any polygon is the distance around its boundary. Perimeter is found by adding together the lengths of the sides.

Perimeter of a Rectangle

A **rectangle** is a polygon with four 90° (right angles) and with each pair of parallel sides the same length (see Figure 1).This means that we can find the perimeter of a rectangle by adding the lengths of the two long side to the lengths of the two shorter side.

The perimeter of a rectangle equals twice the length (l) added to twice the width (w). The formula is written in two forms:

$P = 2l + 2w$ or $P = 2(l + w)$

where

P is the perimeter, l is the length and w is the width of the rectangle.

Note: When finding perimeter, all units must be the same. If the length is measured in feet and the width in yards, one unit must be changed to that of the other.

Example: Find the perimeter of a house that is 30 m long and 16 m wide.

$P = 2l + 2w$

$= 2(30 \text{ m}) + 2(16 \text{ m})$

$= 60 \text{ m} + 32 \text{ m}$

$= 92 \text{ m}$

The perimeter is 92 m.

Example: Find the amount of fencing required to close in a space that is 400 yd wide and 1500 ft long.

Known:

$l = 1500$ ft

$w = 400$ yd $= 1200$ ft   400 yd x3 = 1200 ft

Find perimeter (P)

$P = 2(l + w)$

$= 2\,(1500 \text{ ft} + 1200 \text{ ft})$

$= 2(2700 \text{ ft})$

$= 5400 \text{ ft}$

The space will require 5400 ft of fencing.

<u>**Perimeter of a Square**</u>

A square is a rectangle with all four sides the same length.

To find the perimeter of a square, multiply the length by 4.

Perimeter of a square = $4l$

FINDING THE AREA

The area of a polygon is the measure of the surface inside the boundary. The units of area are squared units.

Area of a Rectangle

The area of a rectangle is the amount of surface enclosed within its boundaries of **length** and **width.**

Example: The area of a room is the amount of floor space it has.

Area is calculated by multiplying the length of the rectangle times its width.

The formula for area is:

$A = lw$

Note: When finding the area of a rectangle, the units used to measure the length and the width must be the same. If the length is in meters, the width must also be in meters. If the units are different, one must be converted to the other before you can multiply.

Example: Find the area of a rectangle that is 52 cm long and 44 cm wide.

(The units are the same so we don't have to convert.)

Draw the rectangle

THREE DIMENSIONAL FIGURES

A closed, solid geometric figure has three dimensions. It has length, width and height or depth. Some solid figures are the cube, the rectangular solid, the cylinder, the cone and the sphere.

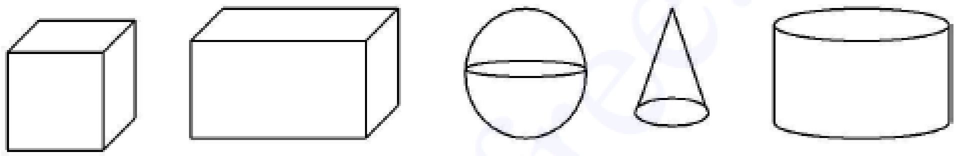


FIGURE 2: Solid Geometric Figures

SURFACE AREA OF THREE DIMENSIONAL FIGURES

The surface area of a three dimensional figure is the combined areas of all the outside surfaces or faces of the figure. When finding the surface area, all measurements must be in the same linear units. The answer will be in square units.

Finding the surface area of a rectangular solid

To find the total area of the outside surface of a rectangular solid, we have to find the areas of each face of the figure.

1. First find the area of the front surface by multiplying the length times the height.
   - The back surface is the same area, so multiply that answer by 2.
2. Next find the area of one side by multiplying the width times the height.
   - Since the opposite side is the same, multiply the answer by 2.
3. Now find the base by multiplying the length times the width.
   - The top is the same as the base, so multiply that answer by 2 also.

Example: Find the total surface area of a cube whose edges measure 10 in.

Known:

Edges of cube = 10 in

Find: A = 6(l

Surface area of cube 2)

$A = 6\,(l^2)$

$= 6(10^2)$

$= 6(100)$

$= 600$ sq in.

Finding the surface area of a cylinder

The surface area of a cylinder consists of the outside curved surface, which is actually a rectangle if it is straightened, and the circular areas at the top and bottom.

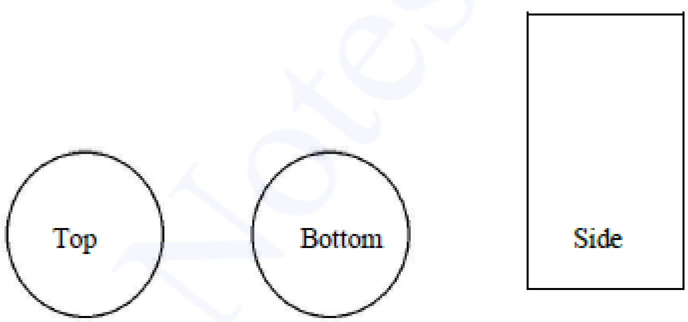


FIGURE 3: Finding the surface area of a cylinder

To find the surface area of a cylinder:

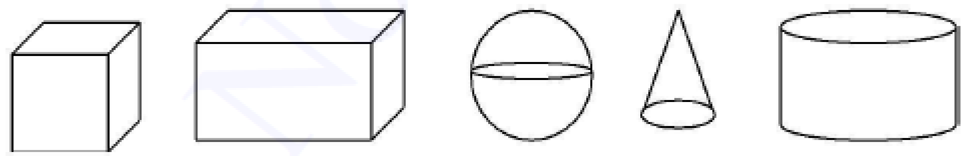
1. Find the area of each of the top and bottom circles.
2. Find the area of the rectangular side:
3. Add the areas together.

1. **To find the area of the top and bottom:** Use the formula $A = \pi r^2$. A cylinder has two circles (the top and the bottom), so we need to find the two areas, $2\pi r^2$.

Note: $\pi = 3.14$

VOLUME OF THREE DIMENSIONAL GEOMETRIC FIGURES

The **volume** or capacity of a solid figure is the amount of space contained within its boundaries. To calculate volume, multiply length times width times depth. Since each linear measurement has a unit, the units in the answer become cubic units. For example, meters x meters x meters equal cubic meters. The short form for cubic units such as cubic inches is in^3 or cu in.



Cube Rectangular solid Sphere Cone Cylinder

FIGURE Solid Geometric Figures

Volume of a rectangular solid

The volume of a rectangular solid equals the length times the width times the height.

$V = lwh$

Example: Find the volume of a rectangular solid 9 m long, 4 m wide and 3 m high.

$V = lwh$

$= 9 \times 4 \times 3$

Volume of a cube

The volume of a cube equals the length of one edge cubed. The formula is:

$V = l^3$

Example: Find the volume of a cube whose length measures 2 m.

$V = l^3$

$= 2^3$

$= 8\ m^3$

Second Moment

If any quantity is multiplied by the distance from the axis s-s twice, we have a second moment. Mass multiplied by a distance twice is called the moment of inertia but is really the second moment of mass. The symbol for both is confusingly a letter I.

$$I = A\,k^2$$

Parallel axis theorems

The moment of inertia of any object about an axis through its center of mass is the minimum moment of inertia for an axis in that direction in space. The moment of inertia about any axis parallel to that axis through the center of mass

Perpendicular axis theorems

For a planar object, the moment of inertia about an axis perpendicular to the plane is the sum of the moments of inertia of two perpendicular axes through the same point in the plane of the object. The utility of this theorem goes beyond that of calculating moments of strictly planar objects. It is a valuable tool in the building up of the moments of inertia of three dimensional objects such as cylinders by breaking them up into planar disks and summing the moments of inertia of the composite disks.

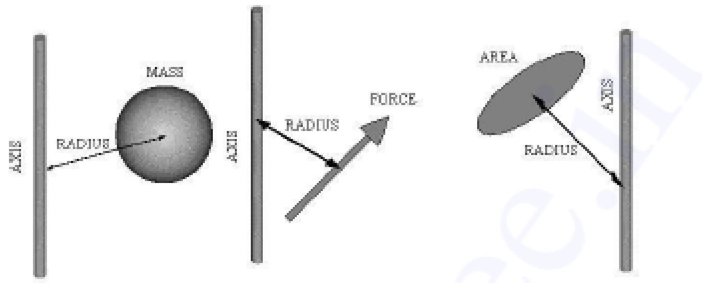
$$I_z = I_x + I_y$$

Composite bodies: If a body is composed of several bodies, to calculate the moment of inertia about a given axis one can simply calculate the moment of inertia of each part around the given axis and then add them to get the mass moment of inertia of the total body.

CENTROIDS AND FIRST MOMENTS OF AREA

A moment about a given axis is something multiplied by the distance from that axis measured at 90^{o} to the axis.

The moment of force is hence force times distance from an axis.
The moment of mass is mass times distance from an axis.
The moment of area is area times the distance from an axis.



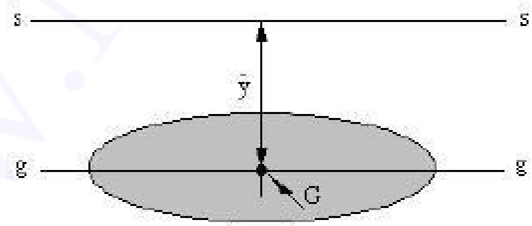
In the case of mass and area, the problem is deciding the distance since the mass and area are not concentrated at one point.

The point at which we may assume the mass concentrated is called the centre of gravity.

The point at which we assume the area concentrated is called the centroid.

Think of area as a flat thin sheet and the centroid is then at the same place as the centre of gravity. You may think of this point as one where you could balance the thin sheet on a sharp point and it would not tip off in any direction.

This section is mainly concerned with moments of area so we will start by considering a flat area at some distance from an axis as shown



The centroid is denoted G and its distance from the axis s-s is y. The axis drawn through G parallel to s-s is the axis g-g. The first moment of area about the axis s-s is the product of area A and distance.

1st moment of area = A

$\bar{y}$ From this we may define the distance y.

$\bar{y}$ = 1st moment of area/Area.

For simple symmetrical shapes, the position of the centroid is obvious.

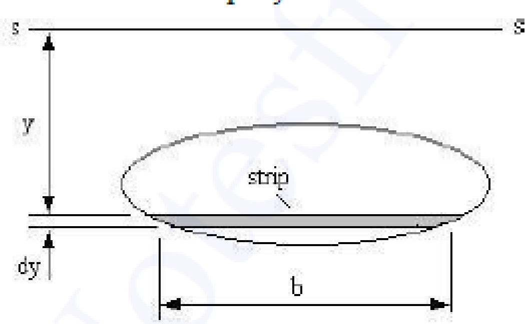
SECOND MOMENTS OF AREAS

If any quantity is multiplied by the distance from the axis s-s twice, we have a second moment. Mass multiplied by a distance twice is called the moment of inertia but is really the second moment of mass. We are concerned here with area only and the area multiplied by a distance twice is the second moment of area. The symbol for both is confusingly a letter I.

The above statement is over simplified. Unfortunately, both the mass and area are spread around and neither exists at a point. We cannot use the position of the centroid to calculate the 2^{nd} moment of area. Squaring the distance has a greater effect on parts further from the axis than those nearer to it. The distance that gives the correct answer is called the **RADIUS OF GYRATION** and is denoted with a letter k. This is not the same as $\bar{y}$.

The simplest definition of the 2^{nd}. moment of area is $I= A k^2$

Whilst standard formulae exist for calculating the radius of gyration of various simple shapes, we should examine the derivations from first principles. We do this by considering the area to be made up of lots of elementary strips of width b and height dy. The distance from the axis s-s to the strip is y.



The area of the strip = dA = b dy

1^{st} moment of area of strip = y dA = by dy

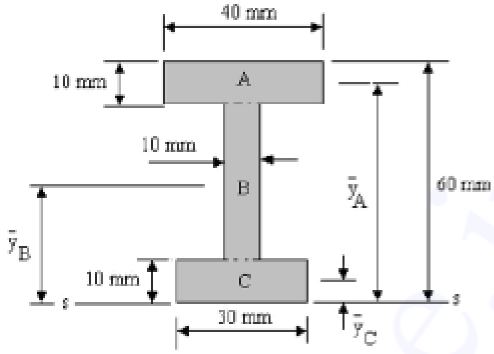
2^{nd} moment of area of strip = y2 dA = b y2 dy

For the whole area, the 2^{nd} moment of area is the sum of all the strips that make up the total area. This is found by integration.

$$\mathbf{I = \int b\, y2\, dy}$$

The limits of integration are from the bottom to the top of the area. This definition is important because in future work, whenever this expression is found, we may identify it as I and use standard formulae when it is required to evaluate it. We should now look at these.

Calculate the 2^{nd} moment of area for the same shape as in EXAMPLE 1.3. about the axis s-s



<u>SOLUTION</u>

The table shows the previous solution with extra columns added to calculate the second moment of area using the parallel axis theorem. In the new column calculate the second moment of area for each part (A, B and C) about each's own centroid using $BD^3/12$. In the next column calculate Ay^2.

Part	Area	$\bar{y}$	$A\bar{y}$	$I_{gg}=BD^3/12$	$A\bar{y}^2$	Iss
A	400	55	22000	3333	1210000	1213333
B	400	30	12000	53333	360000	413333
C	300	5	1500	22500	7500	30000
Total	1100		35500			1656666

The total 2^{nd} moment of area is 1656666 mm^4 about the bottom. We require the answer about the centroid so we now use the parallel axis theorem to find the 2^{nd} moment about the centroid of the whole section.

The centroid is 32.77 mm from the bottom edge.

$$I_{gg} = I_{ss} - A\bar{y}^2$$

$$I_{gg} = 1656666 - 1100 \times 32.77^2$$

$$I_{gg} = 475405.8 \text{ mm}^4 = 475.4 \times 10^{-9} \text{ m}^4$$

Note $1 \text{ m}^4 = 10^{12} \text{ mm}^4$

Unit IV. FRICTION

FRICTION OF FORCE

Friction is the force that opposes the motion of an object. To stop a moving object, a force must act in the opposite direction to the direction of motion. For instance, if a book is pushed across a desk, the book will move. The force of the push moves the book. As the book slides across the desk, it slows down and stops moving. The force that opposes the motion of an object is called friction. Figure 1 shows a typical sketch for friction on a body.

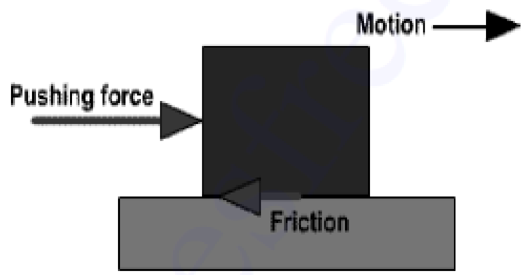


Figure 1 Friction on a body

Example: A block of weight is placed on a rough horizontal plane surface and force P is applied on the horizontal such that the block tends to move.

If P is small, the body will not move as the force of friction acting on a body in the direction opposite to P will be more than P. but if the magnitude of P goes on increasing, a stage comes, when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called *limiting force of friction*. It is denoted by F.

Resolving the forces on the body vertical and horizontal, we get,

$\sum H = 0$	$\sum V = 0$
$-F+P = 0$	$-W+R = 0$
Therefore, $P = F$	Therefore, $R = W$

If the magnitude of P is further increased the body will start moving. The force of Friction, acting on the body when the body is moving, is called as *Kinetic Friction.*

COEFFICIENT OF FRICTION

The coefficient of friction is a number which represents the friction between two surfaces. Between two equal surfaces, the coefficient of friction will be the same. The symbol usually used for the coefficient of friction is m

The maximum frictional force (when a body is sliding or is in limiting equilibrium) is equal to the coefficient of friction × the normal reaction force.

$$F = \mu R$$

Where μ is the coefficient of friction and R is the normal reaction force.

This frictional force, F, will act parallel to the surfaces in contact and in a direction to oppose the motion that is taking/ trying to take place as given in Figure 3.

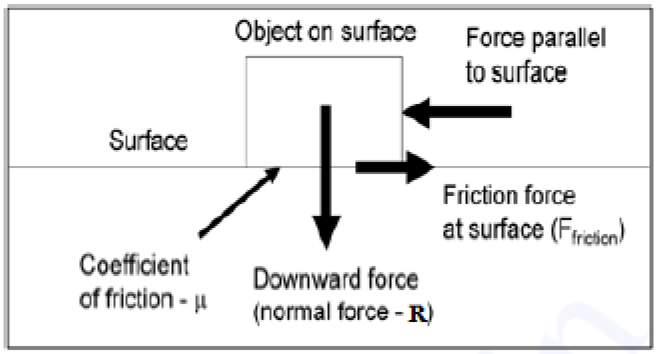


Figure 3 Coefficient of Friction

ANGLE OF FRICTION:

The angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction is called as angle of friction.

Let R be Normal reaction

μ be the coefficient of friction

F be the Force of Friction = μR

In this case the R will not be equal to the weight of the body. The R is obtained by resolving the forces on block horizontally and vertically. The forces P is resolved in two components i.e. $P \cos \theta$ in horizontal direction and $P \sin \theta$ in vertical direction.

$\sum H=0$ $\quad -F+P\cos\theta=0$

Therefore, $\quad F = P\cos\theta$ or $\mu R = P\cos\theta$

$\sum V=0$ $\quad R+P\sin\theta-W = 0$

Therefore, $\quad R = W-P\sin\theta$

From the above equations it is clear that R is not equal to W and the values of W, P and θ are known, R can be obtained which is used to determine μ.

$$\boxed{\mu = F/R}$$

Note:

i. F always equal to μR

ii. R always not equal to W

CONE OF FRICTION:

The right circular cone with vertex at the point of contact of the two bodies, axis in the direction of R and the semi-vertical angle is α. The inverted cone with semi central angle α equal to limiting frictional angle α, is called cone of friction.

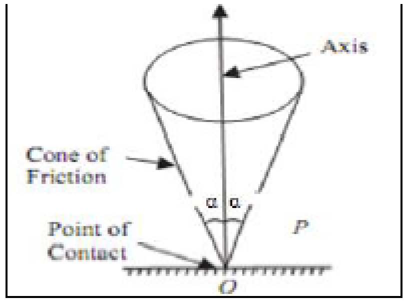


Figure6 Cone of Friction

ANGLE OF REPOSE

It is the angle of inclination (α) of the plane to the horizontal, at which the body just begins to move down the plane. A little consideration will show that the body will begin to move down the plane, if the angle of inclination (α) of the plane is equal to the angle of friction (ϕ). From Fig. 1.18, we find that

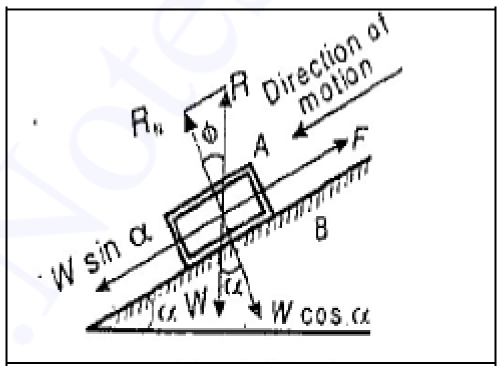


Figure 7 Angle of repose

$W \sin \alpha = F = \mu R = \mu W \cos \alpha$

$\tan \alpha = \mu$

Therefore, $\tan \alpha = \mathbf{F/R}$

Angle of repose is defined as the minimum angle made by an inclined plane with the horizontal such that an object placed on the inclined surface just begins to slide.

- **Relation between Angle of Friction and Angle of Repose**

Let us consider a body of mass 'W' resting on a plane.

Also, consider when the plane makes 'α' angle with the horizontal; the body just begins to move.

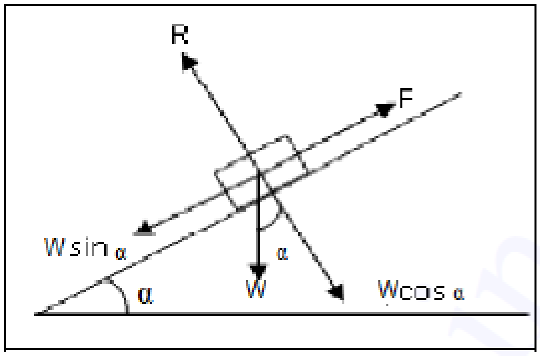


Figure 8 **Relation between φ and α**

Let 'R' be the normal reaction of the body and 'F' be the frictional force. Now α is the inclination of the plane with the horizontal.

Here,

$W \sin \alpha = F = \mu R = \mu W \cos \alpha$

$\tan \alpha = \mu = \tan \varphi$

Therefore, $\alpha = \varphi$

Angle of Friction, φ = Angle of repose, α

LAWS OF SOLID FRICTION or LAWS OF COLUMB FRICTION

1. The force of friction acts in opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as surface is at rest.
3. When the surface is on the point of motion, the force of friction is said to be maximum and this maximum frictional force is called as limiting friction force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and area of the surface in contact.
6. The friction of force is independent of velocity of sliding.

i. Static Friction

Static friction comes into play when a body is forced to move along a surface but movement does not start. The magnitude of static friction remains equal to the applied external force and the direction is always opposite to the direction of motion. The magnitude of static friction depends upon μ_s (coefficient of static friction) and R (normal reaction of the body).

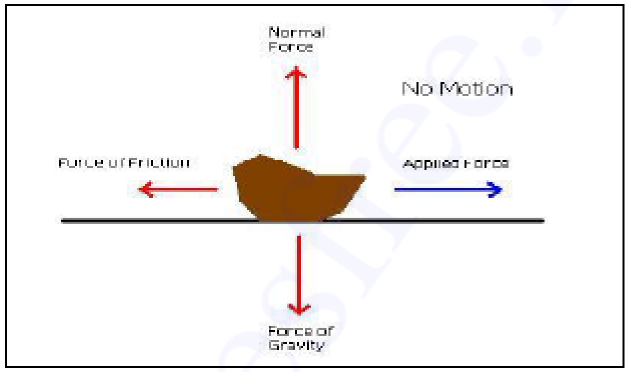


Figure 9 Static Friction

ii. Kinetic Friction

Kinetic friction denoted as μk comes into play when a body just starts moving along a surface. When external applied force is sufficient to move a body along a surface then the force which opposes this motion is called as kinetic frictional force.

Magnitude of kinetic frictional force $F_k = \mu_k$ R

Where μ_k is coefficient of kinetic frictional force and N is the net normal reaction on the body. The magnitude of kinetic frictional force is always less than magnitude of static frictional force. When value of applied net external force F is more than F_k then body moves with a net acceleration and when these forces are equal then body moves with a constant velocity.

2. LADDER FRCITION

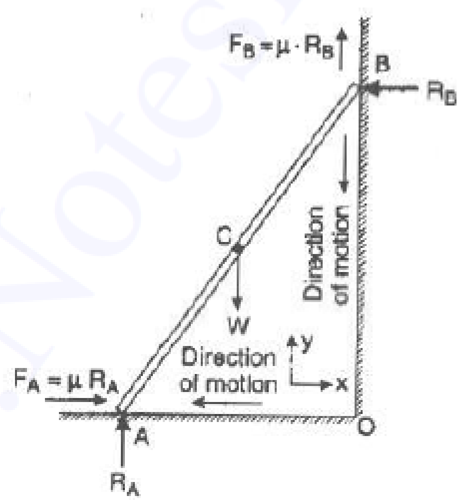
A commonly used ladder has two uprights of bamboo or iron, connected by a number of parallel bars to provide steps. These bars are called as rungs. As shown in figure, a ladder AB rests with its one end B against a vertical wall OB and the other end A on a horizontal plane OA.

In case of slip, the end B will move downwards and A outwards. In this situation of ladder is under the action of following forces.

(1) Normal reactions R_A and R_B

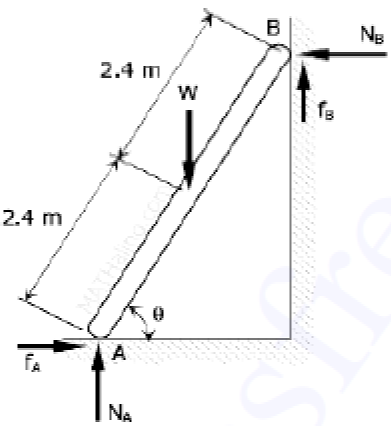
(2) Frictional forces caused by R_A and R_B.

(3) Weight of the ladder = W.



Problems on Ladder Friction:

Problem No.1 A uniform ladder 4.8 m ft long and weighing W N is placed with one end on the ground and the other against a vertical wall. The angle of friction at all contact surfaces is 20°. Find the minimum value of the angle θ at which the ladder can be inclined with the horizontal before slipping occurs.



<u>Solution</u>

Coefficient of friction

$\mu=\tan\phi=\tan 20^\circ$

$\mu=0.364$

Friction forces at each end of the ladder

$F_A=\mu N_A=0.364N_A$

$F_B=\mu N_B=0.364N_A$

$\Sigma H=0$

$N_B=F_A$

$N_B=0.364N_A$

$\Sigma V=0$

$N_A+F_B= W$

$N_A+0.364N_A= W$

$N_A+0.364(0.364N_A) = W$

$1.1325N_A=W$

Unit V- Dynamics of Particles

INTRODUCTION

The dynamics of particles deals with the study of forces acting on a body and its effects, when the body is in motion. It is further divided into Kinematics and kinetics.

Kinematics – The study of motion of body without considering the forces which cause the motion of the body.

Kinetics – The study of motion of body with considering the external forces which cause the motion of the body.

Plane motion – If a particle has no size but mass it is considered to have only plane motion, not rotation. In this chapter the study motion of particles with only plane motion is taken without considering force that cause motion i.e., Kinematics.

The plane motion of the body can be sub divided into two types

(i) Rectilinear motion

(ii) Curvilinear motion

1. RECTILINEAR MOTION *(Straight Line Motion)* - It is the motion of the particle along a straight line.

Example: A car moving on a straight road

A stone falls vertically downwards

A ball thrown vertically upwards

Displacement –The displacement of a moving particle is the change in its position, during which the particle remain in motion. It is the vector quantity, i.e., it has both magnitude and direction. The SI unit for displacement is the metre (m).

Velocity – The rate of change of displacement is velocity. It is the ratio between distances travelled in particular direction to the time taken. It is also a vector quantity, i.e., it has both magnitude and direction. The SI unit for velocity is the metre/second (m/sec) or kilometer/hour (km/h)

Acceleration – The rate of change of velocity is acceleration. It is the ratio between changes in velocity to the time taken. The change in velocity means the difference between final velocity and initial velocity. It is also a vector quantity. The SI unit for acceleration is the metre/second^2 (m/sec^2).

Retardation – The negative acceleration is retardation. It occurs when final velocity is less than initial velocity ($v<u$).

Speed – The distance travelled by a particle or a body along its path per unit time. It is a scalar quantity, i.e., it has only magnitude. The SI unit for speed is the metre/second (m/sec) or kilometer/hour (km/h)

RELATIVE MOTION

A body is said to be in motion if it changes its position with respect to the surroundings, taken as fixed. This type of motion is known as the individual motion of the body. An example of relative motion is how the sun appears to move across the sky, when the earth is actually spinning and causing that apparent motion. Usually, we consider motion with respect to the ground or the Earth. Within the Universe there is no real fixed point. The basis for Einstein's Theory of Relativity is that all motion is relative to what we define as a fixed point.

Relative velocity – Basic concept

Let's consider two motors A and B are moving on a road in same direction moving in uniform speed. Let the uniform velocities of motors A and B be u m/sec and v m/sec respectively (assume $v > u$)

Now, a person standing on the road looks at the motor A and finds that it is going at a speed of u m/sec. Similarly, looks at motor B and finds it is going at a speed of v m/sec separately. But for the driver of motor A, the motor B seems to move faster than him at the rate of only (v - u) m/sec. i.e., the motor A is imagined to be at ret or, the driver of motor A forgets his own motion.

Relative velocity of B with respect to A is (v-u). It is denoted by $V_{B/A}$

$$\therefore V_{B/A} = V_B - V_A = (v - u) \text{ m/sec}$$

Similarly for the driver of motor B, the motor A seems to move slower (assume $u < v$) than him at the rate of only (u - v) m/sec. i.e., the motor B is imagined to be at ret or, the driver of motor B forgets his own motion.

Relative velocity of A with respect to B is (v - u). It is denoted by $V_{A/B}$

$$\therefore V_{A/B} = V_A - V_B = (u - v) \text{ m/sec}$$

MATHEMATICAL EXPRESSION FOR VELOCITY AND ACCELERATION

(i) Velocity, v = ds/dt

(ii) Acceleration, $a = d^2s/dt^2$

Where, s – distance travelled by a particle in a straight line.

t – time taken by the particle to travel the distance 's'

Equation of motion in straight line

Let, u – initial velocity (m/sec)

v – Final velocity (m/sec)

s – Distance travelled by a particle (m)

t – Time taken by the particle to change from u to v (second)

a – acceleration of the particle (m/sec^2)

$$v = u + at$$

$$s = ut + ½$$

$$(at^2)\ v^2 = u^2$$

Note: 1) If a body starts from rest, its initial velocity is zero i.e., u=0

2) If a body comes to rest, its final velocity is zero i.e., v=0

PROBLEMS

Example1. A car is moving with a velocity of 20 m/sec. the car is brought to rest by applying brakes in 6 seconds. Find i) retardation ii) distance travelled by the car after applying brakes.

Given data

u = 20 m/s

v = 0 (car is brought to rest)

t = 6 sec

Solution

i) Retardation or negative acceleration

Using equation of motion, v = u+at

$$0 = 20 + (a*6)$$

$$a = -3.33 \text{ m/sec}^2$$

Retardation = 3.33 m/sec²

ii) Distance travelled

Using equation of motion, $s = ut + 1/2\,(at^2)$

$$= (20*6) + 1/2(3.33*6^2)$$

$$= 60 \text{ m}$$

Distance, s = 60 m

Example2. A train starts from rest and attains a velocity of 45 kmph in 2 minutes, with uniform acceleration. Calculate i) acceleration ii) distance travelled and iii) time required to reach a velocity of 36n kmph.

Given data

Initial velocity, u = 0 (train starts from rest)

Final velocity, v = 45 kmph = 12.5 m/sec

Time taken , t = 2 minutes = 120 seconds

Solution

i) Acceleration, a

Using equation of motion, $v = u + at$

$$A = 0.104 \text{ m/sec}^2$$

ii) Distance travelled in 2 minutes, s

Using equation of motion, $s = ut + 1/2\,(at^2)$

$$S = 748.8 \text{ m}$$

iii) Time required to attain velocity of 36 kmph

u = 0

v = 36 kmph = 10 m/sec

Using equation of motion, $v = u + at$

$$t = 96.15 \text{ sec}$$

2. CURVILINEAR MOTION - It is the motion of the particle along a curved path. It has two dimensions.

Example: A stone thrown into the air at an angle

Throwing paper airplanes in air

There are two systems involved in curvilinear motion. They are

(i) Cartesian systems (rectangular coordinates)

(ii) Polar system (radial coordinates)

CARTESIAN SYSTEMS

It is a rectangular coordinate system which has the horizontal component in X-axis and vertical component in Y-axis.

Horizontal component of velocity, $V_x = dx/dt$

Vertical component of velocity, $V_y = dy/dt$

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

Angle of inclination of velocity with X-axis, $\alpha=\tan^{-1}(V_y/V_x)$

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)}$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1} (a_y/a_x)$

PROBLEMS

Example1. The portion of a particle along a curved path is given by the equations $x=t^2+8t+4$ and $y=t^3+3t^2+8t+4$. Find the i) initial velocity, u ii) velocity of the particle at t=2 sec iii) acceleration of the particle at t=0 and iv) acceleration of the particle at t= 2 sec.

Given data

$x=t^2+8t+4$

$y=t^3+3t^2+8t+4$

Solution

Horizontal component of velocity , $V_x = dx/dt = d(t^2+8t+4)/dt = 2t+8$-------------- (1)

Vertical component of velocity, $V_y = dy/dt = d(t^3+3t^2+8t+4)/dt = 3t^2+6t+8$------- (2)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = d(2t+8)/dt = 2$ ---------------(3)

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = d(3t^2+6t+8)/dt = 6t+6$------ (4)

i) **Initial velocity, u**

Put t = 0 in equation (1) and (2)

$V_x = 2t+8$

Now, $\mathbf{V_x = 8}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 8}$ **m/sec**

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

$= \sqrt{(8^2+8^2)}$

$\mathbf{V = 11.31}$ **m/sec**

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$= \tan^{-1}(8/8)$

$\boldsymbol{\alpha = 45^\circ}$

ii) **Velocity at t= 2 sec**

Put t = 2 seconds in equation (1) and (2)

$V_x = 2t+8$

Now, $\mathbf{V_x = 12}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 32}$ **m/sec**

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

$= \sqrt{(12^2+32^2)}$

$V = 34.17$ m/sec

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$= \tan^{-1}(32/12)$

$\alpha = 69.4^\circ$

iii) **Acceleration at t=0**

Put t = 0 in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2$ m/sec^2

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 6$ m/sec^2

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)} = \sqrt{(2^2+6^2)} = 6.34$ m/sec^2

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(6/2) = 71.56^\circ$

iv) **Acceleration at t = 2 sec**

Put t = 2 sec in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2$ m/sec^2

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 18$ m/sec^2

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)} = \sqrt{(2^2+18^2)} = 18.11$ m/sec^2

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(18/2) = 83.66^\circ$

PROJECTILES

The projectile is an example of curvilinear motion of a particle in plane motion. The motion of a particle is neither vertical nor horizontal, but inclined to the horizontal plane

Definitions

Projectile – A particle projected in space at an angle to the horizontal plane.

Angle of projection means the angle to the horizontal at which the projectile is projected. It is denoted by a.

Velocity of projectile means the velocity with which the projectile is thrown into space. It is denoted by u (m/sec)

Trajectory means the path described by the projectile.

PROBLEMS

Example1. A particle is projected with an initial velocity of 60 m/sec, at an angle of 75° with the horizontal. Determine i) the maximum height attained by the particle ii) horizontal range of the particle iii) time taken by the particle to reach highest point iv) time of flight

Given data

Initial velocity, u = 60 m/sec

Angle of projection, $\alpha = 75°$

Solution

i) the maximum height attained by the particle

$h_{max} = u^2 \sin^2\alpha / 2g = 171.19$ m (take $g = 9.81$ m/sec^2)

ii) horizontal range

$R = u^2 \sin 2\alpha / g = 183.48$ m

iii) time taken to reach highest point

$t = u \sin \alpha / g = 5.9$ sec

iv) time of flight

$T = 2 u \sin \alpha / g = 11.8$ sec

Example2. A particle is projected with an initial velocity of 12 m/sec at an angle α with the horizontal. After sometime the position of the particle is observed by its x and y distances of 6m and 4 m respectively from the point of projection. Find the angle of projection?

Given data

Initial velocity, u = 12 m/sec

Horizontal distance, x = 6 m

Vertical distance, y = 4 m

Solution

If the coordinate points on the projectile path are given, then use equation of trajectory.

Equation of path of projectile (trajectory)

$Y = \tan \alpha\, X - ½ (g^2X/u^2 \cos^2\alpha)$

Put u = 12 m/sec, X = 6 m and Y = 4 m

Take $g = 9.81$ m/sec^2

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Department of Mechanical Engineering

Lecture Notes

Subject Code : ME3391

Subject Name: ENGINEERING THERMODYNAMICS

Sem/Year : 03/II

Regulation : 2021

Unit - I

Basic Concepts And First Law

Basic Concepts - Concepts of continuum - microscopic and macroscopic aproach - path and ~~path~~ point function - Intensive and Extensive properties - total and specific quantities - System and their types (8 mark) - Thermodynamic equilibrium - State, path, process - Quasi-static process - Reversible and Irreversible process - heat and work transfer - Sign convention - Displacement work and other modes of work - Zeroth law of thermodynamics - First law of thermodynamics - Application of closed system (13 mark) - open system (steady flow energy equation) (13 mark) -

Thermodynamics :-

It deals with heat and work transfer in low temperature as well as high temperature. The study of movement of heat energy is called as thermodynamics.

For example - Engine, Air-Conditioner, Turbine.

Microscopic Approach (or) Statistical Thermodynamic :-

In this process the inter molecular structure and the deviation for heatest study is termed as Microscopic Approach, it is also known as Statistical thermodynamics.

Macroscopic Approach (or) classical Thermodynamic :-

In this process the input and output is only considered where the internal changes are neglected, it is known as Macroscopic Approach (or) Classical thermodynamics

Thermodynamic Properties :-

* Force :-

• A Force acting on a body according to Newton's Second law create a motion.

• Every force create a depend upon mass and acceleration.

$$F = ma$$

$$F = kg \times m/s^2$$

$$F = \frac{kgm}{s^2} \qquad F = N$$

$$\left[1N = \frac{1kgm}{s^2} \right]$$

* Density :-

• It is defined as ratio of mass per unit Volume

$$\rho = \frac{m}{V} = \frac{kg}{m^3}$$

$$\rho = kg/m^3$$

* Pressure :-

• It is defined as normal force exerted by a system against unit area.

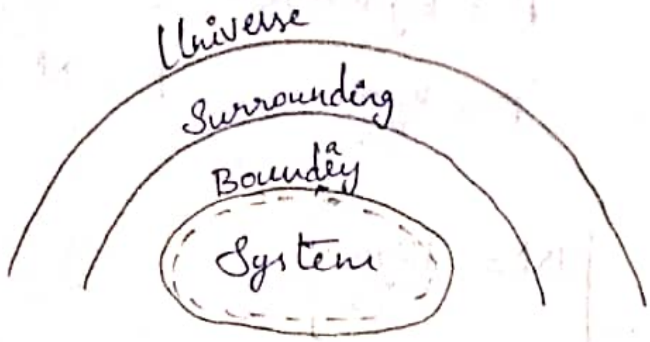
$$p = \frac{F}{A} \qquad p = N/m^2$$

$\left[1 N/m^2 = 1 Pa \right]$ Pa - Pascal

$\left[1 \times 10^5 \frac{N}{m^2} \text{ (or) } Pa = 1 bar \right]$

• the atmospheric pressure is 1.01325 bar

System and Control Volume :- (*) 8mark [Nov, Dec 2016]



* A System is a definite quantity of matter or a region choosed for study.

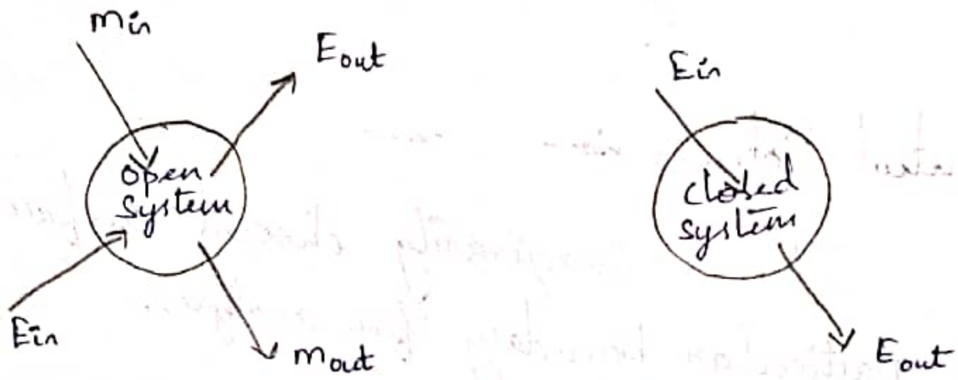
* The Mass of the region outside the system is called Surrounding.

* the Surrounding and System is seperated by Boundary.

* A Boundary is a real or imaginary line, movable or immovable lines which seperates System and Surrounding.

* Above the Surrounding everything is Universe.

Types of System :-



[E - Energy
m - mass]

The System is Sub-divided into three types :
(i) open system
(ii) closed System
(iii) Isolated System

(i) Open System :-

In the System if mass and energy enters and exit the Boundary is termed as open system. eg. compressor

(ii) closed System :-

In the system If only energy enters and exit the System is termed as closed System. eg. Hydraulics

(iii) Isolated System :-

If there is no energy and mass transfer it is Isolated System.

eg. Universe

Control Volume :-

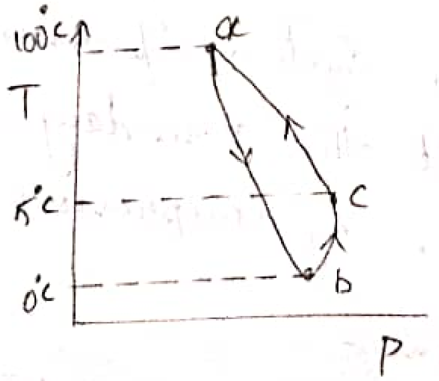
It is a imaginarily choosed surface at particular boundary for analyses.

change of state :-

Any property of a System changes it result in change of state

Path :-

During a change of state it follows a path



Intensive and Extensive Property :-

Intensive Property :-

If a property of a system doesn't depend on mass it is termed as intensive property.

Example : pressure, temperature

Extensive Property :-

If a property of a system depends on mass it is known as extensive property.

Example : density, specific volume.

Homogeneous and Heterogeneous Mixture :-

* Homogeneous Substance throughout the chemical composition has the same physical structure for producing heat energy.

* Heterogeneous if two or more physical structure or chemical composition joints together to produce heat energy.

Thermodynamic Equillibrium:- [May. Jun. 2016, 2014]

• A System is set to exist in State of thermodynamic equillibrium when there is no change in macroscopic property of the System.

• For a system to satisfy thermodynamic equillibrium it should come through three types of equillibrium.

* Mechanical Equillibrium
* Chemical Equillibrium
* Thermal Equillibium

Mechanical Equillibrium :-

If there is no unbalanced forces after the process is completed it is said to be in mechanical equillibrium.

Chemical Equillibrium:-

If there is no chemical change or transfer of chemical after the process is completed it is known as chemical equillibrium.

Thermal Equillibrium:-

If there is no spontaneous change in the property of a System due to heat energy it is termed as thermal Equillibrium.

Non-Equillibrium :-

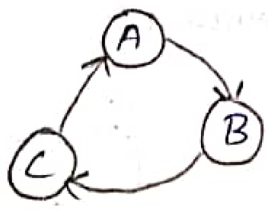
If any one of the condition is violated it is termed as non-equillibrium stage.

Zeroth Law of Thermodynamics :-

"when a body A in thermal equillibrium when body B, also body B is seperately in equillibrium with body C, then A & C will be in equillibrium.

$$A \rightleftharpoons B \qquad B \rightleftharpoons C$$

$$A \rightleftharpoons C$$



First Law of Thermodynamics :-

"When a system undergoes a cyclic process the net heat transfer is equal net work transfer"

$$\oint Q = \oint W$$

$$Q = W + \Delta U$$

Limitation:-

- It does not predict the direction of flow of heat and work.
- Heat and work are mutually convertable.

Perpetual Motion Machine I:-

The machine which violates first law of thermodynamics and produces continuous work for a given heat it is termed as perpetual motion machine I.

Some Important Formula:-

1. $PV = mRT$

2. $\frac{PV}{T} = C$

3. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = C$

[∵ R = gas constant
$R = 0.287$ kJ/kg Kelvin
P = Pressure N/m^2
V = Volume m^3
m = mass kg
T = Temperature k]

3. $R = C_p - C_v$

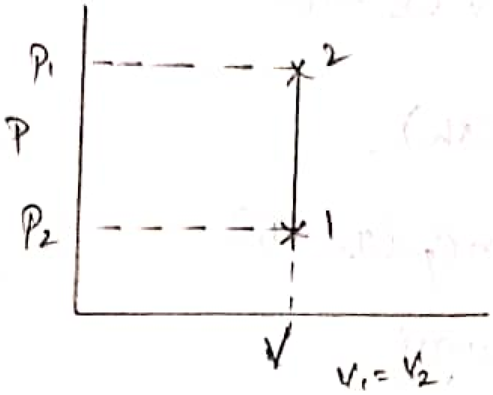
[$C_p = 1.005$ KJ/kg K
$C_v = 0.717$ KJ/kg K]

$\gamma = 1.4$

$n = 1.3$

Constant Volume Process :- [Nov-Dec 2015]

Isochoric



$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_1 = V_2$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$$

work :-

$$W = \int p\,dv$$

$$W = P(V_2 - V_1)$$

$$V_2 = V_1$$

$$\boxed{W = 0}$$

1^{st} law

$$Q = W + \Delta u$$

$$Q = 0 + \Delta u$$

$$\boxed{Q = \Delta u}$$

Heat Q

$$Q = mC_v (T_2 - T_1)$$

Enthalpy (Δh)

$$\Delta h = mC_p (T_2 - T_1)$$

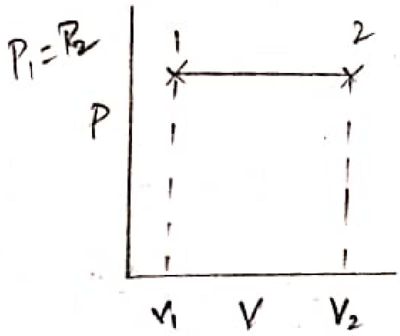
Internal Energy

$$Q = \Delta u$$

$$\Delta u = mC_v (T_2 - T_1)$$

Constant Pressure Process :-

Isobaric



$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

work

$$W = \int P dv$$

$$W = P(V_2 - V_1)$$

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$$PV = mRT$$

$$P(V_2 - V_1) = mR(T_2 - T_1)$$

Heat

$$\boxed{Q = mc_p(T_2 - T_1)}$$

1^{st} law $\therefore R = c_p - c_v$

$$Q = W + \Delta u$$

$$mc_p(T_2 - T_1) = mR(T_2 - T_1) + \Delta u$$

$$mc_p(T_2 - T_1) = m(c_p - c_v)(T_2 - T_1) + \Delta u$$

$$mc_p(T_2 - T_1) = mc_p(T_2 - T_1) - mc_v(T_2 - T_1) + \Delta u$$

$$mc_p(T_2 - T_1) - mc_p(T_2 - T_1) + mc_v(T_2 - T_1) = \Delta u$$

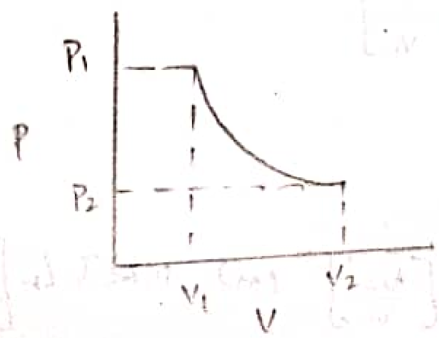
$$\boxed{\Delta u = mc_v(T_2 - T_1)}$$

Enthalpy

$$\boxed{\Delta h = mc_p(T_2 - T_1)}$$

Constant Temperature Process :- [2004, 2008, 2010, 2017]

$$PV = C$$



$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$P_1 V_1 = P_2 V_2$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

work

$$W = \int P dV$$

$$PV = C$$

$$P = \frac{C}{V}$$

$$W = \int \frac{C}{V} dV$$

$$W = C \int \frac{dV}{V}$$

$$= C [\ln V]_{V_1}^{V_2}$$

$$W = C [\ln V_2 - \ln V_1]$$

$$\because \ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

$$W = C \ln\left[\frac{V_2}{V_1}\right]$$

$$PV = mRT$$

$$PV = C$$

$$W = PV \ln\left[\frac{V_2}{V_1}\right] \text{ (or) } mRT_1 \ln\left[\frac{V_2}{V_1}\right]$$

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$$\Delta u = mC_p(T_2 - T_1)$$

$$\Delta u = 0$$

Since Temperature is Constant

1st law

$$Q = W + \Delta u$$

$$Q = W + 0$$

$$Q = W$$

$$Q = PV \ln\left[\frac{V_2}{V_1}\right]$$

Enthalpy

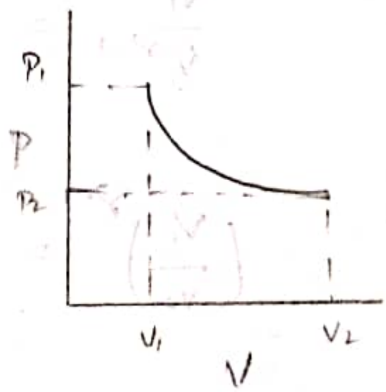
$$\Delta h = mC_p(T_2 - T_1)$$

$$T = C$$

$$\Delta h = 0$$

Reversible Adiabatic or Isentropic :-

$$PV^{\gamma} = C$$



$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\frac{P_1}{P_2} = \frac{V_2^{\gamma}}{V_1^{\gamma}}$$

$$\boxed{\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{\gamma}}$$ relation (1)

$$\frac{PV}{T} = C \quad \text{—} ①$$

$$PV^{\gamma} = C \quad \text{—} ②$$

② ÷ ①

$$\frac{PV^{\gamma}}{PV/T} = C$$

$$PV^{\gamma} \times \frac{T}{PV} = C$$

$$\frac{V^{\gamma} T}{V} = C$$

$$V^{\gamma} V^{-1} T = C$$

$$V^{\gamma-1} T = C$$

$$V_1^{\gamma-1} T_1 = V_2^{\gamma-1} T_2$$

$$\frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{T_2}{T_1}$$

$$\boxed{\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}}$$ relation ②

$$\frac{PV}{T} = C$$

multiply by Power γ.

$$\frac{P^{\gamma} V^{\gamma}}{T^{\gamma}} = C \quad \text{—— (3)}$$

(3) ÷ (2)

$$\frac{\frac{P^{\gamma} V^{\gamma}}{T^{\gamma}}}{PV^{\gamma}} = C$$

$$\frac{P^{\gamma} V^{\gamma}}{T^{\gamma}} \times \frac{1}{PV^{\gamma}} = C$$

$$\frac{P^{\gamma}}{P\,T^{\gamma}} = C$$

$$\frac{P^{\gamma} P^{-1}}{T^{\gamma}} = C$$

$$\frac{P^{\gamma-1}}{T^{\gamma}} = C$$

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\frac{P_1^{\gamma-1}}{P_2^{\gamma-1}} = \frac{T_1^{\gamma}}{T_2^{\gamma}}$$

$$\left[\frac{P_1}{P_2}\right]^{\gamma-1} = \left[\frac{T_1}{T_2}\right]^{\gamma}$$

$$\boxed{\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}}$$ relation ③

relation ① $\Rightarrow \frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{\gamma}$ (or) $\frac{V_2}{V_1} = \left[\frac{P_1}{P_2}\right]^{1/\gamma}$

relation ② $\Rightarrow \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}$ (or) $\frac{V_1}{V_2} = \left[\frac{T_2}{T_1}\right]^{1/\gamma-1}$

relation ③ $\Rightarrow \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}$ (or) $\frac{P_1}{P_2} = \left[\frac{T_1}{T_2}\right]^{\frac{\gamma}{\gamma-1}}$

Work

$$W = \int P dV$$

$$PV^{\gamma} = C$$

$$P = \frac{C}{V^{\gamma}}$$

$$W = \int_{V_1}^{V_2} \frac{C}{V^{\gamma}} dV$$

$$= C \int_{V_1}^{V_2} \frac{1}{V^{\gamma}} dV$$

$$= C \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$= C \left[\frac{V_2^{-\gamma+1}}{-\gamma+1} - \frac{V_1^{-\gamma+1}}{-\gamma+1} \right]$$

$PV^{\gamma} = C$

$$W = PV^{\gamma}\left[\frac{V_2^{-\gamma+1}}{-\gamma+1} - \frac{V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= PV^{\gamma}\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= \left[\frac{P_2 V_2^{\gamma} V_2^{-\gamma+1} - P_1 V_1^{\gamma} V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$W = \left[\frac{P_2 V_2 - P_1 V_1}{-\gamma+1}\right]$$

$$\boxed{W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}}$$

Since adiabatic layer:

$Q = 0$

1st law

$Q = W + \Delta U$

$0 = W + \Delta U$

$W = -\Delta U$

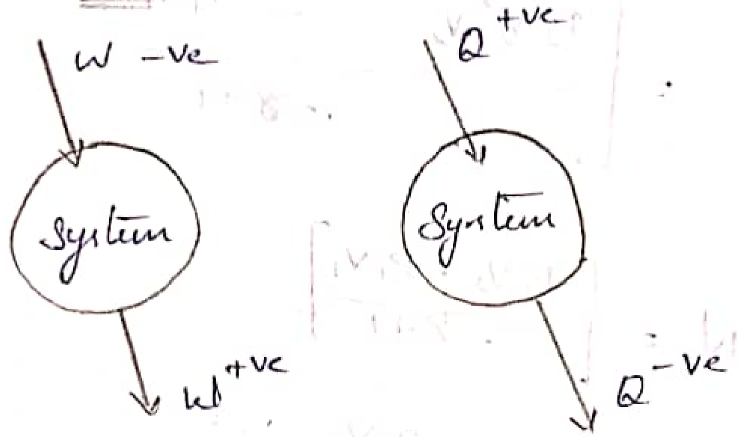
$$\frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = -\Delta U$$

$$\Delta u = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

Enthalpy

$$\Delta h = m c_p (T_2 - T_1)$$

Sign Convention



Type - I

Based on First Law of Thermodynamics

1. A Piston an a cylinder contains a fluid which passes through a complete cycle of four process during the cycle the sum of all heat transfer is -170 kJ the system completes 100 cycles per minute. Find the net rate of output in kW.

Total $Q = -170 \times 100$

$= -17000$ kJ/min

Process	Q (kJ/min)	W (kJ/min)	ΔU (kJ/min)
a-b	0	2170	—
b-c	21000	0	—
c-d	-2100	—	-36000
d-a	—	—	—

Process a-b

$$Q = W + \Delta U$$

$$0 = 2170 + \Delta U$$

$$\boxed{\Delta U = -2170 \text{ kJ/min}}$$

Process b-c

$$Q = W + \Delta U$$

$$21000 = 0 + \Delta U$$

$$\boxed{\Delta U = 21000 \text{ kJ/min}}$$

Process c-d

$$Q = W + \Delta U$$

$$-2100 = W + (-36{,}000)$$

$$\boxed{W = 33900 \text{ kJ/min}}$$

Process a-d = Process (a-b)+(b-c)+(c-d)+(d-a)

$-17000 = 0 + 21,000 - 2100 + d-a$

$$\boxed{Q_{d-a} = -35900 \text{ kJ/min}}$$

$\oint Q = \int W$

$-17000 = 2170 + 0 + 33900 + d-a$

$$\boxed{W_{d-a} = -53070 \text{ kJ/min}}$$

$Q = W + \Delta u$

$-35900 = -53070 + \Delta u$

$\Delta u = -35900 + 53070$

$$\boxed{\Delta u = 17170 \text{ kJ/min}}$$

Net rate of output = W

$\oint Q = \oint W$

$W = -17000$ kg/min

1 kW = 1 kg/s

$W = -\frac{17000}{60}$ kJ/s

$W = -283.33$ kJ/s

$$\boxed{W = 283.33 \text{ kW}}$$

2. A fluid system contains piston and cylinder passed through complete 4 process the sum of all heat transfer during the cycle is 340 kJ ~~when~~ the system contains 200 cycles per minute.

Process	Q (kJ/min)	W (kJ/min)	ΔU (kJ/min)
1 - 2	0	4340	—
2 - 3	42000	0	—
3 - 4	-4200	—	-73200
4 - 1	—	—	—

Total $Q = 340 \times 200$
$= 68000$ kJ/min

Process 1-2

$Q = W + \Delta U$

$0 = 4340 + \Delta U$

$\boxed{\Delta U = -4340 \text{ kJ/min}}$

Process 2-3

$Q = W + \Delta U$

$42000 = 0 + \Delta U$

$\boxed{\Delta U = 42000 \text{ kJ/min}}$

Process 3-4

$$Q = W + \Delta U$$

$$-4200 = W + (-73200)$$

$$W = -4200 + 73200$$

$$\boxed{W = 69000 \text{ kJ/min}}$$

1.

Process 1-4 = Process (1-2) + (2-3) + (3-4) + (4-1)

$$68000 = 0 + 42000 - 4200 + (4-1)$$

$$\boxed{Q_{4-1} = 30200 \text{ kJ/min}}$$

$$\oint Q = \int W$$

$$68000 = 4340 + 0 + 69000 + 4-1$$

$$\boxed{W_{4-1} = -5340 \text{ kJ/min}}$$

$$Q = W + \Delta U$$

$$30200 = -5340 + \Delta U$$

$$\boxed{\Delta U = 35540 \text{ kJ/min}}$$

Not neatly Completed

Type - II

Integral Based Problem

1. A gas of mass 1.5 kg undergoes a quasi static process following a relationship $p = a + bV$ where a and b are constant. the initial and final pressure are 1000 kpa and 200 kpa. the volume are 0.20 m^3 and 1.20 m^3. the specific internal energy of the system is given by $u = 1.5PV - 85$ kJ/kg. Calculate net heat transfer and internal energy of the system. (Δu)

Given data

$P = a + bV$

$P_1 = 1000$ kpa

$P_2 = 200$ kpa

$V_1 = 0.20\ m^3$

$V_2 = 1.20\ m^3$

$u = 1.5PV - 85$ kJ/kg

To find

Q, u

Formula

$Q = W + \Delta u$

$W = \int P\,dV$

$\Delta u = u_2 - u_1$

$$P = a + bV$$

$$P_1 = a + bV_1 \quad \text{— (1)}$$

$$P_2 = a + bV_2 \quad \text{— (1)}$$

$$(1) \Rightarrow 1000 = a + 0.20\,b$$

$$(2) \Rightarrow 800 = a + 1.20\,b$$

$$(-) \quad (-) \quad (-)$$

$$800 = -b$$

$$\boxed{b = -800}$$

Sub in Eqn. (2)

$$800 = a + 1.20(-800)$$

$$a = 200 + 960$$

$$\boxed{a = 1160}$$

$$P = a + bV$$

$$P = 1160 + (-800)V$$

$$P = 1160 - 800V \quad \text{— (3)}$$

$$W = \int_{V_1}^{V_2} P\,dV$$

$$= \int_{V_1}^{V_2} (1160 - 800V)\,dV$$

$$= \left[1160V - \frac{\overset{400}{800}V^2}{2}\right]_{0.2}^{1.2}$$

$$W = \left[1160V - 400V^2\right]_{0.2}^{1.2}$$

$$= \left[1160(1.2) - 400(1.2)^2\right] - \left[1160(0.2) - 400(0.2)^2\right]$$

$$= \left[1392 - 576\right] - \left[232 - 16\right]$$

$$= 816 - 216$$

$$\boxed{W = 600 \text{ kJ}}$$

$$U = 1.5PV - 85$$

$$U_1 = 1.5 P_1 V_1 - 85$$

$$U_2 = 1.5 P_2 V_2 - 85$$

$$\Delta U = U_2 - U_1$$

$$\Delta U = (1.5 P_2 V_2 - 85) - (1.5 P_1 V_1 - 85)$$

$$= 1.5 P_2 V_2 - 85 - 1.5 P_1 V_1 + 85$$

$$= 1.5 P_2 V_2 - 1.5 P_1 V_1$$

$$= 1.5 (P_2 V_2 - P_1 V_1)$$

$$= 1.5 (200(1.2) - 1000(0.2))$$

$$\boxed{\Delta U = 60 \text{ kJ}}$$

1st law

$$Q = W + \Delta U$$

$$= 600 + 60$$

$$\boxed{Q = 660 \text{ kJ}}$$

Result :- $Q = 660$ kJ . $U = 60$ kJ

2. A fluid is confined in a cylinder by a piston the pressure in the fluid is $P = a + bV$ the internal energy of the system is given by $u = 34 + 3.15\,PV$. If the fluid changes from initial state 170 kPa, to 0.03 m^3 to final state of 400 kPa, 0.06 m^3. Find direction and magnitude of heat and work. [May 2016]

Given data

$$P = a + bV$$

$$P_1 = 170 \text{ kPa}$$

$$P_2 = 400 \text{ kPa}$$

$$V_1 = 0.03 \text{ m}^3$$

$$V_2 = 0.06 \text{ m}^3$$

$$u = 34 + 3.15\,PV$$

To Find

Q ; W

Formula

$$Q = W + \Delta u$$

$$W = \int p\,dV$$

$$\Delta u = u_2 - u_1$$

$$P = a + bV$$

$$P_1 = a + bV_1 \quad \text{—— (1)}$$

$$P_2 = a + b\,V_2 \quad \text{—— (2)}$$

$$(1) \Rightarrow 170 = a + 0.03b$$

$$(2) \Rightarrow 400 = a + 0.06b$$

$$-230 = -0.03b$$

$$\boxed{b = 7666.66}$$

Sub in eqn. (1)

$$170 = a + 0.03(7666.66)$$

$$170 = a + 229.99$$

$$\boxed{a = -59.99}$$

$$P = a + bV$$

$$P = -59.99 + 7666.66V \quad \text{— (3)}$$

$$W = \int_{V_1}^{V_2} P\,dV$$

$$= \int_{0.03}^{0.06} (-59.99 + 7666.66V)\,dV$$

$$= \left[-59.99V + \frac{7666.66V^2}{2}\right]_{0.03}^{0.06}$$

$$= \left[-59.99V + 3833.33V^2\right]_{0.03}^{0.06}$$

$$= \left(-59.99(0.06) + 3833.33(0.06)^2\right) - \left[-59.99(0.03) + 3833.33(0.03)^2\right]$$

$$W = 8.550 \text{ kJ}$$

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$$U = 34 + 3.15\,PV$$

$$U_1 = 34 + 3.15\,P_1V_1$$

$$U_2 = 34 + 3.15\,P_2V_2$$

$$\Delta U = U_2 - U_1$$

$$= (34 + 3.15\,P_2V_2) - (34 + 3.15\,P_1V_1)$$

$$= 34 + 3.15\,P_2V_2 - 34 - 3.15\,P_1V_1$$

$$= 3.15\,P_2V_2 - 3.15\,P_1V_1$$

$$= 3.15\,(P_2V_2 - P_1V_1)$$

$$= 3.15\,(400(0.06) - 170(0.03))$$

$$\Delta U = 59.53$$

$$Q = W + \Delta U$$

$$= 8.55 + 59.53$$

$$\boxed{Q = 68.08\ kJ}$$

Type - iii

Formula Based Problems

1. In a vessel of 10kg of oxygen is heated in a reversible constant volume (isochoric) process and the pressure of the oxygen is increased two times of the initial value. The initial temperature is 20°c. Find (i) final temperature change (ii) change in internal energy (iii) change in enthalpy (iv) heat transfer. Assume $R = 0.259\ kJ/kgK$ and $C_v = 0.625\ kJ/kgK$

Given data :-

$m = 10\,kg$

$P_2 = 2 \times P_1$

$T_1 = 20^\circ c + 273.16$

$= 293.16\,k$

$V = C$

$R = 0.2579\,kJ/kgk$

$C_v = 0.6255\,kJ/kgk$

To find

i) $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$\frac{P_1}{2P_1} = \frac{T_1}{T_2}$

$\frac{1}{2} = \frac{293.16}{T_2}$

$T_2 = 586.32\,k$

ii) Internal Energy

$\Delta u = m c_v (T_2 - T_1)$

$\Delta u = 10 \times 0.6255\,(586.32 - 293.16)$

$\Delta u = 1832.25\,kJ$

iii) Enthalpy

$\Delta h = mc_p(T_2 - T_1)$

$R = c_p - c_v$

$0.259 = c_p - 0.625$

$c_p = 0.884$ kJ/kgK

$\Delta h = 10 \times (0.884)(586.32 - 293.16)$

$\boxed{\Delta h = 2591.53 \text{ kJ}}$

iv) Heat Transfer

$Q = \Delta u$

$\boxed{Q = 1832.25 \text{ kJ}}$

Result :-

Final Temperature – 586.32 K

Internal Energy – 1832.25 kJ

Enthalpy – 2591.53 kJ

Heat transfer – 1832.25 kJ

---

2. A 5kg of air at 40°c and 1bar is heated in a reversible Non-flow Constant pressure (Isobaric) process untill the volume is doubled. Find change in volume, work done, change in internal energy, change in enthalpy.

Given data :-

$1\ bar = 1\times10^5\ pa$ or N/m^2

$m = 5\ kg$

$1J = 1N\text{-}m$

$T_1 = 40^\circ c + 273.16$

$= 313.16 k$

$P_1 = 1\ bar$

$P = C$

$V_2 = 2\times V_1$

~~change in Volume~~

Find :- $\Delta V = V_2 - V_1$, W, Δu, Δh

Sol. :- $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$\frac{V_1}{2V_1} = \frac{313.16}{T_2}$

$\frac{1}{2} = \frac{313.16}{T_2}$

$T_2 = 626.32\ K$

i) change in Volume

$P_1 V_1 = mRT_1$

$V_1 = \frac{mRT_1}{P_1}$

$= \frac{5\times0.287\times313.16}{1\times10^2}$

$V_1 = 4.49\ m^3$

$V_1 = \frac{kg \times \frac{kJ}{kgK} \times K}{10^5\ N/m^2}$

$= \frac{kJ\ m^2}{10^5\ N}$

$= \frac{kN\text{-}m\ m^2}{10^5\ N}$

$= \frac{kN\text{-}m\times m^2}{10^2\times kN}$

$V_1 = \frac{m^2}{10^2}$

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$\Delta V = V_2 - V_1$

$\Delta V = 4.49\ m^3$

ii) Work done

~~$W = P(V_2 - V_1$~~

$W = mR(T_2 - T_1)$

$= 5 \times 0.287\ (626.32 - 313.16)$

$W = 449.38\ kJ$

iii) Internal Energy

$\Delta u = mC_v(T_2 - T_1)$

$= 5 \times 0.717\ (626.32 - 313.16)$

$\Delta u = 1124.47\ kJ$

iv) Enthalpy

$\Delta h = mC_p(T_2 - T_1)$

$= 5 \times 1.005\ (626.32 - 313.16)$

$\Delta h = 1576.14\ kJ$

Result :-

change in Volume - $4.49\ m^3$
Work done - $449.38\ kJ$
Internal energy - $1124.47\ kJ$
Enthalpy - $1576.14\ kJ$

3. A 1.5 kg of Certain gas at a pressure of 8 Bar and 20°c accupies a Volume of 0.15 m³. It expands adiabatically to a pressure of 0.9 Bar and Volume 0.73 m³. Determine the work done during the process, gas Content, ratio of specific heat, Value of two specific heat, change in internal energy. change in enthalpy.

Given data :-

$m = 1.5 kg$

$P_1 = 8 bar$

$T_1 = 20°c + 273.16$

$= 293.16 k$

$V_1 = 0.15 m^3$

$P_2 = 0.9 bar$

$V_2 = 0.73 m^3$

To find :-

$W, R, \frac{C_P}{C_V} = \gamma, C_p, C_v, \Delta u, \Delta h$

Formula :-

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$\Delta u = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

$$\Delta h = mC_p(T_2 - T_1)$$

Relation

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$$

× log on both sides

$$\log\left(\frac{P_1}{P_2}\right) = \log\left(\frac{V_2}{V_1}\right)^{\gamma}$$

$$\log\left(\frac{P_1}{P_2}\right) = \gamma \log\left(\frac{V_2}{V_1}\right)$$

$$\frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{V_2}{V_1}\right)} = \gamma$$

$$\frac{\log\left(\frac{8}{0.9}\right)}{\log\left(\frac{0.73}{0.15}\right)} = \gamma$$

$$\boxed{\gamma = 1.38}$$

Work :-

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{(8\times10^2\times0.15) - (0.9\times10^2\times0.73)}{\gamma - 1}$$

$1 bar = 1\times10^5 N/m^2$

$W = 1\times10^5 \frac{N}{m^2} \times m^3$

$W = 1\times10^5 Nm$

$1 kNm = 1 kJ$

$$\boxed{W = 142.89 \text{ kJ}}$$

Internal energy

$$\Delta u = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

$$= \frac{(0.9\times10^2\times0.73) - (8\times10^2\times0.15)}{\gamma - 1}$$

$$\boxed{\Delta u = -142.89\ kJ}$$

Enthalpy

$$\Delta h = mc_p(T_2 - T_1)$$

$$R = c_p - c_v$$

$$\gamma = \frac{c_p}{c_v}$$

$$P_1V_1 = mRT_1$$

$$8\times10^2\times0.15 = 1.5\times R\times 293.16$$

$$\boxed{R = 0.272\ kJ/kgK}$$

$$R = c_p - c_v$$

$$0.272 = c_p - c_v$$

$$\gamma = \frac{c_p}{c_v}$$

$$1.38 = \frac{c_p}{c_v}$$

$$1.38\,c_v = c_p$$

$$0.272 = 1.38C_v - C_v$$

$$0.272 = 0.38\,C_v$$

$$\boxed{C_v = 0.715 \text{ kJ/kgK}}$$

$$C_p = 1.38\,C_v$$

$$C_p = 1.38 \times 0.715$$

$$\boxed{C_p = 0.9867 \text{ kJ/kgK}}$$

Relation:-

$$\left[\frac{P_1}{P_2}\right]^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}$$

$$\left(\frac{8}{0.7}\right)^{\frac{1.38-1}{1.38}} = \frac{293.16}{T_2}$$

$$1.803 = \frac{293.16}{T_2}$$

$$T_2 = 162.59\text{ K}$$

$$\Delta h = mc_p(T_2 - T_1)$$

$$\Delta h = 1.5 \times 0.986\,(162.59 - 293.16)$$

$$\boxed{\Delta h = -193.11 \text{ kJ}}$$

Result:-

$W = 142.89$ kJ, $R = 0.272$ kJ/kgK, $C_p = 0.9867$ kJ/kgK, $C_v = 0.715$ kJ/kgK, $\Delta u = -142.89$ kJ, $\Delta h = -193.11$ kJ, $\gamma = 1.38$

7. A cylinder contains $1 m^3$ of gas at 100 kpa and 100°c the gas is expanding polytropically to a volume of $0.15 m^3$ to a final pressure of 600 kpa. Find (i) mass of the gas. (ii) Value of Index for compression. (iii) change in internal energy. (iv) heat transfer Assume [R = 0.287 kJ/kgk] [γ = 1.4]

Given data:-

$V_1 = 1 m^3$

$P_1 = 100 kpa$

$T_1 = 100°c + 273.16$

$= 373.16 K$

$V_2 = 0.15 m^3$

$P_2 = 600 kpa$

$R = 0.287 kJ/kgK$

$\gamma = 1.4$

To find:-

$m, n, \Delta u, Q$

$P_1 V_1 = m R T_1$

$100 \times 1 = m \times 0.287 \times 373.16$

$m = 0.93 kg$

$m = \frac{kpa \times m^3}{kJ/kgK \times K}$

$= \frac{\frac{kN}{m^2} \times m^3}{kJ/kg}$

$= \frac{kJ}{kJ/kg}$

$m = kg$

Relation

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

$$\frac{100}{600} = \left(\frac{0.15}{1}\right)^n$$

$$\frac{\log\left(\frac{100}{600}\right)}{\log\left(\frac{0.115}{1}\right)} = n$$

$$\boxed{n = 0.94}$$

Internal energy

$$\Delta u = \frac{P_2V_2 - P_1V_1}{n-1}$$

$$= \frac{(600\times0.115)-(100\times1)}{0.94-1}$$

$$\Delta u = 166.66 \text{ kJ}$$

Heat

$$Q = W \times \left(\frac{\gamma-n}{\gamma-1}\right)$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$W = -166.66 \text{ kJ}$$

$$Q = -166.66 \times \left(\frac{1.4-0.94}{1.4-1}\right)$$

$$Q = -191.65 \text{ kJ}$$

Result :-

(i) mass of the gas = 0.93 kg

(ii) value of the Index for compression = 0.94

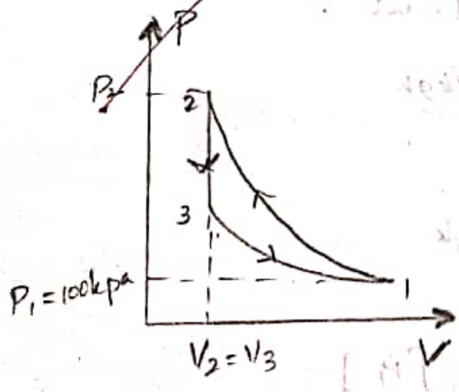
(iii) change in internal energy = 166.66 kJ

(iv) Heat transfer = -191.65 kJ

Type - IV

Mixed Type of Problem

1. A 3 cycle process operating with nitrogen as a working medium has constant temperature (Isothermal) compression at 34°c with initial pressure of 100 kpa then the gas undergoes constant volume heating and it is followed by polytropic process ($PV^n = C$) with n = 1.35. The Isothermal compression requires -67 kJ/kg of work. Assume C_v of nitrogen is 0.731 kJ/kgK and molecular weight of nitrogen is 28. Find (i) Pressure, Volume and temperature around the cycle, (ii) heat in and outs (iii) Net work.



Given data:-

$P_1 = 100$ kpa

$T_1 = 34°C + 273.16$
$= 307.16K$

$n = 1.35$

$W_{A\to C} = -67$ kJ/kg

$C_v = 0.731$ kJ/kgK

$M = 28$

$G = 8.314$

To find

P_1, V_1, T_1

P_2, V_2, T_2

P_3, V_3, T_3

Q_1, Q_2, Q_3

W_1, W_2, W_3

Sol:-

Process 1-2 $PV = C$

$$W = mRT \ln\left[\frac{V_2}{V_1}\right]$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_1V_1 = P_2V_2$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$W = mRT \ln\left[\frac{P_1}{P_2}\right]$$

where
$$R = \frac{\text{uni gas Constant}}{\text{mol. wt}}$$

$$= \frac{8.314 \text{ kJ/kgk}}{28}$$

$$R = 0.2961 \text{ kJ/kgk}$$

$$W = mRT \ln\left[\frac{P_1}{P_2}\right]$$

$$-67 = 0.296 \times 307.16 \times \ln\left[\frac{100}{P_2}\right]$$

$$\frac{-67}{0.296 \times 307.16} = \ln\left[\frac{100}{P_2}\right]$$

$$-0.736 = \ln 100 - \ln P_2$$

$$-0.736 - \ln 100 = -\ln P_2$$

$$-5.34 = -\ln P_2$$

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$\ln P_2 = 5.34$

$P_2 = e^{5.34}$

$\boxed{P_2 = 208.518 \text{ kpa}}$

$\boxed{T_2 = 307.16K \text{ (Since } PV = C)}$

$P_1 V_1 = mRT_1$

$100 \times V_1 = 1 \times 0.296 \times 307.16$

$\boxed{V_1 = 0.90 \; m^3}$

Relation

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\frac{100}{208.51} = \frac{V_2}{0.90}$$

$\boxed{V_2 = 0.43 m^3}$

Process 2-3 $\quad V = C$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$$

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

Process 3-1 $\quad PV^n = C$

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^n$$

$$\frac{P_3}{P_1} = \left[\frac{V_1}{V_3}\right]^n$$

$\because V_2 = V_3 = 0.43$

$\boxed{V_3 = 0.43 m^3}$

$$\frac{P_3}{100} = \left[\frac{0.90}{0.43}\right]^{1.315}$$

$$\boxed{P_3 = 271 \, kpa}$$

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$\frac{208.51}{271} = \frac{307.16}{T_3}$$

$$\boxed{T_3 = 404.15 \, k}$$

$P_1 = 100 \, kpa$, $V_1 = 0.90 \, m^3$, $T_1 = 307.16 \, k$

$P_2 = 208.51 \, kpa$, $V_2 = 0.43 \, m^3$, $T_2 = 307.16 \, k$

$P_3 = 271 \, kpa$, $V_3 = 0.43 \, m^3$, $T_3 = 404.15 \, k$

Process 1-2 $PV = C$

$$Q_{1-2} = mRT_1 \ln\left[\frac{V_2}{V_1}\right]$$

$$Q_{1-2} = -67 \, kJ/kg$$

Process 2-3 $V = C$

$$Q_{2-3} = mC_v (T_3 - T_2)$$

$$= 1 \times 0.73 (404.15 - 307.16)$$

$$Q_{2-3} = 70.89 \, kJ/kg$$

Process 3-1 $PV^n = C$

$$Q_{3-1} = W \times \left(\frac{\gamma - n}{\gamma - 1}\right)$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$W_3 = \frac{P_3V_3 - P_1V_1}{n-1}$$

$$= \frac{(271 \times 0.43) - (100 \times 0.90)}{1.35 - 1}$$

$$W_3 = 75.8\ kJ/kg$$

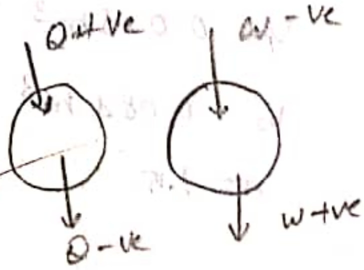
$$Q_{3-1} = 75.8 \times \left(\frac{1.4 - 1.35}{1.4 - 1}\right)$$

$$Q_{3-1} = 9.47\ kJ/kg$$

$W_{1-2} = -67\ kJ/kg$

$W_{2-3} = 0 \quad (V = c)$

$W_{3-1} = 75.8\ kJ/kg$



(iii) $Q_{in} = Q_{2-3} + Q_{3-1}$

$$\boxed{Q_{in} = 80.34\ kJ/kg}$$

$$\boxed{Q_{out} = -67\ kJ/kg}$$

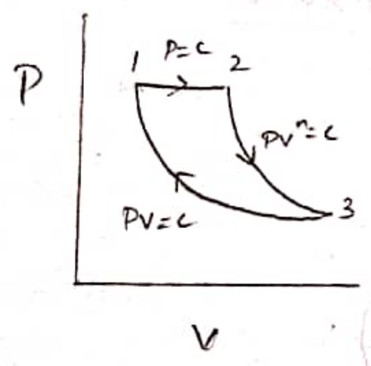
$Network = W_{1-2} + W_{2-3} + W_{3-1}$

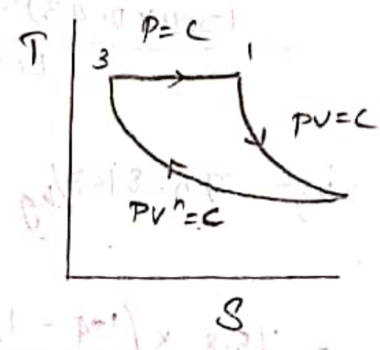
$= -67 + 0 + 75.8$

$$\boxed{Network = 8.8\ kJ/kg}$$

2. The mass of air is initial at 260°C and 700kpa and occupies 0.028 m³ the air is expanded at constant pressure to 0.084 m³ a polytrophic process with n = 1.5 is then expand and followed by a constant temperature process. Find (i) Sketch PV and TS of the cycle (ii) Find heat received and rejected. (iii) Find the efficiency of the cycle.

(i)





Given data:-

$T_1 = 260°C + 273.16$

$= 533.16\ k$

$P_1 = 700 kpa$

$V_1 = 0.028 m^3$

$V_2 = 0.084\ m^3$

$n = 1.5$

To find

$Q_1,\ Q_2,\ Q_3$

$P_1 = P_2 = 700\ kpa$ (P=C)

$T_1 = T_3 = 533.16\ k$ (P=C)

Process 1-2 $P = C$

$$Q_{1-2} = m c_p (T_2 - T_1)$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\frac{0.028}{0.084} = \frac{533.16}{T_2}$$

$$T_2 = 533.16 \times \frac{0.084}{0.028}$$

$$T_2 = 1599.48\ K$$

$$Q_{1-2} = m\ C_p (T_2 - T_1)$$

$$= 1 \times 1.005 (1599.48 - 533.16)$$

$$\boxed{Q_{1-2} = 1071.65\ kJ/kg}$$

Process 2-3 $PV^n = C$ $Q = W \times \left[\frac{\gamma - n}{\gamma - 1}\right]$

$$W = \frac{P_2 V_2 - P_3 V_3}{n-1}$$

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^n$$

$$\frac{700}{P_2} = \left(\frac{0.084}{0.028}\right)^{1.5}$$

$$\frac{700}{P_2} = 5.19$$

$$P_2 = 134.87\ kpa$$

~~Process 2-3~~ ~~$PV^n = c$~~

~~$Q = W \times \left[\frac{\gamma - n}{\gamma - 1}\right]$~~

~~$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$~~

~~$$W = \frac{700(0.028) - 134.87(0.084)}{1.5-1}$$~~

~~$W = 16.54$ kJ~~

~~$Q = 16.54 [$~~

Process 2-3 $PV^n = c$

$$Q_{2-3} = W \times \left[\frac{\gamma - n}{\gamma - 1}\right]$$

$$W_{2-3} = \frac{P_2 V_2 - P_3 V_3}{n-1}$$

$$\frac{P_2}{P_3} = \left[\frac{V_3}{V_2}\right]^n$$

Process 3-4 $PV = C$

$$Q = MRT_3 \ln\left[\frac{V_3}{V_4}\right]$$

or

$$P_3 V_3 \ln\left[\frac{V_3}{V_4}\right]$$

$$\frac{P_3}{P_1} = \frac{V_1}{V_3}$$

Process 2-3 $PV^n = C$

Relation

$$\frac{P_2}{P_3} = \left[\frac{V_3}{V_2}\right]^n$$

$$\frac{700}{P_3} = \left[\frac{V_3}{0.084}\right]^{1.5} \quad \text{— (1)}$$

$$\left[\frac{V_2}{V_3}\right]^{n-1} = \frac{T_3}{T_2}$$

$$\left[\frac{0.084}{V_3}\right]^{1.5-1} = \frac{533.16}{1599.48}$$

$$\frac{(0.084)^{0.5}}{V_3^{0.5}} = 0.33$$

$$\frac{0.289}{V_3^{0.5}} = 0.33$$

$$0.875 = V_3^{0.5}$$

$$V_3 = 0.46 \text{ m}^3$$

sub in (1)

$$\frac{700}{P_3} = \left[\frac{0.46}{0.084}\right]^{1.5}$$

$$\frac{700}{P_3} = 12.81$$

$$P_3 = 54.64 \text{ kPa}$$

$$W_{2-3} = \frac{P_2V_2 - P_3V_3}{n-1}$$

$$= \frac{700(0.084) - 54.64(0.46)}{1.5-1}$$

$$W_{2-3} = 67.33 \text{ kJ}$$

$$Q_{2-3} = W \times \left[\frac{\gamma - n}{\gamma - 1}\right]$$

$$= 67.33\left[\frac{1.4-1.5}{1.4-1}\right]$$

$$= 67.33\left[\frac{-0.1}{0.4}\right]$$

$$\boxed{Q_{2-3} = -16.83 \text{ kJ/kg}}$$

Process 3-1 $\quad PV = C$

$$Q_{3-1} = P_3V_3 \ln\left[\frac{V_3}{V_1}\right]$$

$$= 54.64 \times 0.46 \ln\left[\frac{0.46}{0.028}\right]$$

$$\boxed{Q_{3-1} = 70.12 \text{ kJ/kg}}$$

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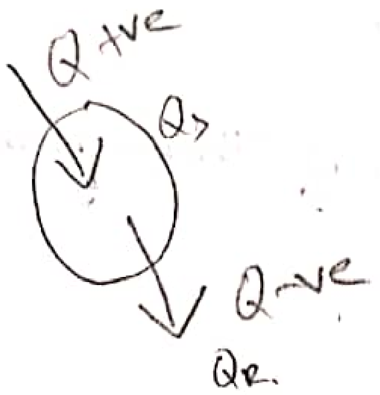


$$Q_S = Q_1 + Q_3$$

$$= 1071.65 + 70.12$$

$$Q_S = 1141.77 \text{ kJ/kg}$$

$$Q_R = -16.83 \text{ kJ/kg}$$

$$\eta = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{1141.77 - 16.83}{1141.77}$$

$$\boxed{\eta = 0.98}$$

∴ the efficiency of the cycle is 98%

good work completed 11/7/22

Unit - I Part - B

Application of Ist law of Thermodynamics of

Open System :-

$$Q = W + \Delta E$$

$$Q = W + E_p + E_k + E_s + E_e + \ldots$$

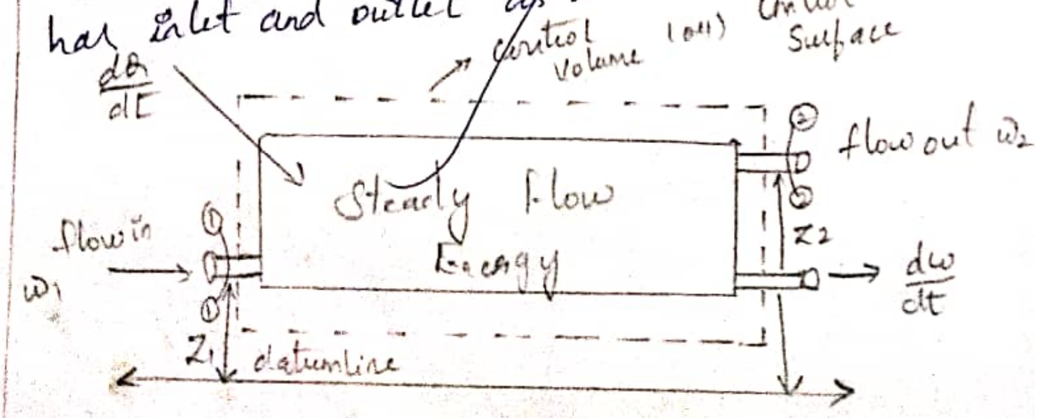
When there is mass transfer across the system boundary it is called as open system. Most of the Engineering applications are open system. for example - compressor, boiler, turbine

Eulerian Approach :-

Fixing a certain region in a fluid space is called as Control Volume. Through which the moving substance flows, the changes in the flow is analyzed inside the control volume this approach is known as Eulerian Approach.

Steady Flow Energy Equation (SFE) :-

Consider any mechanical device which has inlet and outlet as well as work output.



A_1, A_2 - Cross section area of the stream (m^2)

ω_1, ω_2 - mass flow rate (kg/s)

P_1, P_2 - Pressure at inlet and outlet (N/m^2)

v_1, v_2 - Specific Volume (m^3/kg)

u_1, u_2 - Specific internal energy (KJ/kg)

V_1, V_2 - Velocity at inlet and exit (m/s)

Z_1, Z_2 - Datum height

$\frac{dQ}{dt}$ - net rate of heat transfer (kJ/s)

$\frac{dw}{dt}$ - net rate of work transfer (kJ/s)

$$h = u + PV$$

Mass Balance :-

$$\omega_1 = \omega_2$$

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

$$\frac{m^2 \times \frac{m}{s}}{m^3/kg} = \frac{m^2 \times \frac{m}{s}}{m^3/kg}$$

$$\frac{m^3}{s} \frac{kg}{m^3} = \frac{m^3 kg}{s\, m^3}$$

$$\omega_1 = \omega_2$$

Energy Balance:

In a flow process there are two types of energy.

1. external work
2. Internal work

Work Transfer:-

$$W = W_x + P_2 v_2\, dm - P_1 v_1\, dm$$

$$\frac{dw}{dt} = \frac{dW_x}{dt} + P_2 v_2 \frac{dm}{dt} - P_1 v \frac{dm}{dt}$$

$$\frac{dw}{dt} = \frac{dW_x}{dt} + P_2 v_2 \omega_2 - P_1 v_1 \omega_1 \quad \text{--- (1)}$$

Since there is no accumalation of energy by energy theorem. the Total rate of stream entering the control volume is equal to total rate of stream leaving the control Volume.

1st Law of thermodynamics

$$\int Q = \int W \quad \text{--- (2)}$$

Energy conservation theorem

$$\omega_1 e_1 = \omega_2 e_2 \quad \text{--- (3)}$$

(2) + (3)

$$\frac{dQ}{dt} + \omega_1 e_1 = \frac{dw}{dt} + \omega_2 e_2$$

$$\frac{dW}{dt} = \frac{dQ}{dt} + \omega_1 e_1 - \omega_2 e_2$$

$$\frac{dQ}{dt} + \omega_1 e_1 = \frac{dW_x}{dt} + P_2 v_2 \omega_2 - P_1 v_1 \omega_1 + \omega_2 e_2$$

Energy

$$e = e_k + e_p + e_u$$

$$K.E = \frac{1}{2} m v^2$$

$$P.E = mgz$$

$$I.E = \Delta u = u_2 - u_1$$

$$e_1 = \frac{1}{2} v_1^2 + g z_1 + u_1$$

$$e_2 = \frac{1}{2} V_2^2 + g z_2 + u_2$$

$$\frac{dQ}{dt} + \omega_1 \left[g z_1 + \frac{V_1^2}{2} + u_1 \right] = \frac{dW_x}{dt} + P_2 V_2 \omega_2 - P_1 V_1 \omega_1 + \omega_2 \left[g z_2 + \frac{V_2^2}{2} + u_2 \right]$$

$$\frac{dQ}{dt} + \omega_1 \left[g z_1 + \frac{V_1^2}{2} + u_1 + P_1 v_1 \right] = \frac{dW_x}{dt} + \omega_2 \left[g z_2 + \frac{V_2^2}{2} + u_2 + P_2 V_2 \right]$$

$$\omega_1 = \omega_2 , \quad h_1 = u_1 + P_1 v_1$$

$$\omega \left[g z_1 + \frac{V_1^2}{2} + h_1 \right] + \frac{dQ}{dt} = \omega \left[g z_2 + \frac{V_2^2}{2} + h_2 \right] + \frac{dW_x}{dt}$$

for unit conversion

$$\omega \left[\frac{g z_1}{1000} + \frac{V_1^2}{2000} + h_1 \right] + \frac{dQ}{dt} = \omega \left[\frac{g z_2}{1000} + \frac{V_2^2}{2000} + h_2 \right] + \frac{dW_x}{dt}$$

Hence the steady flow Energy Equation is proved

Type - I

1. Air flows steadily at a rate of 0.5 kg/s through an air compressor entering at 7 m/s, 100 kpa and 0.95 m³/kg and existing at 0.19 m³/kg, 5 m/s, 700 kpa the Internal energy of the air leaving is 190 kJ/kg greater than the air entering. Air enters to the cooler at a rate of 58 kW. Find (i) rate of shaft work in kW. (ii) Find the ratio of inlet pipe diameter to outlet pipe diameter [Nov Dec 2014]

Given data :-

$\omega = 0.5\ kg/s$

$V_1 = 7\ m/s$

$P_1 = 100\ kpa$

$v_1 = 0.95\ m^3/kg$

$V_2 = 5\ m/s$

$P_2 = 700\ kpa$

$v_2 = 0.19\ m^3/kg$

$\frac{dQ}{dt} = 58\ kW$

$\delta u = u_2 - u_1$

$\Delta u = 190\ kJ/kg$

$= -190$

* $Z_1 = Z_2$
* $\omega_1 = \omega_2 = 0.5\ kg/s$
* ~~$u_2 = 190 + u_1$~~

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$$w_1\left[\frac{gZ_1}{1000}+\frac{V_1^2}{2000}+P_1v_1+u_1\right]+\frac{dQ}{dt}=$$

$$w_2\left[\frac{gZ_2}{2000}+\frac{V_2^2}{2000}+P_2v_2+u_2\right]+\frac{dw}{dt}$$

$$\left[\frac{V_1^2}{2000}+P_1v_1+u_1\right]+\frac{dQ}{dt}=\left[\frac{V_2^2}{2000}+P_2V_2+u_2\right]+\frac{dw}{dt}$$

$$\left[\left(\frac{V_1^2-V_2^2}{2000}\right)+(P_1V_1-P_2V_2)+(u_1-u_2)\right]+\frac{dQ}{dt}=\frac{dw}{dt}$$

$$\left[\left(\frac{7^2-5^2}{2000}\right)+(100\times0.95)-(700\times0.19)+(-190)\right]+58=\frac{dw}{dt}$$

$$(0.012-38-190)+58=\frac{dw}{dt}$$

$$\frac{dw}{dt}=-169.98\ kJ/kg$$

$$\frac{dw}{dt}=-169.98\times0.5$$

$$\boxed{\frac{dw}{dt}=-84.91\ kW}$$

Mass Balance

$$w_1=w_2$$

$$\frac{A_1V_1}{v_1}=\frac{A_2V_2}{v_2}$$

$$\frac{\pi d_1 l \times V_1}{v_1} = \frac{\pi d_2 l \times V_2}{v_2}$$

$$\frac{d_1 \times 7}{0.95} = \frac{d_2 \times 5}{0.19}$$

$$\frac{d_1}{d_2} = \frac{5}{0.19} \times \frac{0.95}{7}$$

$$\boxed{\frac{d_1}{d_2} = 3.57}$$

Type - V

2. A Turbine operation under a steady flow Condition receiving steam at following state pressure 1.2 MPa, Temperature 188 °C, Enthalpy 2785 kJ/kg and elevation is 3m. the steam leaves at following state pressure 20 kpa and velocity 100 m/s the Enthalpy 2512 kJ/kg and elevation 0m heat is lost to the surrounding at the rate of 0.29 kJ/s. If the rate of steam following through the Turbine is 0.42 kg/s what is the power out of Turbine the inlet velocity is 33.3 m/s.

Given data:

1×10^6 Pa = 1 MPa

$P_1 = 1.2$ MPa $= 1.2 \times 10^3$ kpa $= 1200$ kpa

$T_1 = 188°C$

$h_1 = 2785$ kJ/kg

$Z_1 = 3$ m

$\frac{dQ}{dt} = 0.29$ kJ/s

$P_2 = 20$ kpa

$h_2 = 2512$ kJ/kg

$V_2 = 100$ m/s

$Z_2 = 0$ m

$\omega = 0.42$ kg/s

$V_1 = 33.3$ m/s

$$\omega_1\left[\frac{gZ_1}{1000} + \frac{V_1^2}{2000} + h_1\right] + \frac{dQ}{dt} = \omega_2\left[\frac{gZ_2}{1000} + \frac{V_2^2}{2000} + h_2\right] + \frac{dw}{dt}$$

$$\left[\left(\frac{9.81 \times 3}{1000}\right) + \left(\frac{33.3^2}{2000}\right) + 2785\right] + 0.29 = \left[\frac{100^2}{2000} + 2512\right] + \frac{dw}{dt}$$

$$\omega\left[0.029 + 0.555 + 2785\right] + 0.29 = \omega\left[5 + 2512\right] + \frac{dw}{dt}$$

Net work

$$\omega\left[2785.86 - 2517\right] = \frac{dw}{dt}$$

$$\frac{dw}{dt} = 0.42 \times 268.86$$

$$\boxed{\frac{dw}{dt} = 112.92 \text{ kW}}$$

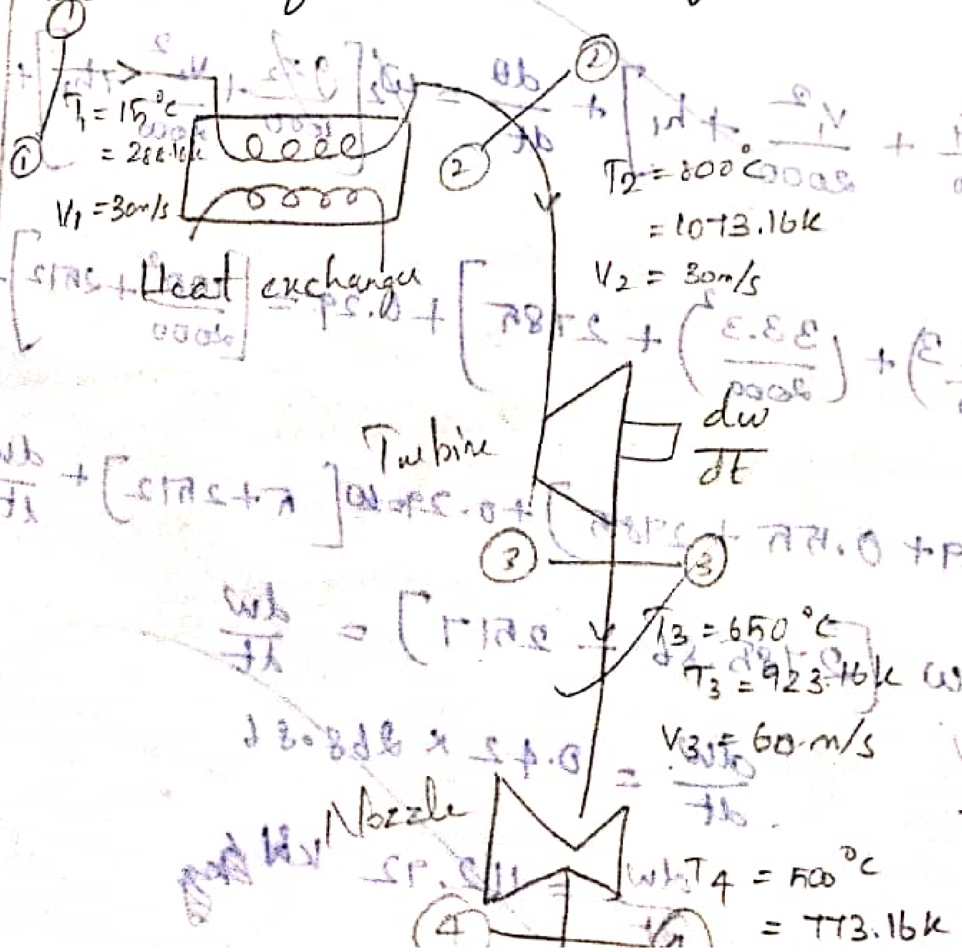
19/1/19.

2. Type - II

[May June 2011]

Combination Problem

1. Air at a temperature of 15 °c passes through an heat exchanger at a velocity of 30 m/s where it is raised to 800 °c it enters into the turbine with the same velocity 30 m/s and expands untill the temperature falls to 650 °c, on leaving the turbine air is taken at a velocity 60 m/s to a nozzle ~~If the air flow rate~~ where it expands untill the temperature falls to 500 °c ~~If~~ the air flows at the rate of 2 kg/s find (i) rate of heat transfer to the air inside the heat exchanger, (ii) Power output from the Turbine assuming no heat loss, (iii) Velocity at the exit of nozzle assuming no heat loss.



To find

(i) $\frac{dQ}{dt}$ (Heat exchanger)

(ii) $\frac{dW}{dt}$ (Turbine)

(iii) $V_4 = ?$

Process 1-2 [Heat Exchanger]

$$\omega\left[\frac{gZ_1}{1000} + \frac{V_1^2}{2000} + h_1\right] + \frac{dQ}{dt} = \omega\left[\frac{gZ_2}{1000} + \frac{V_2^2}{2000} + h_2\right] + \frac{dW}{dt}$$

$Z_1 = Z_2$

$V_1 = V_2$

$$\omega[h_1] + \frac{dQ}{dt} = \omega[h_2] + \frac{dW}{dt}$$

$\frac{dW}{dt} = 0$

$$\frac{dQ}{dt} = \omega[h_2 - h_1]$$

$$\frac{dQ}{dt} = \omega(\Delta h)$$

$$= \omega \times [mcp(T_2 - T_1)]$$

$$= \omega \times [1 \times 1.005(1073.16 - 288.16)]$$

$$\boxed{\frac{dQ}{dt} = 15776.8 \ kJ/s \ (or) \ kW}$$

Process 2-3 [Turbine]

$$\omega\left[\frac{gZ_2}{1000}+\frac{V_2^2}{2000}+h_2\right]+\frac{dQ}{dt}=\omega\left[\frac{gZ_3}{1000}+\frac{V_3^2}{2000}+h_3\right]+\frac{dW}{dt}$$

$$Z_2 = Z_3$$

$$\frac{dQ}{dt}=0$$

$$\omega\left[\frac{V_2^2}{2000}+h_2\right]=\omega\left[\frac{V_3^2}{2000}+h_3\right]+\frac{dW}{dt}$$

$$\frac{dW}{dt}=\omega\left[\frac{V_2^2}{2000}-\frac{V_3^2}{2000}+h_2-h_3\right]$$

$$\frac{dW}{dt}=\omega\left[\frac{V_2^2-V_3^2}{2000}+(h_2-h_3)\right]$$

$$=2\left[\frac{30^2-60^2}{2000}+mC_p(T_2-T_3)\right]$$

$$=2\left[-1.35+1\times1.005(1073.16-923.16)\right]$$

$$=2\left[-1.35+150.75\right]$$

$$\boxed{\frac{dW}{dt}=298.8\ kJ/s\ \text{or}\ kW}$$

Process 3-4 [Nozzle]

$$\omega\left[\frac{gz_3}{1000}+\frac{V_3^2}{2000}+h_3\right]+\frac{dQ}{dt}=\omega\left[\frac{gz_4}{1000}+\frac{V_4^2}{2000}+h_4\right]+\frac{dW}{dt}$$

$$z_3 = z_4$$

$$\frac{dQ}{dt}=0$$

$$\frac{dW}{dt}=0$$

$$\omega\left[\frac{V_3^2}{2000}+h_3\right]=\omega\left[\frac{V_4^2}{2000}+h_4\right]$$

$$\frac{V_4^2}{2000}=\left[\frac{V_3^2}{2000}+(h_3-h_4)\right]$$

$$\frac{V_4^2}{2000}=\left[\frac{60^2}{2000}+c_p(T_3-T_4)\right]$$

$$\frac{V_4^2}{2000}=\left[1.8+1.005(923.16-773.16)\right]$$

$$\frac{V_4^2}{2000}=\left[1.8+150.75\right]$$

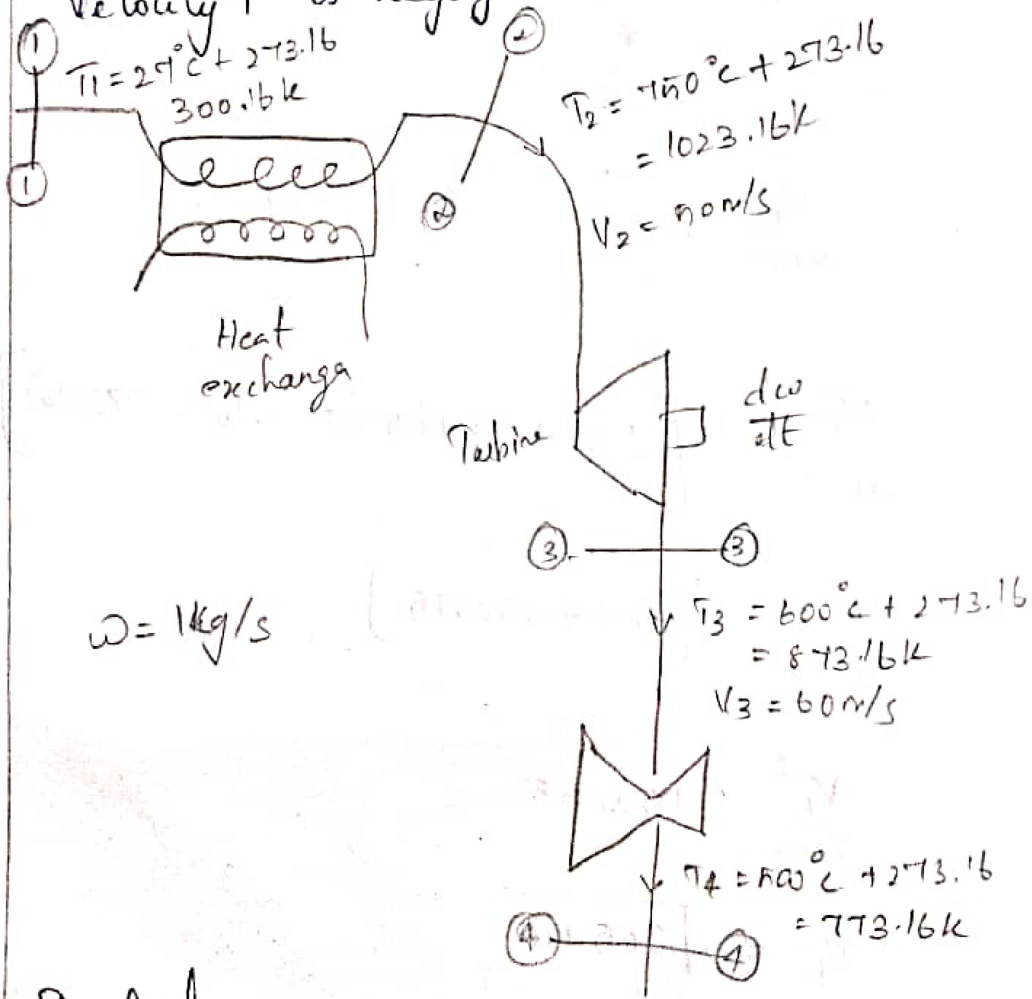
$$\frac{V_4^2}{2000}=152.55$$

$$V_4=\sqrt{305100}$$

$$\boxed{V_4 = 552.35\ m/s}$$

2. In the gas Turbine Insulation air heated inside the heat exchanger upto 750°c from ambient temperature of 27°c hot air then enters into a gas turbine with a Velocity of 50 m/s and leaves at 600°c, Air leaving the turbine enters into a Nozzle at 60 m/s and leaves the nozzle at the temperature of 500°c for a unit mass flow rate. Determine (i) heat transfer in heat exchanger (ii) power output from turbine (iii) Velocity at exit of the nozzle.

Velocity 1 is negligible.



To find

(i) $\frac{dQ}{dt}$ (HX) (ii) $\frac{dw}{dt}$ (Turbine) (iii) V_4

Process 1-2

$$\omega\left[\frac{gZ_1}{1000}+\frac{V_1^2}{2000}+h_1\right]+\frac{dQ}{dt}=\omega\left[\frac{gZ_2}{1000}+\frac{V_2^2}{2000}+h_2\right]+\frac{dW}{dt}$$

$\frac{dW}{dt}=0$

$Z_1=Z_2$

$V_1=0$

$$\omega[h_1]+\frac{dQ}{dt}=\omega\left[h_2+\frac{V_2^2}{2000}\right]$$

$$\frac{dQ}{dt}=\omega\left[\frac{V_2^2}{2000}+h_2-h_1\right]$$

$$=\omega\left[\frac{50^2}{2000}+\Delta h\right]$$

$$=\omega\left[1.25+mc_p(T_2-T_1)\right]$$

$$=1\left[1.25+1.005(1023.16-300.16)\right]$$

$$=1\left[1.25+726.615\right]$$

$$\boxed{\frac{dQ}{dt}=727.86\ kJ/s \text{ or } kW}$$

Process 2-3

$$\omega\left[\frac{gZ_2}{1000}+\frac{V_2^2}{2000}+h_2\right]+\frac{dQ}{dt}=\omega\left[\frac{gZ_3}{1000}+\frac{V_3^2}{2000}+h_3\right]+\frac{dW}{dt}$$

$\frac{dQ}{dt}=0$

$Z_2=Z_3$

$$\omega\left[\frac{V_2^2}{2000}+h_2\right] = \omega\left[\frac{V_3^2}{2000}+h_3\right]+\frac{dw}{dt}$$

$$\frac{dw}{dt} = \omega\left[\frac{V_2^2-V_3^2}{2000}+(h_2-h_3)\right]$$

$$=\omega\left[\frac{50^2-60^2}{2000}+mc_p(T_2-T_3)\right]$$

$$=1\left[-0.55+1.005(1023.16-873.16)\right]$$

$$=1\left[-0.55+150.75\right]$$

$$\boxed{\frac{dw}{dt} = 150.2\ kJ/s \text{ or } kW}$$

Process 3-4

Turbine

$$\omega\left[\frac{gz_3}{1000}+\frac{V_3^2}{2000}+h_3\right]+\frac{dQ}{dt} = \omega\left[\frac{gz_4}{1000}+\frac{V_4^2}{2000}+h_4\right]+\frac{dw}{dt}$$

$$\frac{dQ}{dt}=0$$

$$\frac{dw}{dt}=0$$

$$z_3=z_4$$

$$\omega\left[\frac{V_3^2}{2000}+h_3\right] = \omega\left[\frac{V_4^2}{2000}+h_4\right]$$

$$\frac{V_4^2}{2000} = \omega\left[\frac{V_3^2}{2000}+h_3-h_4\right]$$

$$= \omega\left[\frac{60^2}{2000}+\Delta h\right]$$

$$= 1\left[1.8 + mc_p(T_3 - T_4)\right]$$

$$= 1\left[1.8 + 1.005(873.16 - 773.16)\right]$$

$$= 1\left[1.8 + 100.5\right]$$

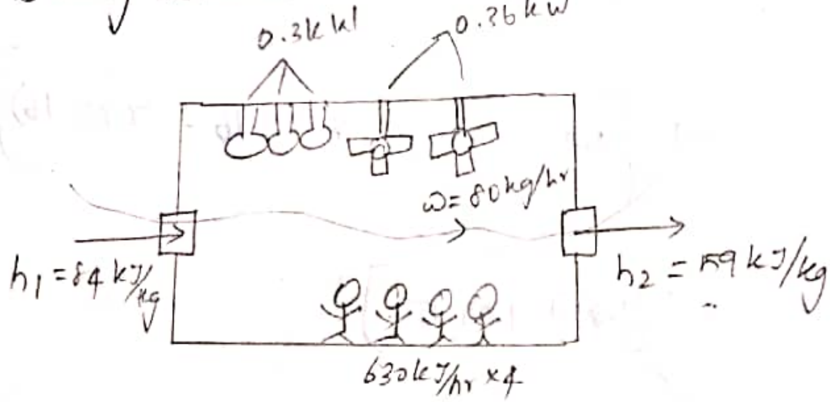
$$\frac{V_4^2}{2000} = 102.3$$

$$V_4^2 = 204600$$

$$\boxed{V_4 = 452.32 \text{ m/s}}$$

Type - III

1. A room of four person has two fans each Consuming 0.18 kW and three 100W lamp ventilation air at a rate of 80 kg/hr, Air enters with an enthalpy of 84 kJ/kg and leaves with an enthalpy of 59 kJ/kg if each person puts out heat at the rate of 630 kJ/hr. Determine the rate at which heat is removed by the air cooler Assuming study state is maintained in a room.



$$\omega\left[\frac{gZ_1}{1000} + \frac{V_1^2}{2000} + h_1\right] + \frac{dQ}{dt} = \omega\left[\frac{gZ_2}{1000} + \frac{V_2^2}{2000} + h_2\right] + \frac{dW}{dt}$$

* $Z_1 = Z_2$
* $V_1 = V_2$

$$\omega[h_1] + \frac{dQ}{dt} = \omega[h_2] + \frac{dW}{dt}$$

For fan

$$2 \text{ fan} = 2 \times 0.18$$
$$= 0.36 \text{ kW or kJ/s}$$

Light

3 light $= 3 \times 100$ W

$= 300$ W

$= 0.3$ KW or kJ/s

Human

4 person $= 4 \times 630$ kJ/hr

$= 4 \times \frac{630}{60 \times 60}$

$= 0.7$ kJ/s

$\omega = 80$ kg/hr

$= \frac{80}{60 \times 60}$

$\omega = 0.02$ kg/s

$$\frac{dQ}{dt} = \frac{dQ}{dt}_{light} + \frac{dQ}{dt}_{fan} + \frac{dQ}{dt}_{human}$$

$= 0.3 + 0.36 + 0.7$

$\frac{dQ}{dt} = 1.36$ kJ/s or kW

$$\omega[h_1] + \frac{dQ}{dt} = \omega[h_2] + \frac{dw}{dt}$$

$$\frac{dw}{dt} = \omega(h_1 - h_2) + \frac{dQ}{dt}$$

$= 0.02\,(84 - 59) + 1.36$

$\boxed{\frac{dw}{dt} = 1.86 \text{ kJ/s or kW}}$

∴ The rate at which heat is removed is 1.86 kJ/s

Type - IV

Mixed Problem

1. Air is compressed by an adiabatic compresser from 100 kpa and 12 °c to a pressure of 800 kpa at a steady state of 0.2 kg/s. If the Isentropic efficiency of the compressor is 80%. Determine exit temperature of the air and Require power input to the compressor.

Given

P_1 - 100 kpa $\qquad \omega = 0.2 \, kg/s$

P_2 - 800 kpa

T_1 = 12°c + 273.16 = 285.16 k

Adiabatic compressor relation

$$\left[\frac{P_1}{P_2}\right]^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}$$

$$\left[\frac{100}{800}\right]^{\frac{1.4-1}{1.4}} = \frac{285.16}{T_2}$$

$$\left(\frac{100}{800}\right)^{0.28} = \frac{285.16}{T_2}$$

$$0.552 = \frac{285.16}{T_2}$$

$$\boxed{T_2 = 516.55 \, k}$$

$$w\left[\frac{gz_1}{1000} + \frac{V_1^2}{2000} + h_1\right] + \frac{dQ}{dt} = w\left[\frac{gz_2}{1000} + \frac{V_2^2}{2000} + h_2\right] + \frac{dW}{dt}$$

$z_1 = z_2$

$V_1 = V_2$

$\frac{dQ}{dt}$ = negligible

$$w[h_1] = w[h_2] + \frac{dW}{dt}$$

$$w[h_1 - h_2] = \frac{dW}{dt}$$

$$\frac{dW}{dt} = w\left[mc_p(T_1 - T_2)\right]$$

$$= 0.2\left[1.005(285.16 - 516.59)\right]$$

$$\frac{dW}{dt} = -46.51 \text{ kW}$$

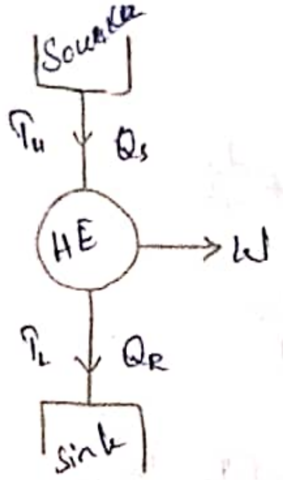
work is supplied to the System

$$\boxed{\frac{dW}{dt} = 46.51 \text{ kW}}$$

Unit - II

Second Law And Availability Analysis

* Heat Engine :-



$$Q_s = W + Q_R$$

$$W = Q_s - Q_R$$

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{W}{Q_s}$$

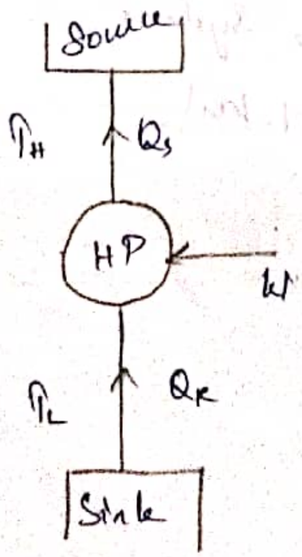
$$\eta = \frac{Q_s - Q_R}{Q_s}$$

In Carnot $\Rightarrow$ $W = T_H - T_L$

$$\eta = \frac{T_H - T_L}{T_H}$$

---

* Heat Pump



$$Q_R + W = Q_s$$

$$W = Q_s - Q_R$$

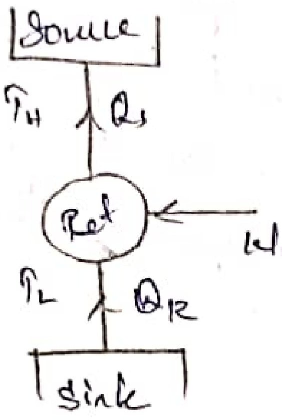
$$\text{cop} = \frac{\text{Desired effect}}{\text{work done}}$$

$$\text{cop} = \frac{Q_s}{Q_s - Q_R}$$

In Carnot $\Rightarrow$ $W = T_H - T_L$

$$\text{cop} = \frac{T_H}{T_H - T_L}$$

* Refrigerator



$$Q_R + W = Q_s$$

$$W = Q_s - Q_R$$

$$COP = \frac{\text{Desired effect}}{\text{work done}}$$

$$COP = \frac{Q_R}{Q_s - Q_R}$$

In Carnot $\Rightarrow W = T_H - T_L$

$$COP = \frac{T_L}{T_H - T_L}$$

where, COP – coefficient of Performance
Q_s – heat Supply
Q_R – heat Rejection

Energy Reservoir :-

A Thermal Energy Reservoir is defined as a large body with infinite space to store a large amount of heat energy. It is divided into two types :- 1. Source , 2. Sink

1. **Source :-**

The Thermal Energy Reservoir from which heat is extracted is known as source.

2. **Sink :-**

A Thermal Energy Reservoir in which remaining heat is store is known as sink.

Carnot Theorem :-

"No heat engine operation in a cyclic process between two fixed temperature can't be more efficient than a Carnot engine or reversible engine which is operating between a temperature limit"

$$\frac{T_H - T_L}{T_H} = \frac{Q_s - Q_R}{Q_s}$$

Kelvin-Planck Statement :-

"It is impossible for heat engine to produce net work in a complete cycle if it exchanges heat only to a body at fixed temperature"

$$\eta = \frac{W}{Q_s} = 100\%$$

$$\eta = \frac{W}{Q_s} = 1$$

$$\boxed{\eta = W = Q_s}$$

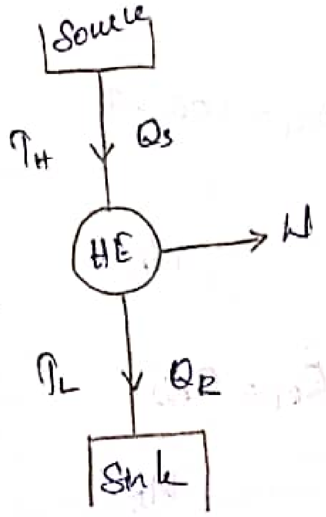
It is impossible

Clausius Statement :

"It is impossible to construct a device which is operating in a cycle which will be producing no effect other than transfer of heat from one body to another body"

---

1. An Inventor claims that he develop the heat engine which would have a heat source of 1000°c and reject heat to sink at 50°c and gives and efficiency of 90 %. Justify wheather the machine can be bought. (4 mark)



Given data

$$T_H = 1000°c = 1273.16k$$

$$T_L = 50°c = 323.16k$$

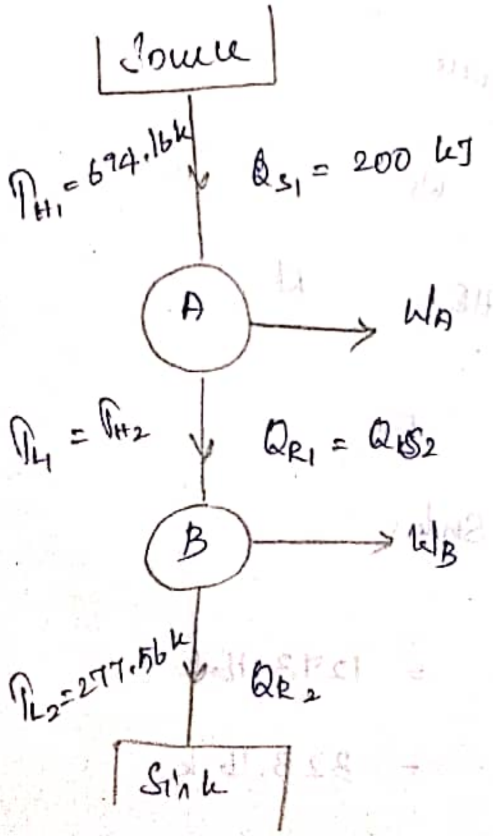
$$\eta_{carnot} = \frac{T_H - T_L}{T_H}$$

$$= \frac{1273.16 - 323.16}{1273.16}$$

$$= 0.74$$

$\eta_{carnot} = 74\%$ ∴ The Inventor claim is wrong

2. Two Reversible Heat Engine A & B are arranged in a Series. A rejecting heat directly to B. Engine receives 200 kJ of heat from a Source temperature of 421 °C while engine B is in communication with cold Sink at a temperature of 4.4 °C. If the work output of A is twice that of B, find (i) Intermediate temperature between A and B, (ii) Efficiency of each engine, (iii) Heat rejected in the cold Sink.



To Find

1. $T_{L_1} = T_{H_2}$

2. η_A, η_B

3. Q_{R_2}

Formula

$$\eta = \frac{Q_s - Q_R}{Q_s} = \frac{W}{Q_s}$$

$$\frac{Q_s - Q_R}{Q_s} = \frac{T_H - T_L}{T_H}$$

Carnot Engine

$$\frac{Q_s - Q_R}{Q_s} = \frac{T_H - T_L}{T_H}$$

$$1 - \frac{Q_R}{Q_s} = 1 - \frac{T_L}{T_H}$$

$$\frac{Q_{R1}}{Q_{s1}} = \frac{T_{L1}}{T_{H1}}$$

$$\frac{Q_{R1}}{200} = \frac{T_{L1}}{694.16}$$

$$Q_{R1} = 0.28\, T_{L1} \quad \text{—— (1)}$$

$$W_A = Q_{S1} - Q_{R1}$$

$$W_A = 200 - 0.28\, T_{L1} \quad \text{—— (2)}$$

$$W_A = 2W_B$$

$$200 - 0.28\, T_{L1} = 2(Q_{S2} - Q_{R2})$$

$$200 - 0.28\, T_{L1} = 2(Q_{R1} - Q_{R2})$$

$$200 - 0.28\, T_{L1} = 2(0.28\, T_{L1} - Q_{R2}) \quad \text{—— (3)}$$

Engine B

$$\frac{Q_{S_2} - Q_{R_2}}{Q_{S_2}} = \frac{T_{H_2} - T_{L_2}}{T_{H_2}}$$

$$\frac{Q_{R_2}}{Q_{S_2}} = \frac{T_{L_2}}{T_{H_2}}$$

$$\frac{Q_{R_2}}{0.28\,T_{L_1}} = \frac{277.56}{T_{L_1}}$$

$$\boxed{Q_{R_2} = 77.71 \text{ kJ}}$$

Sub in ③ Eqn.

$$200 - 0.28\,T_{L_1} = 2(0.28\,T_{L_1} - 77.71)$$

$$200 - 0.28\,T_{L_1} = 0.56\,T_{L_1} - 155.42$$

$$0.56\,T_{L_1} + 0.28\,T_{L_1} = 200 + 155.42$$

$$0.84\,T_{L_1} = 355.42$$

$$T_{L_1} = 423.11 \text{ K}$$

$$\boxed{\therefore T_{L_1} = T_{H_2} = 423.11 \text{ K}}$$

$$\eta_A = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

$$= \frac{200 - 0.28(423.11)}{200}$$

$$= 0.407$$

$$\boxed{\therefore \eta_A = 40.7\%}$$

$$\eta_B = \frac{Q_{S_2} - Q_{R_2}}{Q_{S_2}}$$

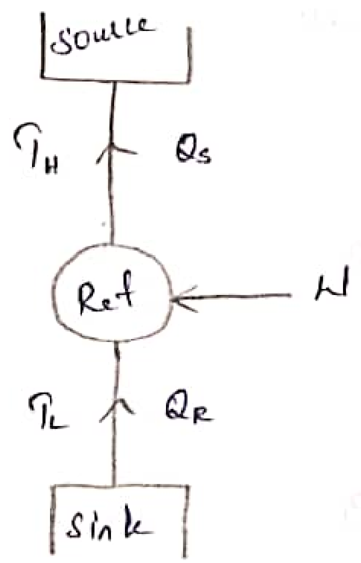
$$= \frac{0.28\,(423.11) - 77.71}{0.28\,(423.11)}$$

$$= 0.344$$

$$\boxed{\eta_B = 34.4\%}$$

---

3. A Inventor claims to develop a refrigerator unit which maintains refrigerator space at $-6^\circ C$ by operating in a room where temperature is $27^\circ C$ COP is 8.5. Find out whether the claims is correct or incorrect.



$$COP = \frac{T_L}{T_H - T_L}$$

$T_L = -6^\circ C = 267.16\,K$

$T_H = 27^\circ C = 300.16\,K$

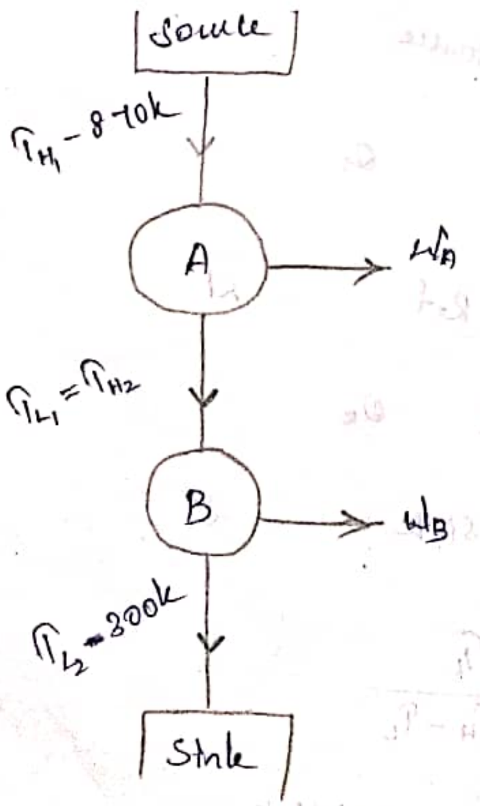
$$Cop = \frac{267.16}{300.16 - 267.16}$$

$$Cop = 8.09$$

∴ the claim is Incorrect

---

4. Two Carnot Engine A & B are operated in Series A recieves heat at 870 K and rejects to a reservoir at a temperature T. B recieves heat rejected by first engine and then rejects to Sink at 300K. In case of (i) equal work output for both cycle find the intermediate temperature (ii) If the System has same efficiency find the intermediate temperature.



Given data :-

$T_{H_1} - 870k$ $\quad$ $T_{L_2} - 300k$

$T_{L_1} = T_{H_2}$

Case (i)

$$W_A = W_B$$

$$\frac{W}{Q_S} = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}} = \frac{T_{H_1} - T_{L_1}}{T_{H_1}}$$

$$T_{H_1} - T_{L_1} = T_{H_2} - T_{L_2}$$

$$870 - T_{L_1} = T_{L_1} - 300$$

$$870 + 300 = 2T_{L_1}$$

$$\boxed{T_{L_1} = 585\ K}$$

Case (ii)

$$\eta_A = \eta_B$$

$$\frac{T_{H_1} - T_{L_1}}{T_{H_1}} = \frac{T_{H_2} - T_{L_2}}{T_{H_2}}$$

$$\frac{870 - 585}{870} = \frac{585 - 300}{585}$$

$$\frac{870 - T_{L_1}}{870} = \frac{T_{L_1} - 300}{T_{L_1}}$$

$$-\frac{T_{L_1}}{870} = -\frac{300}{T_{L_1}}$$

$$\frac{T_{L_1}}{870} = \frac{300}{T_{L_1}}$$

$T_{L_1} =$

$$T_{L_1}(870 - T_{L_1}) = 870(T_{L_1} - 300)$$

$$870T_{L_1} - T_{L_1}^2 = 870T_{L_1} - 261000$$

$$T_{L_1}^2 = 261000$$

$$\boxed{T_{L_1} = 510.8\ K}$$

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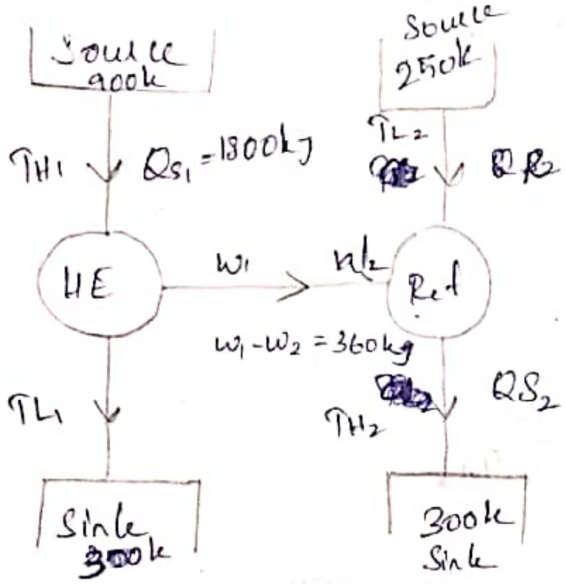
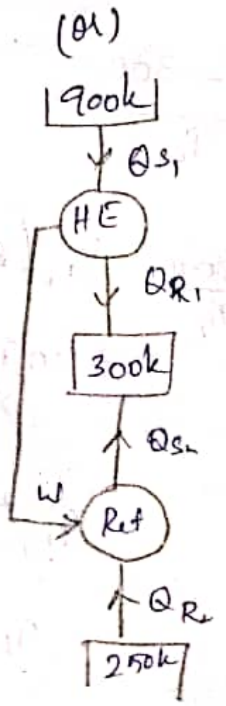
5. A Reversiable heat Engine operating between reservoirs at 900k and 300k drives a refrigerator operating between 300k and 250k refrigerator the Heat engine receives 1800kJ of heat from 900k reservoir the Net work output from the Combined engine refrigerator is 360 kJ. Find the Heat transfer to the refrigerator and net heat rejected to the reservoir at 300k.



(or)



To find

1. $Q_{R_1} + Q_{R_2}$

2. Q_{S_2}

Heat Engine

$$\eta = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}} = \frac{T_{H_1} - T_{L_1}}{T_{H_1}}$$

$$\eta = \frac{900 - 300}{900}$$

$$\eta = 0.66 = 66\%$$

$$\eta = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

$$0.66 = \frac{1800 - Q_{R_1}}{1800}$$

$$\boxed{Q_{R_1} = 612 \text{ kJ}}$$

Refrigerator

$$COP = \frac{Q_{R_2}}{Q_{S_2} - Q_{R_2}} = \frac{T_{L_2}}{T_{H_2} - T_{L_2}}$$

$$COP = \frac{T_{L_2}}{T_{H_2} - T_{L_2}} = \frac{250}{300 - 250} = 5$$

$$COP = \frac{Q_{R_2}}{Q_{S_2} - Q_{R_2}}$$

$$5 = \frac{Q_{R_2}}{Q_{S_2} - Q_{R_2}}$$

Heat Engine

$$\eta = \frac{W_1}{Q_{S_1}}$$

$$0.66 = \frac{W1}{1800}$$

$$W_1 = 1188 \text{ kJ}$$

$$W_1 - W_2 = 360$$

$$1188 - W_2 = 360$$

$$W_2 = 828 \text{ kJ}$$

$$W_2 = Q_{S_2} - Q_{R_2}$$

$$\text{ㄅ} = \frac{Q_{R_2}}{Q_{S_2} - Q_{R_2}}$$

$$\text{ㄅ} = \frac{Q_{R_2}}{W_2}$$

$$\text{ㄅ} = \frac{Q_{R_2}}{828}$$

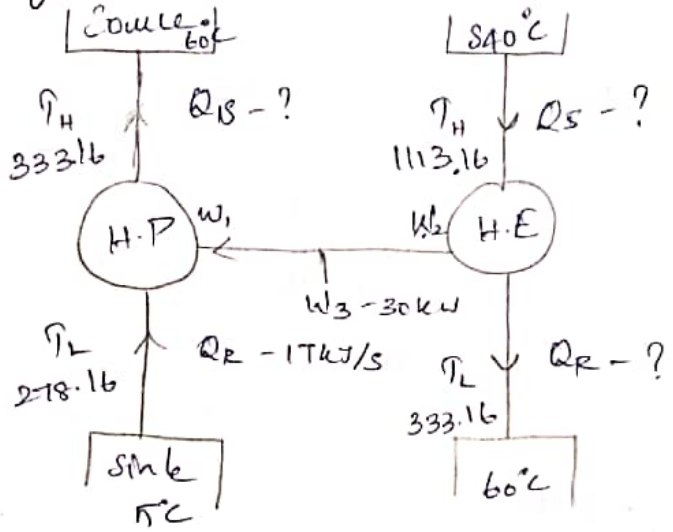
$$\boxed{Q_{R_2} = 4140 \text{ kJ}}$$

$$W_2 = Q_{S_2} - Q_{R_2}$$

$$828 = Q_{S_2} - 4140$$

$$\boxed{Q_{S_2} = 4968 \text{ kJ}}$$

6. A Heat Pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C. A Heat engine driven by a source of 840°C and rejects heat to a reservoir at 60°C, the reversiable heat engine in addition to driving the heat pump also a drives a machine at 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, Find the rate of heat supply from 840°C source, rate of heat rejection to the 60°C sink



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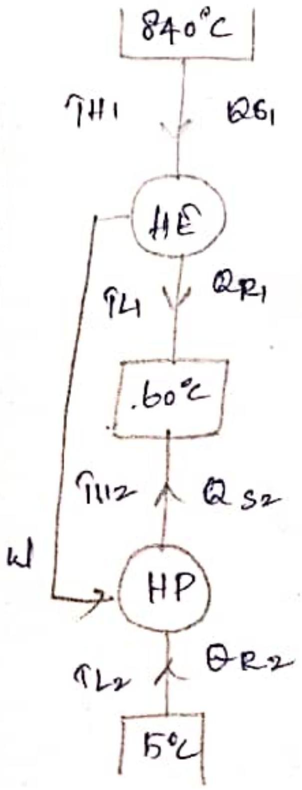
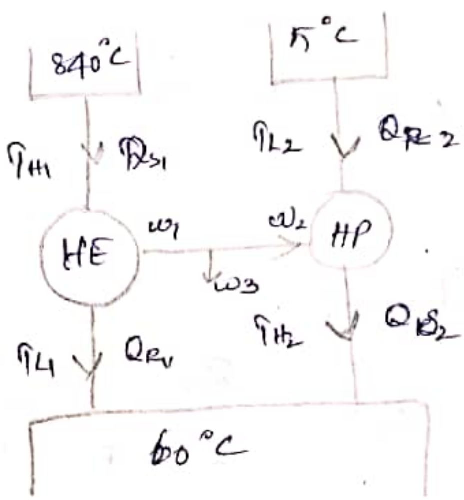




Given data :-

To find

$T_{H_1} = 1113.16\,k$

$Q_{S_1} = ?$

$T_{H_2} = T_{L_1} = 333.16\,k$

$Q_{R_1} = ?$

$T_{L_2} = 278.16\,k$

$Q_{S_2} = ?$

$Q_{R_2} = 17\,kJ/s$

$\omega_3 = 30\,kW$

Engine

$$\eta = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}} = \frac{T_{H_1} - T_{L_1}}{T_{H_1}}$$

$$\eta = \frac{1113.16 - 333.16}{1113.16}$$

$$\eta = 0.70$$

$$\eta = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

$$0.70 = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

Heat pump

$$COP = \frac{Q_{S_2}}{Q_{S_2} - Q_{R_2}} = \frac{T_{H_2}}{T_{H_2} - T_{L_2}}$$

$$COP_{pump} = \frac{333.16}{333.16 - 278.16}$$

$COP_{hp} - 6.04$

$$6.04 = \frac{Q_{s_2}}{Q_{s_2} - 17}$$

$$6.04\,(Q_{s_2} - 17) = Q_{s_2}$$

$$6.04\,Q_{s_2} - 102.68 = Q_{s_2}$$

$$5.04\,Q_{s_2} = 102.68$$

$$\boxed{Q_{s_2} = 20.37\ kJ/s}$$

$$0.70 = \frac{Q_{s_1} - 20.37}{Q_{s_1}}$$

$$0.70\,Q_{s_1} = Q_{s_1} - 20.37$$

$$0.70\,Q_{s_1} - Q_{s_1} = -20.37$$

$$0.3\,Q_{s_1} = 20.37$$

$$Q_{s_1} = 67.9$$

$$W_2 = Q_{s_2} - Q_{R_2}$$

$$= 20.37 - 17$$

$$W_2 = 3.37\ kJ/s$$

$$W_1 = W_2 + W_3$$

$$= 3.37 + 30$$

$$W_1 = 33.37\ kW \text{ (or) } kJ/s$$

$$0.70 = \frac{Q_{S_1} - Q_{R1}}{Q_S}$$

$$0.70\, Q_{S1} = Q_{S_1} - Q_{R1}$$

$$0.70\, Q_{S_1} = W_1$$

$$0.70\, Q_{S_1} = 33.37$$

$$\boxed{Q_{S_1} = 47.67 \text{ kJ/s}}$$

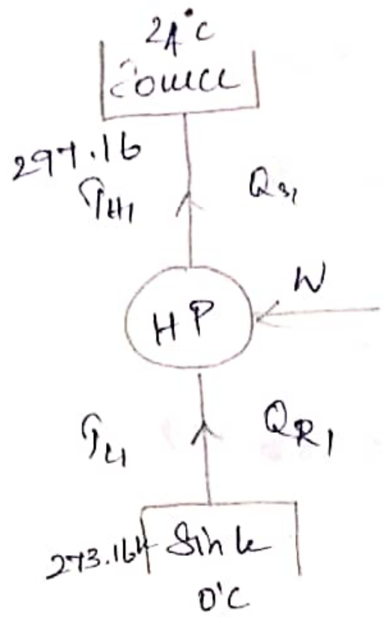
$$W_1 = Q_{S1} - Q_{R1}$$

$$33.37 = 47.67 - Q_{R1}$$

$$\boxed{Q_{R1} = 14.3 \text{ kJ/s}}$$

---

7. An office room was heated by a electrical resistance heater consuming 1200 kWhr of electrical energy in a winter month. Instead of this heater the same room is heated by a heat pump which is having 20% of COP of Carnot COP, the room temperature is 24°C and surrounding is 20°C if the heat supply from the surrounding by the heat pump is 0.65 kW. Determine COP and money saved per month. Assuming cost of electricity is ₹ 1.75 kWhr.



Carnot Actual

$$Cop = \frac{T_{H1}}{T_{H1} - T_{L1}} \neq \frac{Q_{S1}}{Q_{S1} - Q_{R1}}$$

$$Cop_{hp} = \frac{297.16}{297.16 - 273.16}$$

$$Cop_{hp} = 12.37 \times \frac{20}{100}$$

$$Cop_{hp} = 2.47 \text{ (actual)}$$

Electrical heater

1200 kwh.

Total expense on electrical heater = 1200 × 1.75

Rs = 2100

$$Cop_{hp} = \frac{Q_{S1}}{Q_{S1} - Q_{R1}}$$

$$2.47 = \frac{0.65}{0.65 - Q_{R1}}$$

$2.47 (0.65 - Q_{R1}) = 0.65$

$1.60 \ldots - 2.47 Q_{R1} = 0.65$

$2.47 Q_{R1} = 0.95$

$Q_{R1} = 0.38\ldots \text{ kW}$

$W = Q_{S1} - Q_{R1}$

$= 0.65 - 0.38\ldots$

$= 0.27 \text{ kJ} \times 60 \times 60$

$W = 0.27 \times 60 \times 60$

$= 9[illegible] \text{ kW} \times 1.75 \text{ kWhr}$

$Rs = 1701$

Total Saving $= 2100 - 1701$

$Rs = 399$

Entropy :-

It is Impossible by any procedure, No matter how idealised the system is it can't be brought to zero temperature in any finite number of operations. "It is commanly called third law of thermodynamics"

Entropy is an index of unavailability or degradation of energy; It can also be a molecular disorder or random function of a system. It is denoted by ΔS.

$$\Delta S = \frac{\text{Over all heat transfer}}{\text{Temperature}}$$

$$\boxed{\Delta S = \frac{dQ}{T}}$$ — clausius Inequality

$$ds = \frac{dQ}{T} \quad (\text{or}) \quad S_2 - S_1 = \frac{dQ}{T}$$

Entropy change for Ideal Gas

* In terms of temperature and Volume:-

$$S = f(T, V)$$

1^{st} law

$$Q = W + \Delta u$$

differentiate

$$dQ = dW + du$$

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$$dQ = Pdv + mc_v \Delta T$$

$\div$ by T

$$\frac{dQ}{T} = \frac{Pdv}{T} + \frac{mc_v \Delta T}{T}$$

$$ds = \frac{Pdv}{T} + \frac{mc_v \Delta T}{T}$$

$$PV = mRT$$

$$P = \frac{mRT}{V}$$

$$ds = \frac{mRT\,dv}{VT} + mc_v \frac{\Delta T}{T}$$

$$ds = \frac{mR\,dv}{V} + mc_v \frac{\Delta T}{T}$$

$$ds = mR \int_{V_1}^{V_2} \frac{1}{V} dv + mc_v \int_{T_1}^{T_2} \frac{1}{T} dt$$

$$ds = mR \left[\ln V_2 - \ln V_1\right] + mc_v \left[\ln T_2 - \ln T_1\right]$$

$$= mR \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right] + mC_v \ln\left[\frac{T_2}{T_1}\right] \quad \text{— (1)}$$

* In terms of Pressure and Temperature:-

$$S = f(P, T)$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{P_1}{T_1} \times \frac{T_2}{P_2} = \frac{V_2}{V_1}$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} \times \frac{T_2}{T_1}$$

① Eqn.

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2} \times \frac{T_2}{T_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + mR \ln\left[\frac{T_2}{T_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$R = C_p - C_v$$

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + m.C_p \ln\left[\frac{T_2}{T_1}\right] - mc_v \ln\left[\frac{T_2}{T_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + mc_p \ln\left[\frac{T_2}{T_1}\right] \quad \text{—②}$$

* In terms of Pressure and Volume :-

$$S = f(P, V)$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \times \frac{V_2}{V_1}$$

Sub ① eqn

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right] + m C_v \ln\left(\frac{V_2}{V_1} \times \frac{P_2}{P_1}\right)$$

$$S_2 - S_1 = mR \ln\left(\frac{V_2}{V_1}\right) + mc_v \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{P_2}{P_1}\right]$$

$$R = C_p - c_v$$

$$S_2 - S_1 = mc_p \ln\left[\frac{V_2}{V_1}\right] - mc_v \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{P_2}{P_1}\right]$$

$$S_2 - S_1 = mc_p \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{P_2}{P_1}\right] \quad \text{—③}$$

Important formula :-

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + mc_p \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mc_p \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{P_2}{P_1}\right]$$

Constant Volume Process [V=c]

Eqn (3)

$$S_2 - S_1 = mc_v \ln\left[\frac{P_2}{P_1}\right] + mc_p \ln\left[\frac{V_2}{V_1}\right]$$

$$S_2 - S_1 = mc_v \ln\left[\frac{P_2}{P_1}\right]$$

Eqn (1)

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mc_v \ln\left[\frac{T_2}{T_1}\right]$$

Constant Pressure Process [P=c]

eqn (2)

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + mc_p \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mc_p \ln\left[\frac{T_2}{T_1}\right]$$

Eqn (3)

$$S_2 - S_1 = mc_p \ln\left[\frac{V_2}{V_1}\right] + mc_v \ln\left[\frac{P_2}{P_1}\right]$$

$$S_2 - S_1 = mc_p \ln\left[\frac{V_2}{V_1}\right]$$

Constant Temperature Process $[PV = C]$

Eqn (1)

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right] + mC_v \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{V_2}{V_1}\right]$$

Eqn (2)

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right] + mC_p \ln\left[\frac{T_2}{T_1}\right]$$

$$S_2 - S_1 = mR \ln\left[\frac{P_1}{P_2}\right]$$

Adiabatic Process $[PV^{\gamma} = C]$

$$S_2 - S_1 = 0$$

Polytropic Process $[PV^n = C]$

$$S_2 - S_1 = mc_v\left[\frac{\gamma - n}{n}\right] \ln\left[\frac{P_1}{P_2}\right]$$

---

1. In Isothermal Process 1000 kJ/s of work is done by the system at a temperature of 200°C what is entropy change for the process.

Given data :-

$$W = dQ = 1000 \text{ kJ/s}$$

$$T = 200^\circ C = 473.16 K$$

$$\Delta S = \frac{dQ}{T}$$

$$PV = C$$

$$W = Q$$

$$\Delta S = \frac{1000}{473.16}$$

$$\boxed{\Delta S = 2.11 \ kJ/kgK}$$

---

2. Air in a closed vessel at fixed volume $0.15 m^3$ exerts a pressure of 12 bar at 250 °c. If the vessel is cooled so that pressure falls to 3.5 bar Determine (i) Final Temperature (ii) Heat transfer (iii) change in Entropy [Apr 13] 13 Mark

Given data :-

$$V = C$$

$$V_1 = 0.15 m^3 = V_2$$

$$P_1 = 12 \text{ bar} = 12 \times 10^5 \ N/m^2$$

$$T_1 = 250 °c = 523.16 K$$

$$P_2 = 3.5 \text{ bar} = 3.5 \times 10^5 \ N/m^2$$

To find

T_2, Q, ΔS

Formula :-

$$Q = mc_v [T_2 - T_1]$$

$$S_2 - S_1 = mc_v \ln\left[\frac{P_2}{P_1}\right]$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{12}{523.16} = \frac{3.5}{T_2}$$

$$\boxed{T_2 = 152.58\,k}$$

$$P_1V_1 = mRT_1$$

$$12 \times 0.15 = m \times 0.287 \times 523.16$$

$$m = \frac{12 \times 10^2 \times 0.15}{0.287 \times 523.16}$$

$$m = 1.1\,kg$$

$$m = \frac{bar \times m^3}{\frac{kJ}{kgK} \times K}$$

$$= \frac{10^2 \frac{kN}{m^2} \times m^3}{kJ/kg}$$

$$= \frac{10^2 kJ}{kJ/kg}$$

$$m = 10^2\,kg$$

Heat Transfer

$$Q = mc_v (T_2 - T_1)$$

$$= 1.1 \times 0.717 \; (152.58 - 523.16)$$

$$\boxed{Q = -292.27\;kJ}$$

Entropy

$$s_2 - s_1 = mc_v \ln\left[\frac{P_2}{P_1}\right]$$

$$s_2 - s_1 = 1.1 \times 0.717 \ln\left(\frac{3.5}{12}\right)$$

$$\boxed{s_2 - s_1 = -0.97\,kJ/k}$$

3. 5 m^3 of air at 27°C is compressed upto 6 bar polytropic were initial pressure is 2 bar it follows $PV^{1.3} = C$ It is subsequently expanded adiabatically to 2 bar Consider the two process are reversible. determine (i) Net work (ii) Net Heat Transfer (iii) change in entropy (iv) Plot PV, TS Diagram.

Given data :-

$V_1 = 5 m^3$

$T_1 = 27°C = 300.16 K$

$P_1 = 2 bar$

$P_2 = 6 bar$

Compression follow $PV^{1.3} = C$

$n = 1.3$ (polytropic)

Expansion $PV^{\gamma} = C$

$\gamma = 1.4$ (adiabatic)

$P_3 = 2 bar$

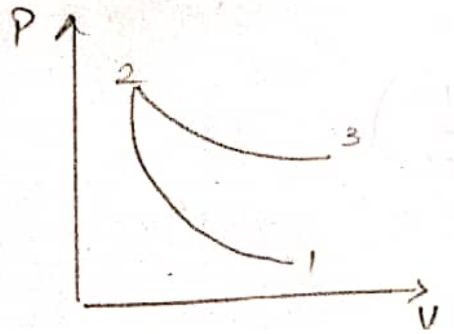
To Find

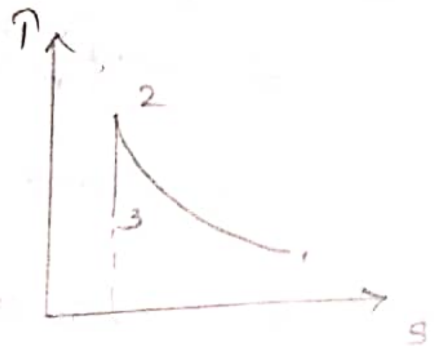
Network

Net heat

ΔS

PV - TS





Process 1-2 Polytropic

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$W = \frac{mR(T_1 - T_2)}{n-k}$$

Relation

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^n$$

$$\left[\frac{P_1}{P_2}\right]^{1/n} = \frac{V_2}{V_1}$$

$$\left[\frac{2}{6}\right]^{1/1.3} = \frac{V_2}{5}$$

$$V_2 = 2.14 \, m^3$$

work

$$W = \frac{P_1V_1 - P_2V_2}{n-1} \times 10^2$$

$$= \frac{2(5) - 6(2.14)}{1.3-1} \times 10^2$$

$$W = -946.6 \, kJ$$

Heat Transfer

$$Q = W \times \left(\frac{\gamma - n}{\gamma - 1}\right)$$

$$= -946.6 \left(\frac{1.4-1.3}{1.4-1}\right)$$

$$Q = -236.65 \, kJ$$

Entropy

$$S_2 - S_1 = mc_v \left(\frac{\gamma - n}{n}\right) \ln \left[\frac{P_1}{P_2}\right]$$

$$= 0.717 \left(\frac{1.4 - 1.3}{1.3}\right) \ln \left[\frac{2}{6}\right)$$

$$S_2 - S_1 = -0.060 \text{ kJ/kg K}$$

Process 2-3 Adiabatic

$$S_2 - S_1 = 0$$

$$Q = 0$$

$$W = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1}$$

relation

$$\frac{P_2}{P_3} = \left[\frac{V_3}{V_2}\right]^{\gamma}$$

$$\left[\frac{P_2}{P_3}\right]^{1/\gamma} = \frac{V_3}{V_2}$$

$$\left[\frac{6}{2}\right]^{1/1.4} = \frac{V_3}{2.14}$$

$$V_3 = 4.69 \text{ m}^3$$

work

$$W = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1}$$

$$= \frac{6(2.14) - 2(4.69)}{1.4 - 1} \times 10^2$$

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$W = 8.65 \times 10^2 \, kJ$

$W = 865 \, kJ$

Result

Net Work $= W_{12} + W_{2-3}$

$= -946.6 + 865$

Network $= -81.6 \, kJ$

Net Heat $= -236.65 \, kJ$

Entropy $= -0.060 \, kJ/kgK$

---

4. 5 kg of air at 2 bar and 30°C is compressed to 24 bar according to the process $PV^{1.2} = C$ it is followed by constant volume expansion process where temperature is kept at 30°C. Find volume and temperature of polytropic process and Entropy change in the process.

Given data :-

$m = 5 \, kg$

$P_1 = 2 \, bar$

$T_1 = 30°C = 303.16 \, K$

$P_2 = 24 \, bar$

$T_3 = 30°C = 303.16 \, K$

Compression follow $PV^{1.2} = C$ polytropic

$n = 1.2$

expansion $V = C$ constant volume

Find

P_2 · V, T_2, ΔS

Process 1-2 polytropic

relation

$$\left[\frac{P_1}{P_2}\right]^{\frac{n-1}{n}} = \frac{T_1}{T_2}$$

$$\left[\frac{2}{24}\right]^{\frac{1.2-1}{1.2}} = \frac{303.16}{T_2}$$

$$\boxed{T_2 = 458.7\ K}$$

$$P_1V_1 = mRT_1$$

$$2 \times V_1 = 5 \times 0.287 \times 303.16$$

$$V_1 = \frac{5 \times 0.287 \times 303.16}{2 \times 10^2}$$

$$\boxed{V_1 = 2.17\ m^3}$$

relation

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{n}$$

$$\left[\frac{2}{24}\right]^{\frac{1}{1.2}} = \frac{V_2}{2.17}$$

$$\boxed{V_2 = 0.27 m^3}$$

$$S_2 - S_1 = mc_v \left[\frac{\gamma - n}{n}\right] \ln \left[\frac{P_1}{P_2}\right]$$

$$= 5 \times 0.717 \left[\frac{1.4 - 1.2}{1.2}\right] \ln \left[\frac{2}{24}\right]$$

$$\boxed{S_2 - S_1 = -1.48 \text{ kJ/K}}$$

Process 2-3 $V = C$

$$S_2 - S_1 = mc_v \ln \left[\frac{T_3}{T_2}\right]$$

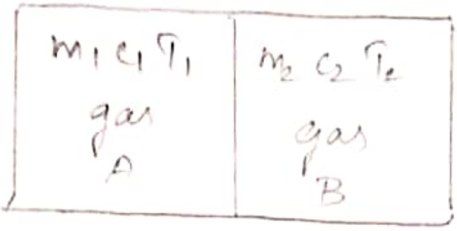
$$= 5 \times 0.717 \ln \left[\frac{303.16}{458.7}\right]$$

$$\boxed{S_2 - S_1 = -1.48 \text{ kJ/K}}$$

Entropy for Open System :-

Entropy for mixing of two fluids :-

Assume a reservoir which is divided into two sub-system where it is filled with quantity of mass each quantity of mass at sub-system are maintained at two different temperatures and two different specific heat values. So the intermediate temperature will be depending on mass, specific heat and temperature.



$$T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

$$\Delta S = 2 m_1 c_1 \ln \left[\frac{\left(\frac{T_1 + T_2}{2}\right)}{\sqrt{T_1 - T_2}} \right]$$

Entropy Balance Equation :-

Entropy entering the system + Entropy generated with in the System = Entropy leaving the System + Entropy change in the surrounding

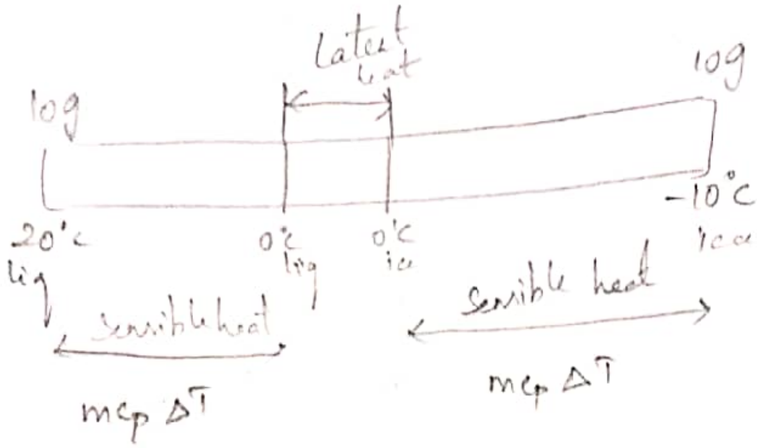
$$S_{in} + S_{gen} = S_{out} + S_{surr}$$

$$S_{in} - S_{out} + S_{gen} = S_{surr}$$

$$\boxed{S_{sys} + S_{gen} = S_{surr}}$$

---

1. 10 gram of water at 20°c is converted into Ice at −10°c at atmospheric pressure (P = C). Assuming specific heat of liquid water at 4.2 J/gK and that of Ice is half of the value. Taking the latent heat of fusion of Ice at 0°c is 335 J/g. Calculate Total Entropy of the System.



Given Data :-

$m = 10g$

$T_1 = 20°c$

$T_2 = -10°c$

$C_{p_{liq}} = 4.2 \ J/gK$

$C_{p_{ice}} = 2.1 \ J/gK$

$L = 335 \ J/g$

$S_{sys.} + S_{gen} = S_{surr}$

$S_{sys.} = \frac{dQ}{T}$

$dQ = Q_{(20° - 0°c)} + Q_{(0° - 0°c)} + Q_{(0° - -10°c)}$

$= mc_p \Delta T + mL + mc_p \Delta T$

$= m \left[c_p (T_2 - T_1) + L + c_p (T_2 - T_1) \right]$

$= 10 \left[4.2 (293.16 - 293.16) + 335 + 2.1 (263.16 - 273.16) \right]$

~~dQ = 4400~~

$dQ = 2300\ J$

$$ds_{sys} = \frac{dQ}{T}$$

$$= \frac{2300}{273.16}$$

$ds_{sys} = 8.42\ J/k$

~~Liq~~ Liq

$$\Delta S = mc_{p_{liq}} \ln\left[\frac{T_2}{T_1}\right]$$

$$\Delta S = 10 \times 4.2 \ln\left[\frac{273.16}{293.16}\right]$$

$\Delta S = -2.96\ J/k$

Ice

$$\Delta S = mc_{p_{ice}} \ln\left[\frac{T_2}{T_1}\right]$$

$$= 10 \times 4.2 \ln\left[\frac{263.16}{273.16}\right]$$

$\Delta S = -0.78\ J/k$

$S_{system} + S_{gen} = S_{surr}$

$S_{surr} = 8.42 - 2.96 - 0.78$

$S_{surr} = 4.68\ J/k$

1. the Velocity and Enthalpy of fluid at the inlet of the nozzle are 50 m/s and 2800 kJ/kg the enthalpy at the exist of nozzle is 2600 kJ/kg nozzle is horizontal and insulated so that no heat transfer takes place. Find (i) Velocity of the fluid at exist of the nozzle (ii) mass flow rate If area of inlet of nozzle is 0.09 m^2 and specific volume 0.185 m^3/kg (iii) Exist area of the nozzle If specific Volume at exist of nozzle is 0.495 m^3/g,

Given data :-

$V_1 = 50 m/s$

$h_1 = 2800 kJ/kg$

$h_2 = 2600 kJ/kg$

$a_1 = 0.09 m^2$

(i)

$$\omega_1\left[\frac{gz_1}{1000} + \frac{V_1^2}{2000} + h_1\right] + \frac{dQ}{dt} = \omega_2\left[\frac{gz_2}{1000} + \frac{V_2^2}{2000} + h_2\right] + \frac{dw}{dt}$$

$\frac{dQ}{dt} = 0$ $\frac{dw}{dt} = 0$

$z_1 = z_2$ $\omega_1 = \omega_2$

$$\frac{50^2}{2000} + 2800 = \frac{V_2^2}{2000} + 2600$$

$$\frac{V_2^2}{2000} = 201.25$$

$$V_2 = 634.42 \text{ m/s}$$

(ii)

$$w = \frac{A_1 V_1}{v_1}$$

$$= \frac{0.09 \times 50}{0.185}$$

$$w = 24.32 \text{ kg/s}$$

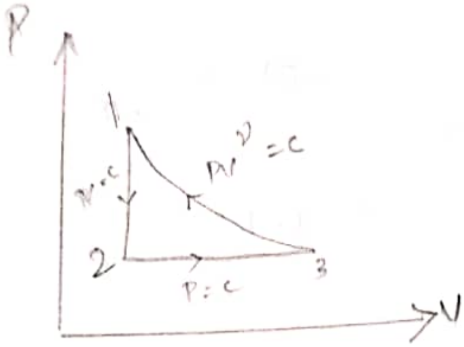
(iii)

$$w = \frac{A_2 V_2}{v_2}$$

$$24.32 = \frac{A_2 \times 634.42}{0.495}$$

$$A_2 = 0.0189 \text{ m}^2$$

). 90 kJ of heat is Supply to the System at a Constant Volume the system rejects 95 kJ of heat constant pressure and 18 kJ of work is done on it If system brought to original state by adiabatic process Determine (i) adiabatic work (ii) value of internal energy at all states If initial value is 105 kJ



Given data:

$Q_1 - 90\,kJ$

$Q_2 = 95\,kJ$

$W_2 - 18\,kJ$

Process$_{1-2}$

$Q_1 = W_1 + \Delta U_1$

$90 = 0 + \Delta U$

$\Delta U = 90\,kJ$

$U_2 - U_1 = 90$

$U_2 - 105 = 90$

$\boxed{U_2 = 195\,kJ}$

$\boxed{U_1 = 105\,kJ}$

Process$_{2-3}$

$Q_2 = W_2 + \Delta U$

$-95 = -18 + \Delta U$

$\Delta U = -77$

$u_3 - 195 = ~~\ldots~~ - 77$

~~$u_3 = \ldots$~~ $\boxed{u_3 = 118\ kJ}$

Process 3-1

$Q_3 = W_3 + \Delta u$

$0 = W_3 + u_1 - u_3$

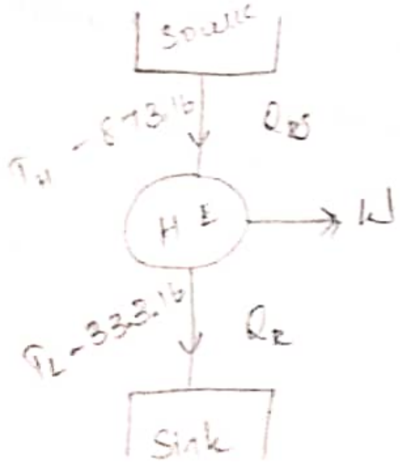
$0 = W_3 + (105 - 118)$

~~$W_3 = \ldots$~~

$\boxed{W_3 = 13\ kJ}$

---

3. A Heat Engine operates between a source at 600 [°C and a] sink 60 °c. Determine least rate of rejection per ~~kJ~~ kW net output of the Engine.



$$\eta = \frac{T_H - T_L}{T_H}$$

$$= \frac{873.16 - 333.16}{873.16}$$

$\eta = 0.61$.

Ideal

$$\eta = \frac{Q_s - Q_R}{Q_s}$$

$$0.61 = \frac{W}{Q_s}$$

Ideal $Q_s = 1.63$

$W = Q_s - Q_R$

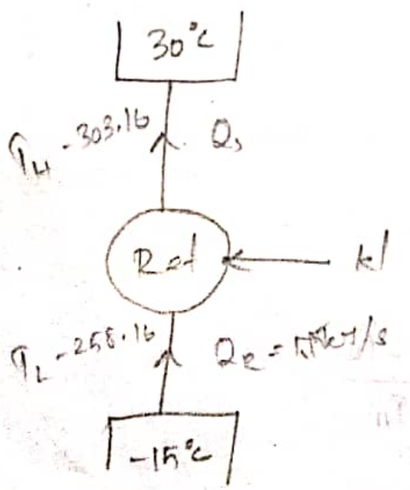
$1 = 1.63 - Q_R$

$-Q_R = 1 - 1.63$

$$\boxed{Q_R = 0.63 \text{ kW}}$$

---

4. A Domestic food freezer maintain temperature of −15°c the ambient air is 30°c If heat leaks into freezer at a continuous rate of 1.75 kJ/s what is the least power necessary to pump the heat outside continuously.



$$Cop = \frac{T_L}{T_H - T_L}$$

$$= \frac{258.16}{303.16 - 258.16}$$

$$Cop = 5.73$$

$$Cop = \frac{Q_R}{Q_S - Q_R}$$

$$5.73 = \frac{1.75}{Q_S - 1.75}$$

$$Q_S - 1.75 = \frac{1.75}{5.73}$$

$$Q_S = 2.05$$

$$W = Q_S - Q_R$$

$$W = 2.05 - 1.75$$

$$\boxed{W = 0.30\ kJ/s}$$

Drive expression for Isobaric and Isochoric

---

6. A Gas undergoes Thermodynamic cycle consisting of flowing process. Process 1-2 Constant Pressure $P_1 = 1.4$ bar, $V_1 = 0.028\ m^3$, $W = 10.5$ kJ.

Process 2-3 Compression, $PV = C$, $u_3 = u_2$.

Process 3-1 $V = C$ $u_1 - u_3 = -26.4$ kJ

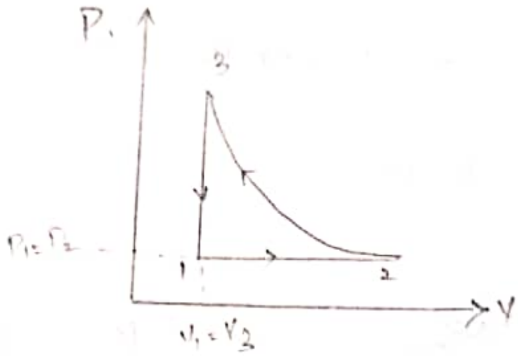
Find (i) sketch the PV Diagram (ii) calculate Network for the cycle

(iii) Calculate heat transfer for process 1-2

Show that $\Sigma Q = \Sigma W$

Net work = -8.2...

$Q_{1-2} = 36.9$ k...



Process 1-2   Constant Pressure

$P_1 = 1.4$ bar $V_1 = 0.028 m^3$ $W = 10.5 kJ$

$$W = P(V_2 - V_1)$$

$$10.5 = 1.4 \times 10^2 (V_2 - 0.028)$$

~~$10.5 = 1.4 \times 10^2 V_2 - 0.039$~~ $10.5 = 1.4 \times 10^2 V_2 - 3.92$

~~$V_2 = ...m^3$~~ $V_2 = 0.103$

$P_1 = P_2$

~~Process 2-3   PV = C~~

$P_2 = 1.4$ bar

~~$W = P_2 V_2 \ln \left[\frac{V_3}{V_2} \right]$~~

~~...~~

$$P_1 V_1 = m R T_1$$

$$10^2 \times 1.4 \times 0.028 = 1 \times 0.287 T$$

~~...~~ $T_1 = 13.68$ K

relation

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\frac{0.028}{0.103} = \frac{13.68}{T_2}$$

$$T_2 = 50.32k$$

<u>Process</u> 2-3 $PV=C$

~~$P_2V_2 = mRT_2$~~

~~$P_2 \times 0.103 = 1 \times 0.287 \times 50.32$~~

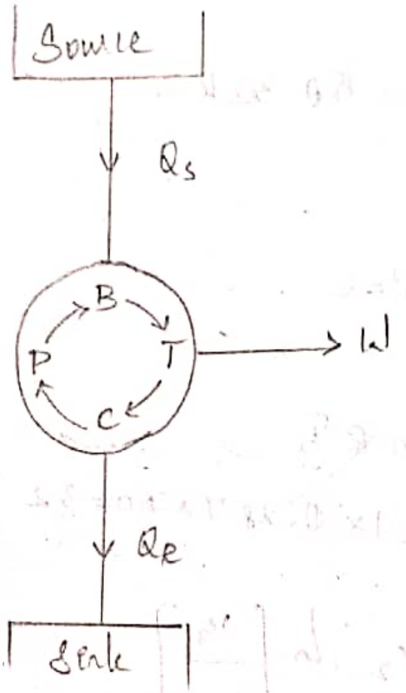
$$W = P_2V_2 \ln\left[\frac{V_3}{V_2}\right]$$

$$W = 1.4 \times 10^2 \times 0.103 \ln\left[\frac{V_3}{0.103}\right]$$

Unit - III

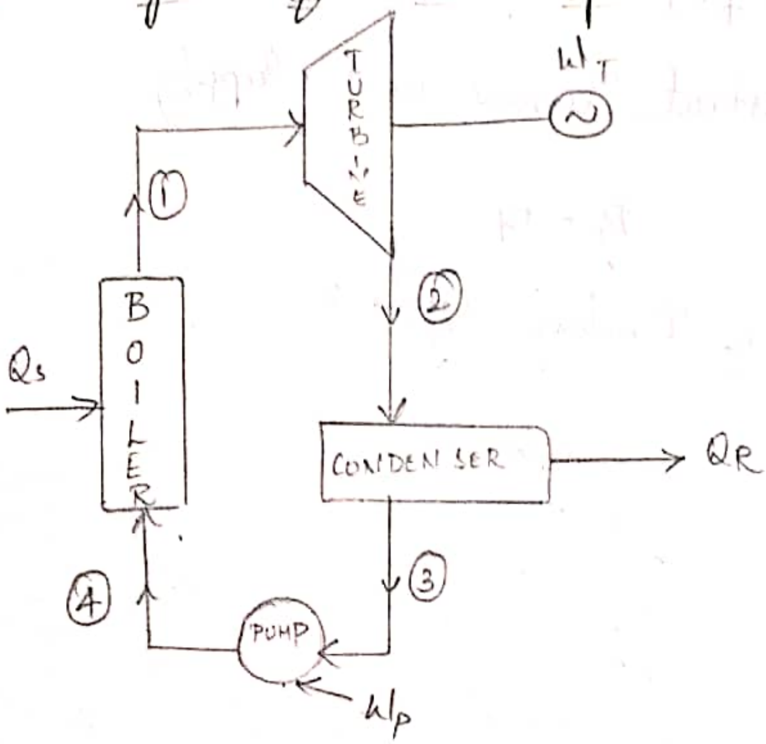
Phase change Process of A Pure Substance

Rankine Cycle :-



Rankine cycle is called as Vapour power cycle. It is also assumed as hypothetical cycle or Ideal cycle. It consists of four main parts (i) Boiler, (ii) Turbine, (iii) condenser, (iv) Pump.

Line Diagram of Rankine Cycle :-



Process 1-2

Isentropic Expansion of A Turbine (or) Isentropic Reversible Adiabatic Process

$$S_1 = S_2$$

Process 2-3

Constant Pressure Heat Rejection Process

$$P_2 = P_3$$

Process 3-4

Isentropic Compression or Reversible Adiabatic Process

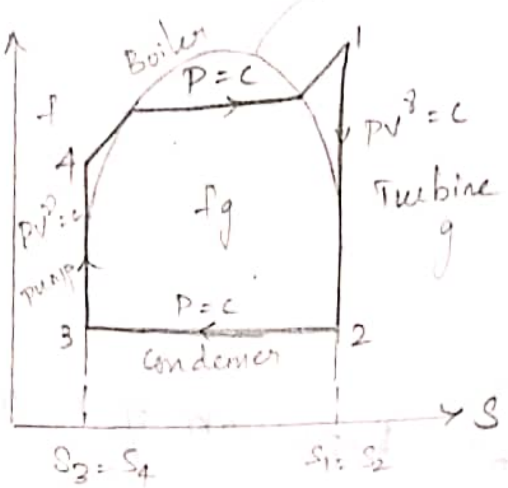
$$S_3 = S_4$$

Process 4-1

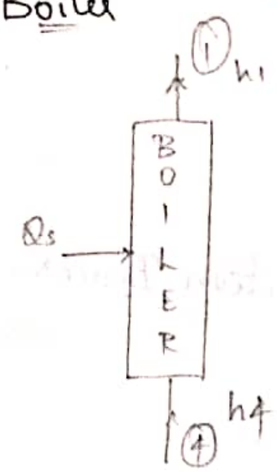
Constant Pressure heat Supply

$$P_1 = P_4$$

T-S of Rankine Cycle :-



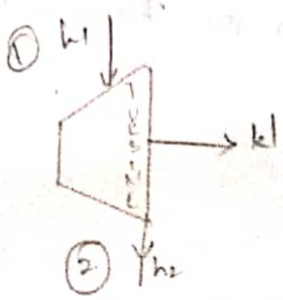
Boiler



$$h_4 + Q_S = h_1$$

$$Q_s = h_1 - h_4 \quad \text{—— (1)}$$

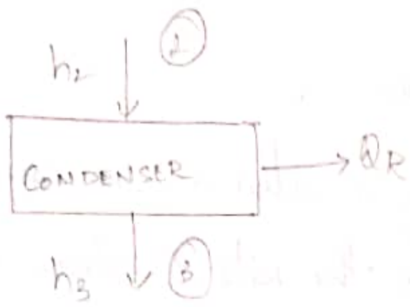
Turbine



$$h_1 = W + h_2$$

$$W = h_1 - h_2 \quad \text{—— (2)}$$

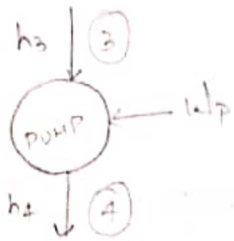
Condenser



$$h_2 = Q_R + h_3$$

$$Q_R = h_2 - h_3 \quad \text{—(3)}$$

Pump



$$h_3 + W_p = h_4$$

$$W_p = h_4 - h_3$$

$$\eta_{rankine} = \frac{\text{output}}{\text{Input}} = \frac{W}{Q_s}$$

$$\eta_{rankine} = \frac{h_1 - h_2}{h_1 - h_4}$$

Considering pump

$$\eta_{rankine} = \frac{W_T - W_p}{Q_s}$$

$$= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Pump work is negligible when compare to the amount Of work obtained by the Systen

$$h_4 - h_3 = V.f_2 (P_1 - P_2) \times 10^2$$

Steam rate or SSC :-

the capacity of a steam is expressed as steam rate, It is the rate of steam flow required to produce shaft output.

$$SSC = \frac{3600}{W_T - W_P} \text{ kg/kW-hr}$$

Heat Rate :-

the cycle efficency is expressed in terms of heat rate. which is rate of heat input recieve to produce unit work output

$$HR = \frac{3600}{\eta_{rankine}} \text{ kg/kW-hr}$$

1. A steam Boiler generates steam at 30bar 300°c at the rate of 2 kg/s the steam is expanded Isentropically in a turbine to a condenser pressure of 0.05 bar. It condensed at constant pressure process and pumped back into the Boiler. Find (i) Draw the plant diagram & T-S diagram of the rankine cycle, (ii) Find the heat supply in the Boiler, (iii) Determine the quality of the steam after expansion. (iv) what is the power generated by the turbine, (v) Find the rankine cycle efficiency considering pump work.

Given data :-

$P_1 = 30$ bar

$T_1 = 300°C$

$P_2 = 0.05$ bar

$w = 2$ kg/s

To find :-

T-s, plant

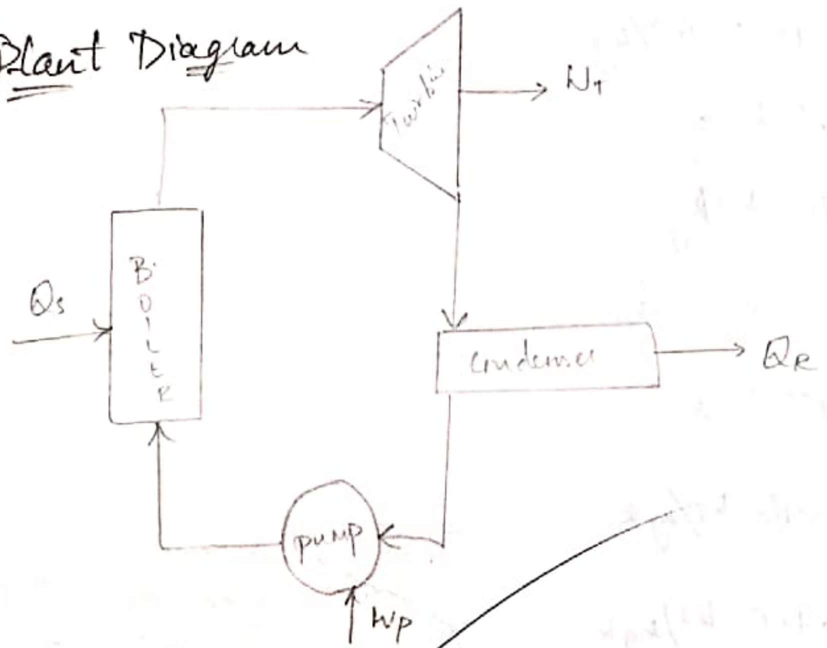
Q_s

x

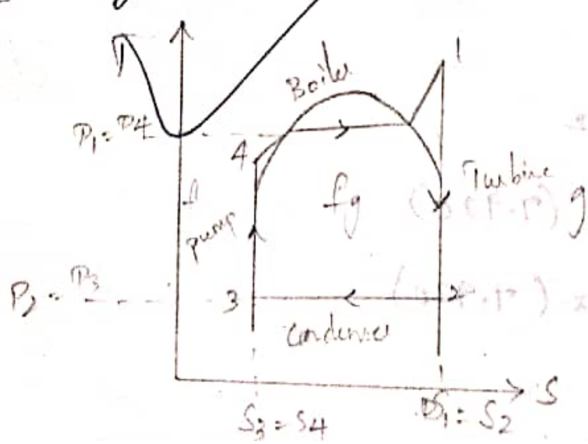
P

η

Plant Diagram



T-s Diagram



$P_1 = 30$ bar

$T_1 = 300^\circ C$

from steam table

$T_{sat} = 233.8^\circ C$

$T_1 > T_{sat}$

the steam is superheated

$v_1 = 0.08116 \ m^3/kg$

$h_1 = 2995.1 \ kJ/kg$

$s_1 = 6.542 \ kJ/kgK$

$P_2 = 0.05$ bar

$V_f = 0.001005 \ m^3/kg$

$h_f = 137.8 \ kJ/kg$

$h_g = 2561.6 \ kJ/kg$

$h_{fg} = 2423.8 \ kJ/kg$

$S_f = 0.476 \ kJ/kgK$

$S_g = 8.396 \ kJ/kgK$

$S_{fg} = 7.920 \ kJ/kgK$

$S_1 = S_2$

$S_2 = S_{f_2} + x S_{fg_2}$

$S_2 = 0.476 + x(7.920)$

$6.542 = 0.476 + x(7.920)$

$\boxed{x = 0.76}$

$$h_2 = h_{f_2} + x h_{fg_2}$$

$$h_2 = 137.8 + (0.76)(2423.8)$$

$$h_2 = 1979.8 \text{ kJ/kg}$$

$$h_3 = h_f = 137.8 \text{ kJ/kg}$$

Pump

$$W_p = h_4 - h_3 = V_{f_2}(P_1 - P_2) \times 10^2$$

$$h_4 - 137.8 = 0.001005\,(30 - 0.05) \times 10^2$$

$$h_4 = 140.8 \text{ kJ/kg}$$

$$\eta_{rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$= \frac{(2995.1 - 1979.8) - (140.8 - 137.8)}{(2995.1 - 140.8)}$$

$$\boxed{\eta_{rankine} = 0.35}$$

Heat Supply

$$Q_s = h_1 - h_4$$

$$Q_s = 2995.1 - 140.8$$

$$\boxed{Q_s = 2854.3 \text{ kJ/kg}}$$

Turbine Power

$$h_1 = W_T + h_2$$

$$W_T = h_1 - h_2$$

$$= 2995.1 - 1979.8$$

$$W_T = 1015.3 \text{ kJ/kg}$$

$\frac{kJ}{kg} \times \frac{kg}{s}$

$1 kW = 1 kJ/s$

$$W_T = 1015.3 \times 2$$

$$\boxed{W_T = 2030.6 \text{ kW}}$$

Result :-

* The heat Supply in the Boiler is 2854.3 kJ/kg
* the quality of steam after expansion is 0.76
* The Power generated by the turbine is 2030.6 kW
* The rankine cycle efficiency is 35%.

---

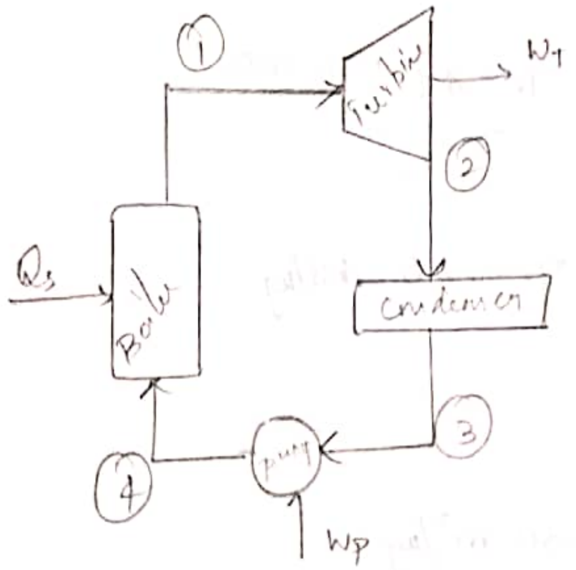
2 A steam at a pressure of 2.5 Mpa and 500°C expanded in a turbine to a condenser pressure of 0.05 Mpa. Determine Rankine cycle efficiency and Specific Steam Consumption, If the steam is reduced to 1 Mpa and kept at same temperature determine the efficiency.

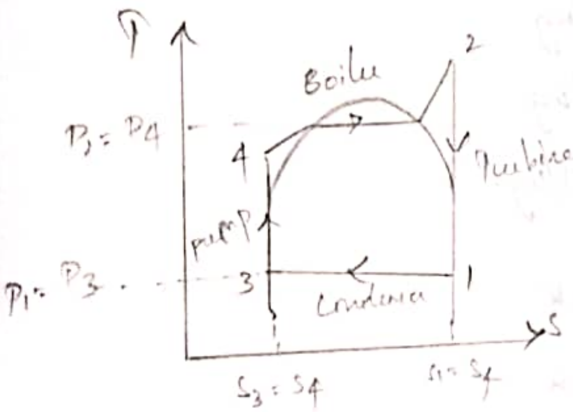
Given data

$P_1 = 2.5\ Mpa = 2.5 \times 10^6\ pa = 25\ bar$

$T_1 = 500°c$

$P_2 = 0.05\ Mpa = 0.05 \times 10^6\ pa = 0.5\ bar$





$P_1 = 25\ bar$

$T_1 = 500°c$

from steam table

$T_{sat} = 233.8\ °c$

$T_1 > T_{sat}$

The Steam is Superheated

$$h_1 = \frac{3462.9 + 3460.6}{2}$$

$$h_1 = 3461.75 \text{ kJ/kg}$$

$$S_1 = \frac{7.344 + 7.305}{2}$$

$$S_1 = 7.3245 \text{ kJ/kg}$$

$P_2 = 0.5$ bar

$V_f = 0.001030 \text{ m}^3/\text{kg}$

$h_f = 340.6$ kJ/kg

$h_g = 2646.0$ kJ/kg

$h_{fg} = 2305.4$ kJ/kg

$S_f = 1.091$ kJ/kg k

$S_g = 7.595$ kJ/kg k

$S_{fg} = 6.504$ kJ/kg k

$$S_1 = S_2$$

$$S_2 = S_{f_2} + x S_{fg_2}$$

$$7.3245 = 1.091 + x(6.504)$$

$$x = 0.95$$

$$h_2 = h_{f_2} + x\, h_{fg_2}$$

$$= 340.6 + 0.95\,(2305.4)$$

$$h_2 = 2530.73 \text{ kJ/kg}$$

$$h_3 = h_f = 340.6 \text{ kJ/kg}$$

$$h_4 - h_3 = V_{f2}\,(P_1 - P_2) \times 10^2$$

$$h_4 - 340.6 = 0.001030\,(25 - 0.5) \times 10^2$$

$$h_4 = 343.12 \text{ kJ/kg}$$

$$\eta_{rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

WT WP

$$= \frac{(3461.75 - 2530.73) - (343.12 - 340.6)}{(3461.75 - 343.12)}$$

$$\boxed{\eta_{rankine} = 0.29 = 29\,\%}$$

$$SSC = \frac{3600}{W_T - W_P}$$

$$= \frac{3600}{(3461.75 - 2530.73) - (343.12 - 340.6)}$$

$$\boxed{SSC = 3.87 \text{ kg/kW-hr}}$$

$P_1 = 10$ bar

$T_1 = 500°C$

from steam table

$T_{sat} = 179.9°C$

$T_1 > T_{sat}$

The steam is Super heated

$h_1 = 3478.3$ kJ/kg

$S_1 = 7.763$ kJ/kgk

$P_2 = 0.5$ bar

$V_f = 0.001030$ m³/kg

$h_f = 340.6$ kJ/kg

$h_g = 2646.0$ kJ/kg

$h_{fg} = 2305.4$ kJ/kg

$S_f = 1.091$ kJ/kgk

$S_g = 7.595$ kJ/kgk

$S_{fg} = 6.504$ kJ/kgk

$S_1 = S_2$

~~[illegible]~~

~~[illegible] kJ/kg~~

~~[illegible]~~ $S_2 = Sf_2 + x Sfg_2$

$7.763 = 1.091 + x\, 6.504$

$x = 1.0$[illegible]

$$h_2 = h_{f_2} + x\, h_{fg_2}$$

$$= 340.6 + (1.0)(2305.4)$$

$$h_2 = 2646 \text{ kJ/kg}$$

$$h_3 = h_f = 340.6 \text{ kJ/kg}$$

$$h_4 - h_3 = V_{f_2}(P_1 - P_2) \times 10^2$$

$$h_4 - 340.6 = 0.001030(10 - 0.5) \times 10^2$$

$$h_4 = 341.57 \text{ kJ/kg}$$

$$\eta_{rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$= \frac{(3478.3 - 2646) - (341.57 - 340.6)}{(3478.3 - 341.57)}$$

$$\boxed{\eta_{rankine} = 0.26 = 26\%}$$

$$SSC = \frac{3600}{W_T - W_P}$$

$$= \frac{3600}{(3478.3 - 2646) - (341.57 - 340.6)}$$

$$\boxed{SSC = 4.33 \text{ kg/kW-hr}}$$

3. Consider a steam power plant operation on ideal Rankine cycle the steam enters the turbine at 3 MPa and 623 K and it is condensed in a condenser at 10 kpa pressure. Find (i) the Thermal efficiency of the plant. (ii) If the steam is superheated to 873 K instead of 623 K what is the thermal efficiency (iii) If boiler pressure is raised to 15 MPa and temperature kept at 873 K find thermal efficiency.

Given data :-

$P_1 = 3\ Mpa = 30\ bar$

$T_1 = 623\ k = 350\ ^\circ C$

$P_2 = 10\ kpa = 0.1\ bar$

$P_1 = 30\ bar$

$T_1 = 350\ ^\circ C$

from steam table

$T_{sat} = 233.8\ ^\circ C$

$T_1 > T_{sat}$

The steam is super heated steam

$h_1 = 3117.5\ kJ/kg$

$v_1 = 0.09053\ m^3/kg$

$S_1 = 6.747\ kJ/kg\,k$

$P_2 = 0.1$ bar

$V_f = 0.001010$ m³/kg

$V_g = 14.675$ m³/kg

$h_f = 191.8$ kJ/kg

$h_g = 2584.7$ kJ/kg

$h_{fg} = 2392.9$ kJ/kg

$S_f = 0.649$ kJ/kgK

$S_g = 8.151$ kJ/kgK

$S_{fg} = 7.502$ kJ/kgK

$S_1 = S_2$

$S_2 = S_{f_2} + x\, S_{fg_2}$

$6.747 = 0.649 + x\, 7.502$

$x = 0.81$

$h_2 = h_{f_2} + x\, h_{fg_2}$

$h_2 = 191.8 + 0.81\,(2392.9)$

$h_2 = 2130.64$ kJ/kg

$h_3 = h_f = 191.8$ kJ/kg

$h_4 - h_3 = V_{f_2}\,(P_1 - P_2) \times 10^2$

$h_4 - 191.8 = 0.001010\,(30 - 0.1) \times 10^2$

$h_4 = 194.81$ kJ/kg

$$\eta_{rankine} = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_4)}$$

$$= \frac{(3117.5-2130.04)-(194.81-191.8)}{(3117.5-194.81)}$$

$$\boxed{\eta_{rankine} = 0.33 = 33\%}$$

Case (ii)

$P_1 = 30$ bar

$T_1 = 873$ k $= 600°K$

$P_2 = 0.1$ bar

$P_1 = 30$ bar

$T_1 = 600°C$

from steam table

$T_{sat} = 233.8$

$T_1 > T_{sat}$

The steam is super heated steam

$h_1 = 3681.0$ kJ/kg

$S_1 = 7.508$ kJ/kg k

$P_2 = 0.1$ bar

$S_1 = S_2$

$S_2 = S_{f_2} + x S_{fg_2}$

Scanned with OKEN Scanner

$7.508 = 0.649 + x(7.502)$

$x = 0.8491$

$h_2 = h_{f_2} + x h_{fg_2}$

$= 191.8 + 0.94 (2392.9)$

$h_2 = 2369.33 \text{ kJ/kg}$

$h_3 = h_f = 191.8 \text{ kJ/kg}$

$h_4 - h_3 = V_{f_2} (P_1 - P_2) \times 10^2$

$h_4 - 191.8 = 0.001010 (30 - 0.1) \times 10^2$

$h_4 = 194.81 \text{ kJ/kg}$

$$\eta_{rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$= \frac{(3681.0 - 2369.33) - (194.81 - 191.8)}{(3681.0 - 194.81)}$$

$\eta_{rankine} = 0.37 = 37\%$

Case (iii)

$P_1 = 150 \text{ Bar}$

$T_1 = 873 \text{ K} = 600°C$

$P_2 = 0.1 \text{ bar}$

$P_1 = 150$ bar

$T_1 = 600°C$

from steam table

$T_{sat} = 342.1 °K$

$T_1 > T_{sat}$

The steam is Super heated steam

$h_1 = 3579.8$ kJ/kg

$S_1 = 6.676$ kJ/kg K

$P_2 = 0.1$ bar

$S_1 = S_2$

$S_2 = S_{f_2} + x S_{fg_2}$

$6.676 = 0.649 + x\ 7.502$

$x = 0.80$

$h_2 = h_{f_2} + x\ h_{fg_2}$

$h_2 = 191.8 + 0.80 (2392.9)$

$h_2 = ~~\text{[illegible]}~~ 2106.12$ kJ/kg

$h_3 = h_f = 191.8$ kJ/kg

$h_4 - h_3 = V_{f_2} (P_1 - P_2) \times 10^2$

$h_4 - 191.8 = 0.001010 (150 - 0.1) \times 10^2$

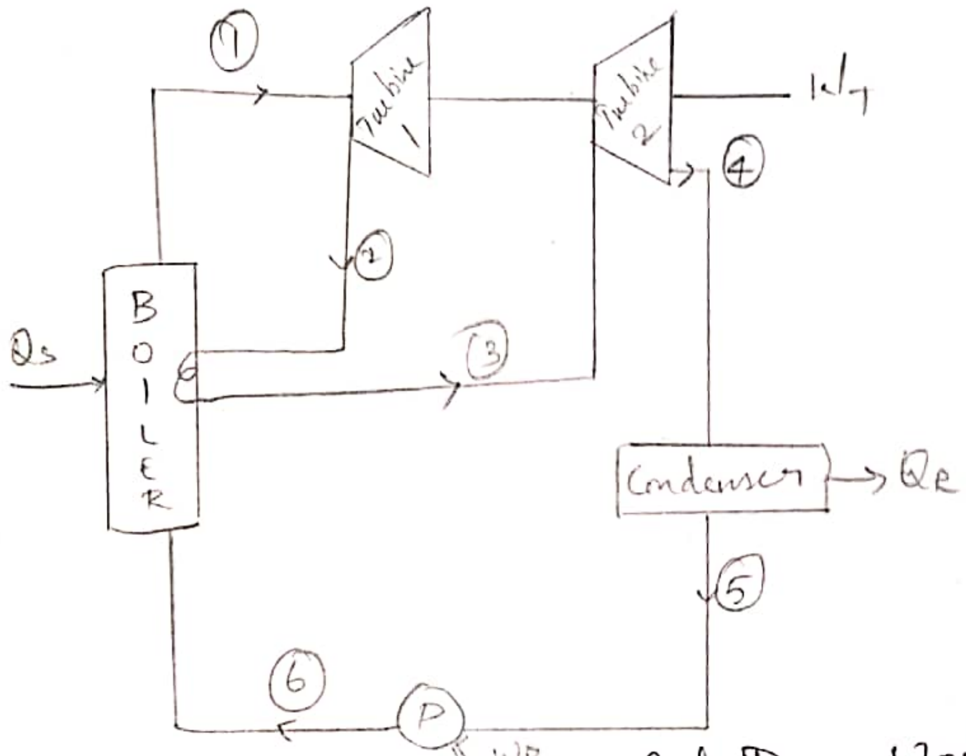
$h_4 = 206.93$ kJ/kg

$$\eta_{rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

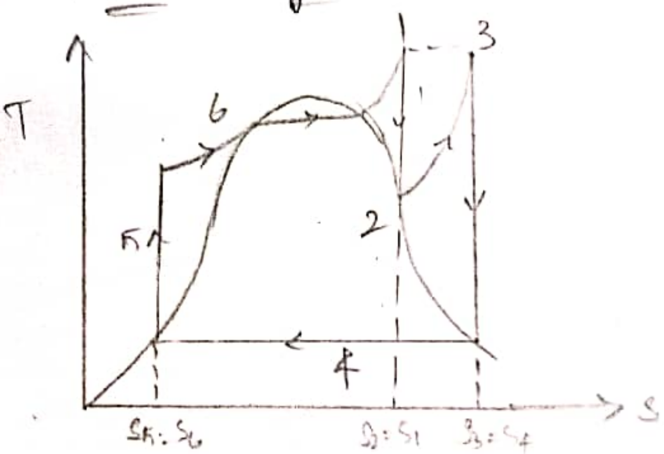
$$= \frac{(3579.8 - 2106.12) - (206.93 - 191.8)}{(3579.8 - 206.93)}$$

$$\boxed{\eta_{rankine} = 0.43 = 43\%}$$

Reheat - Rankine Cycle

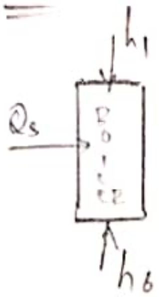


Reheat Rankine cycle with feel Pump Work



$$\eta_{reheat} = \frac{\text{Output}}{\text{Input}} = \frac{W_T - W_P}{Q_s} = \frac{W_{T_1} + W_{T_2} - W_P}{Q_{s_1}}$$

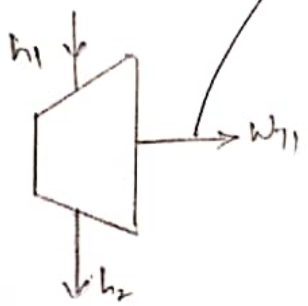
Boiler



$$h_6 + Q_{s_1} = h_1$$

$$Q_{s_1} = h_1 - h_6$$

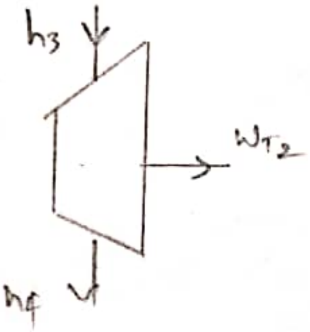
Turbine 1



$$h_1 = W_{T_1} + h_2$$

$$W_{T_1} = h_1 - h_2$$

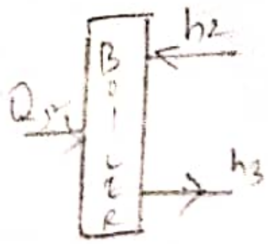
Turbine 2



$$h_3 = W_{T_2} + h_4$$

$$W_{T_2} = h_3 - h_4$$

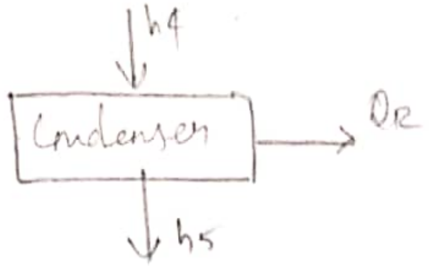
Heater 2



$$h_2 + Q_{s_2} = h_3$$

$$Q_{s_2} = h_3 - h_2$$

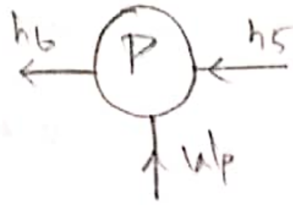
Condenser



$$h_4 = Q_R + h_5$$

$$Q_R = h_4 - h_5$$

Pump



$$h_5 + W_p = h_6$$

$$W_p = h_6 - h_5$$

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

---

1. A steam Power plant operating between a theoritical Reheat Ranking cycle, steam at a boiler 150 bar 550°C expands through a high pressure turbine. It is then reheated at a constant pressure of 40 bar 550°C and expands through a low pressure turbine to a condenser pressure of 0.1 bar. Draw T-S diagram. Find (i) Quality of the steam, (ii) cycle efficiency, (iii) Steam rate.

Given data:-

$P_1 = 150$ bar

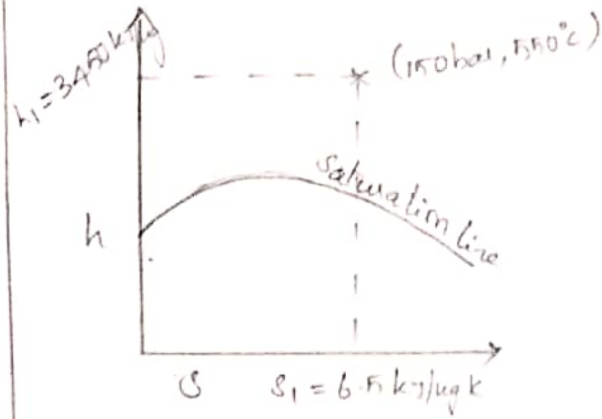
$T_1 = 550$ °C

$P_2 = P_3 = 40$ bar

$T_2 = 550$ °C

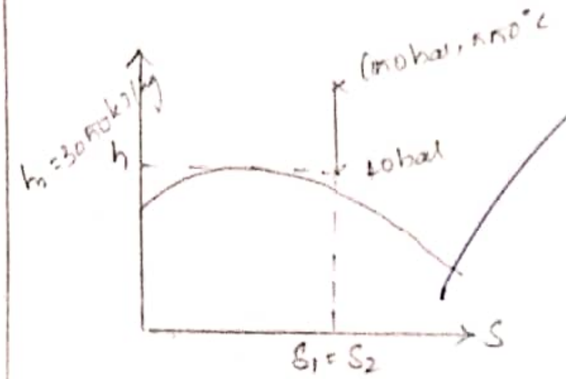
$P_4 = 0.1$ bar

By Using Moillier chart



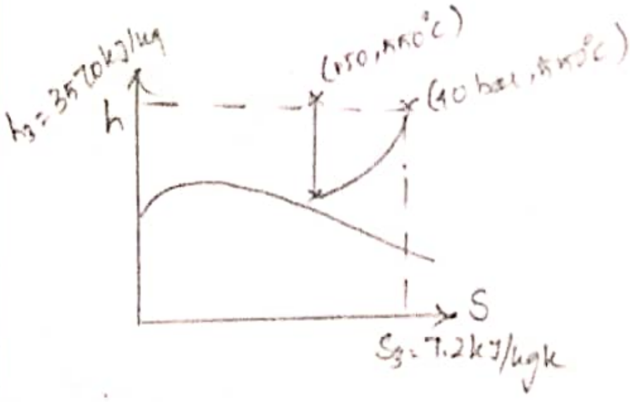
$h_1 = 3450$ kJ/kg

$S_1 = 6.5$ kJ/kgk



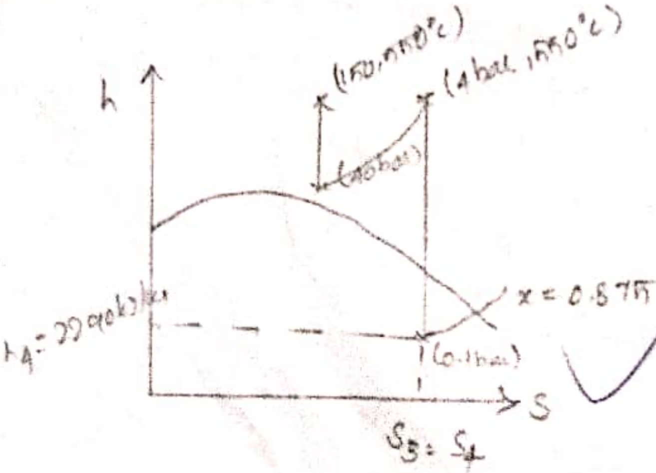
$h_2 = 3050$ kJ/kg

$S_1 = S_2 = 6.5$ kJ/kg



$h_3 = 3570$ kJ/kg

$S_3 = 7.2$ kJ/kgk



$h_4 = 2290$ kJ/kg

$S_3 = S_4 = 7.2$ kJ/kg

$x = 0.875$

$$x = \frac{0.85 + 0.9}{2}$$

$x = 0.875$

By using Steam Tables

$h_5 = h_{f4}$ (0.1 bar)

$h_5 = 191.8$ kJ/kg

Pump work

$(h_6 - h_5) = V_{f4}(P_1 - P_3) \times 10^2$

$h_6 - 191.8 = 0.001010\,(150 - 40) \times 10^2$

$h_6 = 202.9$ kJ/kg

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3570 - 2290) - (202.9 - 191.8)}{(3450 - 202.9) + (3570 - 3050)}$$

$\eta_{reheat} = 0.44 = 44\%$

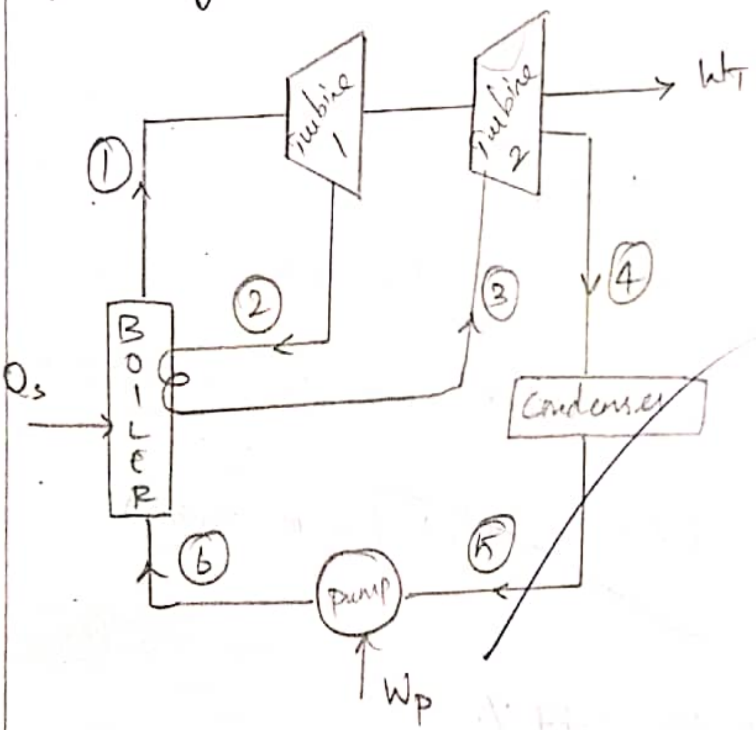
The quality of the steam is $x = 87.5\%$ of steam
12.5 % of water

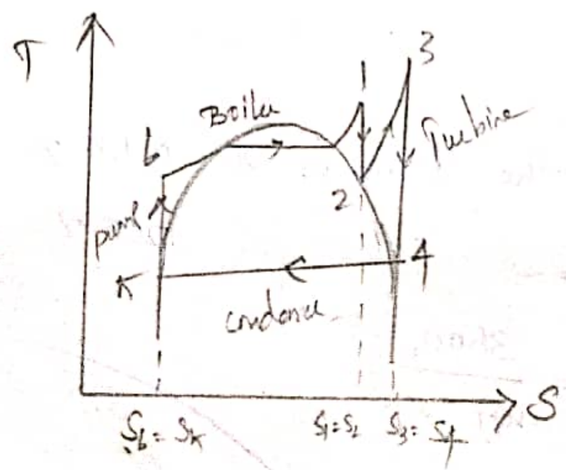
$$SSC = \frac{3600}{W_T - W_P}$$

$$= \frac{3600}{(3450 - 3050) + (3570 - 2290) - (202.9 - 191.8)}$$

SSC = 2.157 kg/kWh-hr

2. A Steam power plant operating on Ideal Reheat Rankine cycle the Steam enters the high pressure Turbine at 3 Mpa and 400°c after expansion to 0.6 Mpa the steam is reheated to 350°c and then expanded in a low pressure turbine and then the Condenser pressure is 10 kpa. Determine (i) thermal efficiency of the cycle, (ii) quality of the Steam.





Given data:-

$P_1 = 3\,Mpa = 30\,bar$

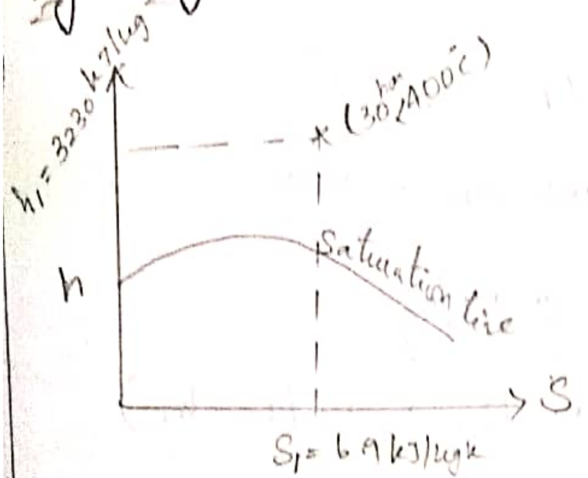
$T_1 = 400°c$

$P_3 = 0.6\,Mpa = 6\,bar$

$T_3 = 350°c$

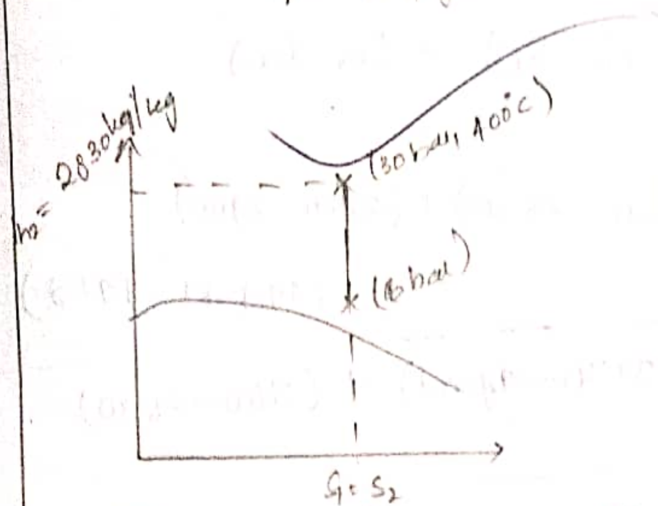
$P_4 = 10\,kp = 0.1\,bar$

By using Moiller chart



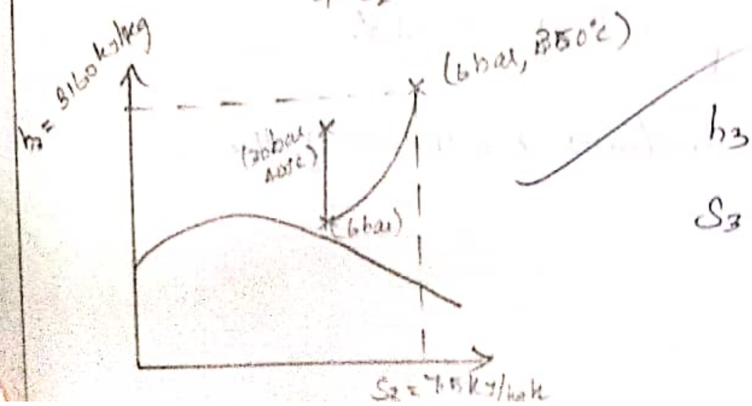
$h_1 = 3230\,kJ/kg$

$S_1 = 6.9\,kJ/kgk$



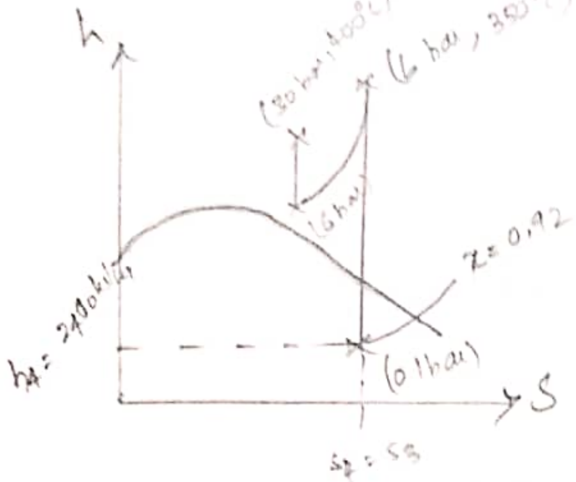
$h_2 = 2830\,kJ/kg$

$S_1 = S_2 = 6.9\,kJ/kgk$



$h_3 = 3160\,kJ/kg$

$S_3 = 7.5\,kJ/kgk$



$h_4 = 2400 \text{ kJ/kg}$

$S_3 = S_4 = 7.05 \text{ kJ/kgK}$

$x = 0.92$

By using steam table

$$h_5 = h_{f4} = 191.8 \text{ kJ/kg}$$

$$h_6 - h_5 = V_{f4}(P_1 - P_3) \times 10^2$$

$$h_6 - 191.8 = 0.001010 (30 - 0.1) \times 10^2$$

$$h_6 = 194.81 \text{ kJ/kg}$$

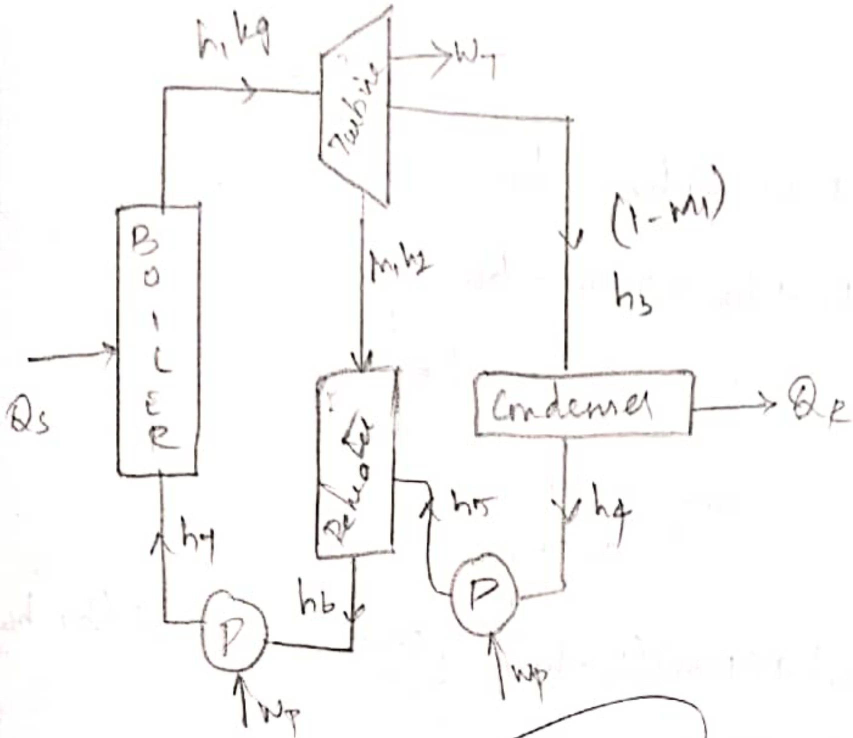
$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

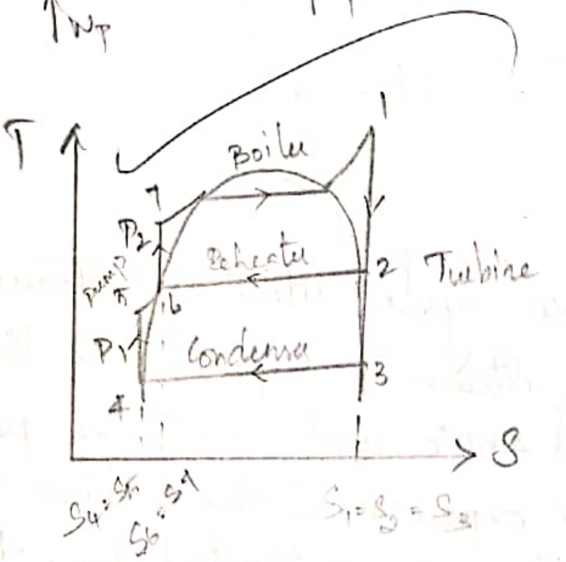
$$= \frac{(3230 - 2830) + (3160 - 2400) - (194.81 - 191.8)}{(3230 - 194.81) + (3160 - 2830)}$$

$$\boxed{\eta_{reheat} = 0.34 = 34\%}$$

The quality of steam $x = 92\%$ of the

Regenerative Rankine Cycle :





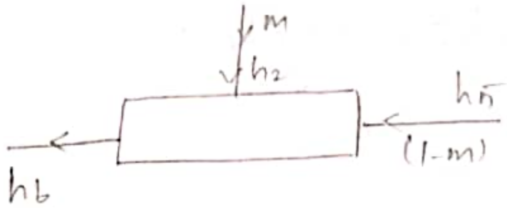
$$W_t = (h_1 - h_2) + (1-m)(h_2 - h_3)$$

$$W_p = (1-m)(h_5 - h_4) + (h_7 - h_6)$$

$$W_p = (1-m) \times v_{f4} (P_5 - P_4) \times 10^2 + v_{f6} (P_7 - P_6) \times 10^2$$

$$Q = h_1 - h_7$$

$$\eta = \frac{W_T - W_P}{Q_S}$$



$$m h_2 + (1-m) h_5 = h_6$$

$$m h_2 + h_5 - m h_5 = h_6$$

$$m(h_2 - h_5) = h_6 - h_5$$

$$m = \frac{h_6 - h_5}{h_2 - h_5}$$

$$\eta_{regenerative} = \frac{\left[(h_1-h_2) + (1-m)(h_2-h_3)\right] - \left[(1-m)(h_5-h_4) + (h_7-h_6)\right]}{(h_1-h_7)}$$

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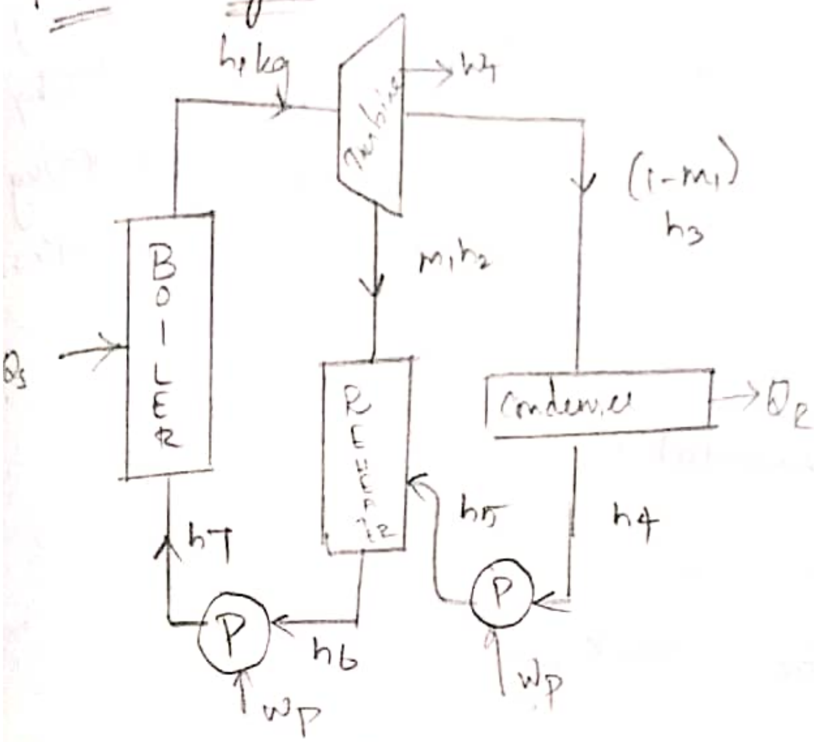
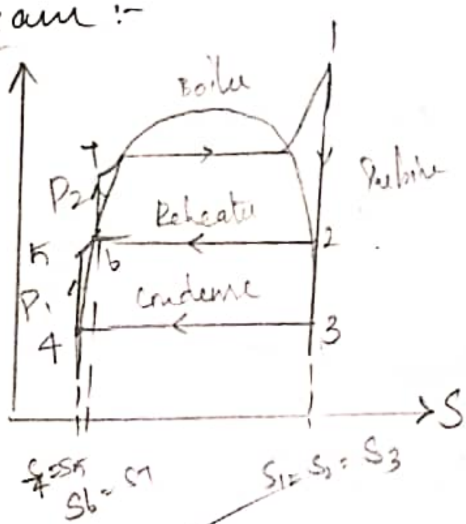
1. A Regenerative cycle utilizes steam as the work fluid. Steam is supplied to the turbine at 40 bar and 450°c and condenser pressure is 0.03 bar after expansion in the turbine to 3 bar some of the steam is extracted from the turbine for heating the feed water from the condenser in an open heater the pressure in the Boiler is 4 bar and the rate of fluid leaving the heater is saturated liquid water at 3 bar, Assuming Isentropic heat drop in the turbine compute the efficiency of the cycle.

Plant Diagram :-



T-S Diagram :-



Given Data :-

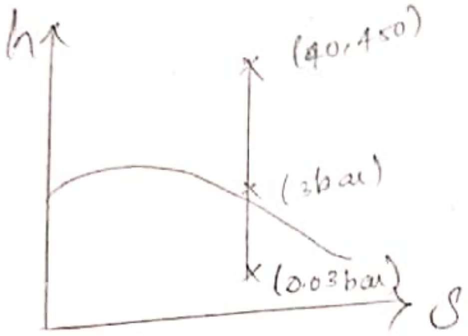
$P_1 = 40$ bar

$T_1 = 450^\circ C$

$P_3 = 0.03$ bar

$P_2 = 3$ bar

Using Mollier chart



$h_1 = 3330$ kJ/kg

$h_2 = 2700$ kJ/kg

$h_3 = 2050$ kJ/kg

$S_1 = S_2 = S_3 = 6.91$ kJ/kg

By Using Steam table

$P_3 = 0.03$ bar

$h_4 = h_{f3} = 101.$ kJ/kg

$P_2 = 3$ bar

$h_6 = h_{f2} = 561.5$ kJ/kg

Pump work

$W_{P_1} = (1-m)\, V_{f4} (P_5 - P_4) \times 10^2$

$W_{P_2} = V_{f6} (P_7 - P_6) \times 10^2$

$$m = \frac{h_6 - h_5}{h_2 - h_5}$$

Pump (2)

$h_7 - h_6 = (0.001074)(40-3) \times 10^2$

$h_7 - h_6 = 3.97$

$h_7 = 3.97 + 561.5$

$h_7 = 565.41$ kJ/kg

$h_5 = 103$ kJ/kg (assuming)

$$m = \frac{561.5 - 103}{2700 - 103}$$

$m = 0.176$ kg

$$\eta_{regenerative} = \frac{\left[(h_1 - h_2) + (1-m)(h_2 - h_3)\right] - \left[(1-m)(h_5 - h_4) + (h_7 - h_6)\right]}{(h_1 - h_7)}$$

$$= \frac{\left[(3330 - 2700) + (1 - 0.176)(2700 - 2050)\right] - \left[(1 - 0.176)(103 - 101) + (565.41 - 561.5)\right]}{(3330 - 565.4)}$$

$= 0.419$

$$\boxed{\eta_{regenerative} = 41.9\%}$$

Part-B(A)

1. A Vessel of Volume $0.04\ m^3$ contains a mixture of saturated water and steam at a temperature of 250°C the mass of the liquid present is 9 kg. Find the Pressure, Mass, Specific Volume, enthalpy, entropy and Internal Energy.

Given data

$V = 0.04\ m^3$

$T = 250°C$

$m_f = 9\ kg$

Solution

$T = 250°C$

By steam table

$P = 39.77\ bar$

$m = m_f + m_g$

$m = 9 + m_g$

$V = 0.04\ m^3$

$V_f = 0.001251\ m^3/kg$

$V_g = 0.050037\ m^3/kg$

Total Volume occupied by liquid

$V_f = m \times V_f$

$V_f = 9 \times 0.001251$

$V_f = 0.011259\ m^3$

Total Volume

$$V = V_f + V_g$$

$$0.04 = 0.011259 + V_g$$

$$V_g = 0.028741 \text{ m}^3$$

$$m_g = \frac{V_g \ (m^3)}{v_g \ (m^3/kg)}$$

$$m_g = \frac{0.02874}{0.050037}$$

$$m_g = 0.57 \text{ kg}$$

$$m = m_f + m_g$$

$$= 9 + 0.57$$

$$\boxed{m = 9.57 \text{ kg}}$$

Specific Volume of mixture (m^3/kg)

$$v = \frac{V}{m}$$

$$v = \frac{0.04}{9.57}$$

$$\boxed{v = 4.17 \times 10^{-3} \ m^3/kg}$$

Enthalpy

$h = h_f + x h_{fg}$

$V = V_f + x V_{fg}$

$4.17 \times 10^{-3} = 0.00125 + x (V_g - V_f)$

$4.17 \times 10^{-3} = 0.00125 + x (0.050037 - 0.001251)$

$x = 0.059$

$h = h_f + x h_{fg}$

$h = 1085.8 + (0.059)(1714.6)$

$h = 1186.96$ kJ/kg

$s = s_f + x s_{fg}$

$= 2.794 + (0.059)(3.277)$

$S = 2.942$ kJ/kgK

$u = h - PV$

$= 1186 - (397742 \times 4.17 \times 10^{-3})$

$u = 1170.3$ kJ/kg

A Reheat cycle operating between 30 bar and 0.04 bar has a Superheat and reheat temperature of 450 °C. The first expansion takes place till the steam is dry saturated and then reheat is given. ~~neglecting~~ ~~feed pu~~ Determine the Ideal cycle efficiency.

Given Data :-

$P_1 = 30$ bar

$P_2 = 0.04$ bar

$T_1 = 450$ °C

$h_1 = 3345$

$S_1 = 7.06$ kJ/kgK

$h_2 = 2720$

$h_3 = 3380$

$h_4 = 2480$

$h_{f4} = h_5 = 121.4$ kJ/kg

$V_f = 0.001004$ m³/kg

$(h_6 - h_5) = V_{f4}(P_1 - P_3) \times 10^2$

$h_6 - 121.4 = 0.001004 (30 - 0.04) \times 10^2$

$h_6 = 124.40$ kJ/kg.

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) \cdot (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$$\eta_{reheat} = 0.39$$

$$= 39\%$$

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2. Consider a steam engine power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 16 MPa and 873 k and is condensed in the condenser at a pressure 10 kPa. If the moisture content of the steam at the exit of the low pressure turbine is not to exceed 10.4 percent Determine the (i) pressure at which the steam should be reheated and (ii) thermal efficiency of the cycle. Assume that the steam is reheated to the inlet temperature of the high-pressure turbine.

$P_1 = 16\ MPa = 160\ bar$

$T_1 = 873\ k = 600^\circ C$

$P_3 = 0.1\ bar$

$x = 100 - 10.4$

$x = 0.896 \approx 0.9$

$h_1 = 3575$

$S_1 = 6.6$

$h_2 = 2480$

$h_3 = 3700$

$h_4 = 2880$

$h_{f4} = h_5 = 191.8$

$V_{f4} = 0.001010$

$h_6 - h_5 = V_{f4} \ (P_1 - P_2) \times 10^2$

$h_6 - 191.8 = 0.001010 \ (160 - 0.1) \times 10^2$

$h_6 = 207.94$

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$\eta_{reheat} = 0.41$

$= 41\%$

Unit-4

Ideal And Real Gases, Thermodynamic Relation

Q. Maxwell's Equations:

Maxwell's Equations relate the entropy to the three directly measurable properties such as P, V, T for pure simple compressible substance.

From first law of thermodynamics

$$\boxed{Q = W + \Delta u}$$

rearranging the Parameters

$$Q = \Delta u + W \qquad W = pdv, \quad Q = Tds$$

$$Tds = du + pdv$$

$$du = Tds - pdv \quad \text{—} (1)$$

we know that $\boxed{h = u + pv}$

differentiate

$$dh = du + d(pv)$$

$$[\because d(uv) = udv + vdu]$$

$$dh = du + pdv + vdp.$$

sub eqn. (1) in the above.

$$dh = Tds - pdv + pdv + vdp.$$

$$dh = Tds + vdp \quad \text{—} (2)$$

~~Lorem text~~

By Helmotz's function

$$\boxed{a = u - Ts.}$$

$$da = du - d(Ts)$$

$$da = du - Tds - sdT$$

Sub eqn. (1)

$$da = Tds - Pdv - Tds - SdT$$

$$da = -Pdv - Tds \quad \text{—— (3)}$$

By Gibbs function

$$\boxed{G = h - Ts}$$

$$dg = dh - d(Ts)$$

$$dg = dh - Tds - sdT$$

Sub eqn. (2)

$$dg = Tds + vdp - Tds - sdT$$

$$dg = Vdp - sdT \quad \text{—— (4)}$$

By Inverse Exact Differential,

Eqn. (1) $du = Tds - pdv$

$$\boxed{\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v} \quad \text{—— (5)}$$

Eqn. (2) $dh = Tds + Vdp$

$$\boxed{\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial s}\right)_P} \quad \text{—— (6)}$$

Eqn. (3)

$$da = -Pdv - sdT$$

$$+\left(\frac{\partial P}{\partial T}\right)_v = +\left(\frac{\partial s}{\partial v}\right)_T$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T} \quad \text{——} \quad (7)$$

Eqn. (4)

$$dg = vdp - sdT$$

$$\boxed{\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T} \quad \text{——} \quad (8)$$

There equations 5, 6, 7, 8 are maxwell's equations

Energy Equations

The Energy Equations are internal energy, enthalpy and entropy.

(i) Internal Energy (U)

we know that the internal energy

$$du = Tds - pdv$$

Let the entropy (s) is the function of Temperature (T) and specific volume (v)

$$s = f(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

$$du = T\,ds - p\,dv$$

Sub. ds in du equation,

$$du = T\left[\left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv\right] - p\,dv$$

$$du = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_v dv - P\,dv \quad \text{--- (1)}$$

Assume $du = c_v\,dT +$ constant

$$\boxed{c_v = T\left(\frac{\partial s}{\partial T}\right)_v}$$

From maxwell equations,

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

Sub c_v & $\left(\frac{\partial s}{\partial v}\right)_T$ value in eqn. (1)

$$du = c_v\,dT + T\left(\frac{\partial P}{\partial T}\right)_v dv - p\,dv \quad \text{--- (2)}$$

∴ The Internal energy, $du = c_v\,dT + T\left(\frac{\partial P}{\partial T}\right)_v dv - p\,dv$

From eqn. (2) the internal energy in terms of measurable properties such as P, v, T and c_v can be determined.

(iii) Enthalpy Relations

we know that

$$dh = Tds + vdp$$

Let the entropy (S) is the function of temperature (T) and Pressure (P).

$$S = f(T, P)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

sub ds in dh equation

$$dh = T\left[\left(\frac{\partial s}{\partial t}\right)_p dT + \left(\frac{\partial s}{\partial P}\right)_T dp\right] + vdp$$

$$dh = T\left(\frac{\partial s}{\partial t}\right)_p dT + T\left(\frac{\partial s}{\partial P}\right)_T dp + vdp \quad \text{—} (1)$$

Assume $dh = C_p dT + \text{constant}$

$$\boxed{C_p = T\left(\frac{\partial s}{\partial t}\right)_p}$$

from maxwell equation

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Sub in eqn. (1)

$$dh = C_p\, dT - T\left(\frac{\partial v}{\partial T}\right)_p dp + vdp \quad \text{—} (2)$$

from eqn. (2) the enthalpy in terms of measurable properties such as P, V, T and C can be calculated.

(iii) Entropy Relations (Tds Equations)

* Entropy as a function of T and P:

$$S = f(T, P)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

we know that

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial T}\right)_P = \frac{C_p}{T}$$

from maxwell equation

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

Sub

$$dS = \frac{C_p}{T} dT - \left(\frac{\partial V}{\partial T}\right)_P dP$$

× T on both side

$$\boxed{T\,dS = C_p\,dT - T\left(\frac{\partial V}{\partial T}\right)_P dP} \quad \text{——— (1)}$$

It is known as the first form of entropy equation (or) the first Tds equation.

* Entropy as a function of T and v :

$$S = f(T, V)$$

$$ds = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dv$$

w.k.T

$$C_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{C_v}{T}$$

from Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

sub

$$ds = \frac{C_v}{T} dT + \left(\frac{\partial P}{\partial T}\right)_v dv$$

×T on both side

$$T ds = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv \quad \text{--- (2)}$$

It is known as the Second form of entropy equation (or) Second Tds equation,

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* Entropy as a function of P and V :

$$S = f(P, V)$$

w.k.T

$$\frac{PV}{T} = C$$

$$PV = CT$$

$$S = f(C, T)$$

$$C = \text{Constant}$$

$$S = f(T)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial T}\right)_V dT$$

w.k.T

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_P$$

$$\frac{C_p}{T} = \left(\frac{\partial s}{\partial T}\right)_P$$

$$C_v = T\left(\frac{\partial s}{\partial T}\right)_V$$

$$\frac{C_v}{T} = \left(\frac{\partial s}{\partial T}\right)_V$$

sub

$$ds = \frac{C_p}{T} dT + \frac{C_v}{T} dT$$

$\times T$ on both side

$$\boxed{T\,ds = C_p\,dT + C_v\,dT} \quad \text{--- (3)}$$

It is known as the third form of entopy equation (or) third $T\,ds$ equation

(X) 4 mark

Ratio of Specific Heat Capacities

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_v$$

Ratio of specific heats $\gamma = \frac{C_p}{C_v}$

$$= \frac{T\left(\frac{\partial S}{\partial T}\right)_p}{T\left(\frac{\partial S}{\partial T}\right)_v}$$

$$\boxed{\gamma = \left(\frac{\partial S}{\partial T}\right)_p \left(\frac{\partial T}{\partial S}\right)_v}$$

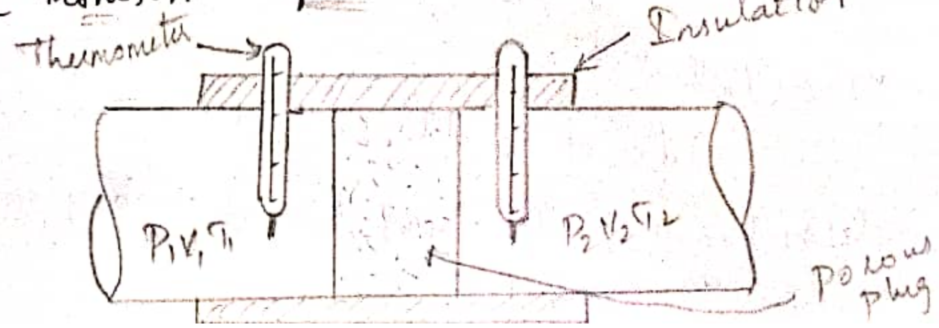
(X) 13 Mark

Joule-Thomson Coefficient :-

Joule-Thomson Coefficient is defined as the ratio of change in temperature to change in pressure at constant enthalpy. It is denoted by μ

$$\mu = \left(\frac{\partial T}{\partial P}\right)_h$$

Joule Thomson Experiment

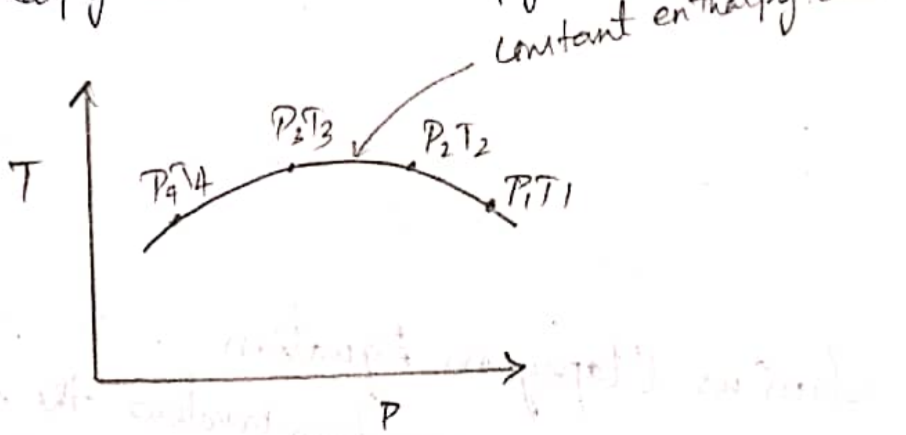


energy eqn. $gz_1 + \frac{V_1^2}{2} + h_1 + Q = gz_2 + \frac{V_2^2}{2} + h_2 + W$

Since there is no considerable change velocity, $V_1 = V_2$ and $z_1 = z_2$. $Q = 0$, $W = 0$, $V_1 = V_2$ and $z_1 = z_2$ are applied in the above equation.

$$h_1 = h_2$$

Enthalpy at Inlet = Enthalpy at Outlet



Case (i)

μ will be positive

This throttling process produces the cooling effect since the temperature reduces

case (ii)

μ ~~and~~ is negative

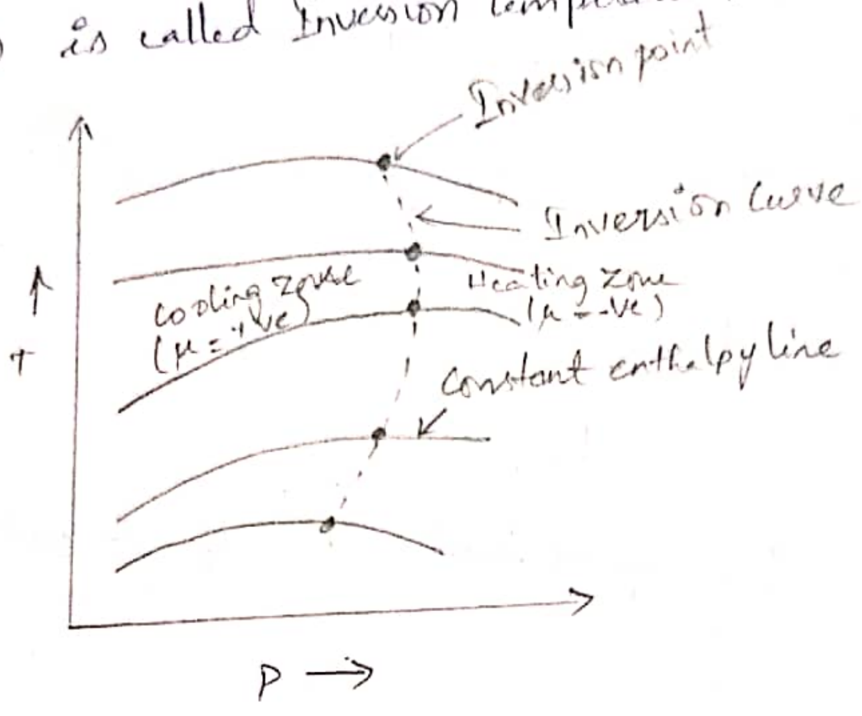
This throttling process produces the heating effect since the temperature increases.

Case (iii)

μ is zero the temperature of the gas will remain constant with throttling process.

Dedicated work
Completed

$\mu = 0$ is called Inversion temperature.



Inversion curve

Clausius Clapeyron Equation

Clapeyron equation involves the relationship between saturation pressure and saturation temperature.

Let, the entropy is a function of temperature and volume

$$S = f(T, V)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_V dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

when the phase is changing from saturated liquid to saturated vapour, the temperature remains constant. $[\because dT = 0]$

$$\partial s = \left(\frac{\partial s}{\partial v}\right)_T dv$$

from Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

sub

$$ds = \left(\frac{\partial P}{\partial T}\right)_v dv$$

$\left(\frac{\partial P}{\partial T}\right)$ - the slope of saturation curve

Integration on both side between saturated liquid (f) and saturated vapour (g)

$$\int_{S_f}^{S_g} ds = \frac{dP}{dT} \int_{V_f}^{V_g} dv$$

$$[s]_{S_f}^{S_g} = \frac{dP}{dT} [v]_{V_f}^{V_g}$$

$$S_g - S_f = \frac{dP}{dT} (V_g - V_f)$$

$$\frac{dP}{dT} = \frac{S_g - S_f}{V_g - V_f}$$

$$\frac{dP}{dT} = \frac{S_{fg}}{V_{fg}} \qquad \left[\because S_g - S_f = S_{fg},\ V_g - V_f = V_{fg}\right]$$

from second law of thermodynamics

$$ds = \frac{dQ}{T}$$

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for Constant pressure process

$$dQ = dh$$

$$ds = \frac{dh}{T}$$

$$\therefore S_{fg} = \frac{h_{fg}}{T}$$

Sub

$$\frac{dP}{dT} = \frac{S_{fg}}{V_{fg}}$$

$$\frac{dP}{dT} = \frac{h_{fg}}{V_{fg}\, T}$$

this equation is known as clausius-clapeyron (or) clapeyron equation.

At very low pressures, $V_g = V_{fg}$ is assumed.

$$\frac{dP}{dT} = \frac{h_{fg}}{T \times V_g}$$

W.K.T $\quad PV_g = RT$

$$V_g = \frac{RT}{P}$$

Sub $\quad \frac{dP}{dT} = \frac{h_{fg}\, P}{T \times RT}$

$$\frac{dP}{dT} = \frac{h_{fg} P}{RT^2}$$

$$\frac{dP}{P} = \frac{h_{fg}}{R} \frac{dT}{T^2}$$

Integrate on both side

$$\int_1^2 \frac{dp}{P} = \frac{h_{fg}}{R} \int_1^2 \frac{dT}{T^2}$$

$$\int_1^2 \frac{dP}{P} = \frac{h_{fg}}{R} \int_1^2 T^{-2} dT$$

$$\ln (P)_1^2 = \frac{h_{fg}}{R} \left[\frac{T^{-2+1}}{-2+1}\right]_1^2$$

$$\ln (P)_1^2 = \frac{h_{fg}}{R} \left[\frac{T^{-1}}{-1}\right]_1^2$$

$$\ln P_2 - \ln P_1 = \frac{h_{fg}}{R} \left[-\frac{1}{T}\right]_1^2$$

$$\ln P_2 - \ln P_1 = \frac{h_{fg}}{R} \left(\left[-\frac{1}{T_2}\right] - \left[-\frac{1}{T_1}\right]\right)$$

$$\ln \left[\frac{P_2}{P_1}\right] = \frac{h_{fg}}{R} \left[-\frac{1}{T_2} + \frac{1}{T_1}\right]$$

$$\ln \left(\frac{P_2}{P_1}\right) = \frac{h_{fg}}{R} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

∴ this equation is also known as Clausius-Clapeyron equation.

Problem :-

1. A Vessel of Volume 0.3 m^3 Contains 15 kg of air at 303k. Determine the pressure exerted by the air using
   1. Perfect gas equation
   2. Van der Waals equation
   3. Generalised compressibility chart.

Take critical temperature of air is 132.8 k and critical pressure of air is 37.7 bar.

Perfect Gas Equation :-

$$PV = mRT$$

$$P \times 0.3 = 15 \times 0.287 \times 303$$

$$\boxed{P = 4348.05 \text{ kpa}}$$

Vanderwalls Equations :-

$$\left[P + \frac{a}{v^2}\right](v-b) = RT$$

$$a = \frac{27 R^2 T_c^2}{64 P_c} = \frac{27(0.28)^2 (132.8)^2}{64(37.7) \times 10^2}$$

$$a = 0.162$$

$$b = \frac{RT_c}{8 P_c} = \frac{0.28 \times 13.8}{8 \times 37.7 \times 10^2}$$

$$b = 1.26 \times 10^{-3}$$

Specific Volume $v = \frac{V}{m} = \frac{0.3}{15}$

$$v = 0.02\ m^3/kg$$

$$\left(P + \frac{0.163}{(0.02)^2}\right)(0.02 - 1.26\times10^{-3}) = 0.287\times303$$

$$P + \frac{0.163}{(0.02)^2} = \frac{86.96}{0.018}$$

$$P + \frac{0.163}{(0.02)^2} = 4640.39$$

$$\boxed{P = 4232.89\ kpa}$$

Generalized Compressibility Chart :

$$T_r = \frac{T}{T_c} = \frac{303}{132.8}$$

$$T_r = 2.28$$

$$P_c V_c = R T_c$$

$$V_c = \frac{R T_c}{P_c}$$

$$V_r = \frac{V}{V_c} = \frac{V}{\frac{R T_c}{P_c}}$$

$$= \frac{0.02 \times 37.7\times10^2}{0.287\times132.8}$$

$$V_r = 1.97$$

from Compressibility chart

$$Z = 0.98$$

$$Z = \frac{PV}{RT}$$

$$P = \frac{0.98 \times RT}{V}$$

$$= \frac{0.98 \times 0.287 \times 303}{0.02}$$

$$\boxed{P = 4261.08 \text{ kpa}}$$

Unit - 5

Gas Mixture And Psychrometry

Psychrometry:

It is the Branch of Science which deals with moist air (mixing of dry air and water Vapour) Is known as Psychrometry. the air of high moisture Content can be measure using psychrometry properties

Psychrometry Properties

1. Dry Bulb Temperature:

The Temperature measure with normal Thermometer is called as Dry Bulb Temperature. It is denoted as t_d.

2. Wet Bulb Temperature:

Temperature measure by a thermometer by covering with wet cloths or cotton wool is called as wet bulb temperature. It is denoted by t_w.

3. Dew Point Temperature:

The Dew Point temperature is the temperature at which water Vapour present in the air begin to Condense, This temperature is called dew Point Temperature.

4. Wet Bulb Depression :

It is the difference between Dry Bulb Temperature and Wet Bulb Temperature

WBD = DBT − WBT

5. Dew Point Depression :-

It is the difference between Dry Bulb Temperature and Dew Point Temperature

DPD = DBT − DPT

6. Specific Humidity (or) Humidity Ratio (or) Moisture Content (ω) :

2 mark

It is defined as mass of water vapour present in 1 kg of Dry air. It is the ratio of mass of water vapour to mass of Dry air

$$\omega = \frac{\text{Mass of water Vapour}}{\text{mass of dry air}} = \frac{m_v}{m_a}$$

$$\omega = 0.622 \frac{P_v}{P_b - P_v}$$

P_b - Barometric pressure

P_v - Partial pressure of water Vapour

7. Saturation Ratio (μ):

It is the ratio of specific humidity of moist air to the specific humidity of saturated air at same Dry bulb temperature.

$$\mu = \frac{\omega}{\omega_s}$$

$$\mu = \frac{P_v}{P_s}\left[\frac{P_b - P_s}{P_b - P_v}\right]$$

8. Relative Humidity (ϕ):

It is the ratio of mass of water vapour present in the moist air to mass of water vapour present in the saturated air.

$$\phi = \frac{P_v}{P_s}$$

9. Total Enthalpy:

Total Enthalpy of moist air is the sum of enthalpy of dry air and enthalpy of water vapour associated with dry air

$$h = C_p t_d + \omega h_g$$

where, C_p - Specific heat at constant pressure
1.005 kJ/kgK

t_d - Dry Bulb Temperature

ω - Specific humidity

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h_g - Specific enthalpy of air corresponding to Dry Bulb Temperature.

Dalton's Law of Partial Pressure

~~Daltons~~ It states that total pressure of mixture of gases is some of partial pressure of each component.

$$P_b = P_a + P_v$$

$$P_v = P_{sw} - \frac{(P_b - P_{sw})(t_d - t_w)}{1527.4 - 1.3 t_w}$$

Important Formula

1. Density of air $= \rho_a = \frac{1}{V_g}$

2. Specific Humidity of Saturated Air $= \omega = 0.622 \left[\frac{P_s}{P_b - P_s}\right]$

3. Relative Humidity $\phi = \frac{P_v}{P_s} = \frac{m_v}{m_s}$

4. Mass of water Vapour $m_v = \frac{PV}{RT}$

1. Moist air at 45°c dry bulb temperature and 30 W Calculate vapour pressure, dew Point, specific humidity, relative humidity, degree of saturation, vapour density, Enthalpy of mixture.

(i) Vapour pressure :-

$$P_v = P_{sw} - \frac{(P_b - P_{sw})(t_d - t_w)}{1527.4 - 1.3 t_w}$$

$P_b = 1.01325$ bar

$$P_v = 0.04242 - \frac{(1.01325 - 0.04242)(45-30)}{1527.4 - 1.3(30)}$$

$$\boxed{P_v = 0.0326 \text{ bar}}$$

$P_v \simeq 0.03360$

(ii) Due point temperature :-

$$\boxed{DPT = 26°C}$$

(iii) Specific Humidity :-

$$\omega = 0.622 \left[\frac{P_v}{P_b - V}\right]$$

$$= 0.622 \left[\frac{0.0326}{1.01325 - 0.0326}\right]$$

$$\boxed{\omega = 0.0206 \text{ kJ/kg of dry air}}$$

(iv) Relative Humidity

$$\phi = \frac{P_v}{P_s}$$

$$\phi = \frac{0.0326}{0.09582}$$

$$\boxed{\phi = 34\%}$$

(v) Degree of Saturation :

$$\mu = \frac{P_v}{P_s}\left[\frac{P_b - P_s}{P_b - P_v}\right]$$

$$\boxed{\mu = 0.318}$$

(vi) Vapour Density :

$$\rho_g = \frac{1}{V_g}$$

DBT = V_g = 15.276 m^3/kg

$$\rho_g = \frac{1}{15.276}$$

$$\boxed{\rho_g = 0.065 \text{ kg/m}^3}$$

(vii) Enthalpy

$$H = C_p t_d + W h_g$$

$$H = 1.005(45) + 0.0206(2583.3)$$

$$\boxed{H = 98.44 \text{ kJ/kg}}$$

Adibatic Mixing of Air Stream :

* Human air conditioning will require mixing of two streams, which is a basic principle of centralized air conditioning.

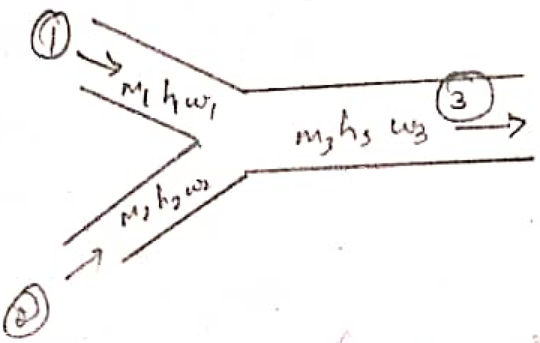
* It is applied for large buildings, like malls process plants and hospital etc...

* The principle of operation is just mixing of two streams

* The air conditioned air is mixed with outside air and supply to living room.

Assumptions :

* Surrounding is small
* Process is fully adibatic
* There is no work interaction
* Change in kinetic and potential Energies are negligible.



Enthalpy Balanced :-

$$m_1 h_1 + m_2 h_2 = m_3 h_3$$

$$m_1 h_1 + m_2 h_2 = (m_1 + m_2) h_3$$

$$m_1 h_1 + m_2 h_2 = m_1 h_3 + m_2 h_3$$

$$m_1 h_1 - m_1 h_3 = m_2 h_3 - m_2 h_2$$

$$m_1 (h_1 - h_3) = m_2 (h_3 - h_2)$$

$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3}$$

Humidity Balanced :-

$$m_1 \omega_1 + m_2 \omega_2 = m_3 \omega_3$$

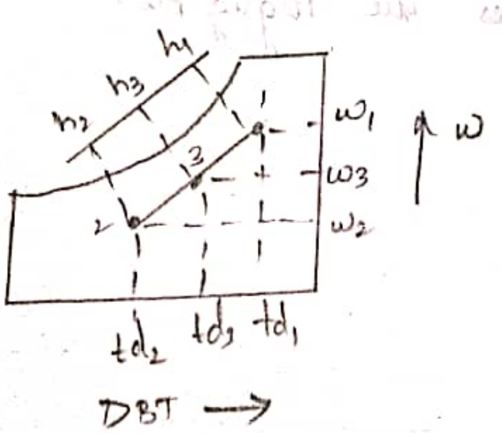
$$m_1 \omega_1 + m_2 \omega_2 = (m_1 + m_2)\, \omega_3$$

$$m_1 \omega_1 + m_2 \omega_2 = m_1 \omega_3 + m_2 \omega_3$$

$$m_1 \omega_1 - m_1 \omega_3 = m_2 \omega_3 - m_2 \omega_2$$

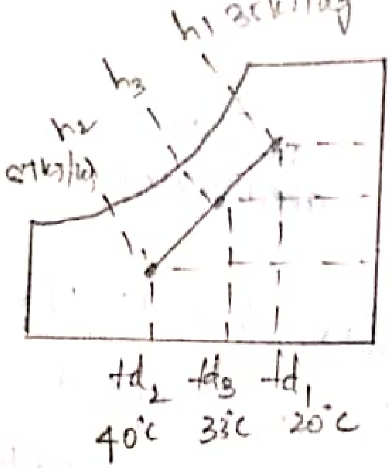
$$m_1 (\omega_1 - \omega_3) = m_2 (\omega_3 - \omega_2)$$

$$\frac{m_1}{m_2} = \left(\frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}\right)$$



Problem (Pg. 5.13)

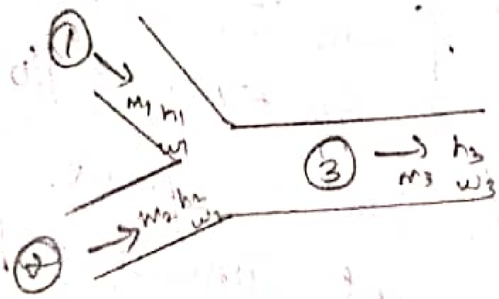
1. Air at 20°, 40% RH is mixed adiabatically with air 40°c, 40% RH in ratio of 1 kg of former with 2kg of latter (on dry bas). find final condition of air



$\omega_1 = 0.0058$ kJ/kg

$\omega_3 = 0.0142$ kJ/kg

$\omega_2 = 0.0185$ kJ/kg



$m_1 + m_2 = m_3$

$td_2 = 40^\circ C$

$RH_2 = 40$

$m_2 = 2\,kg$

$$\frac{m_1}{m_2} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}$$

$$\frac{1}{2} = \frac{\omega_3 - 0.0185}{0.0058 - \omega_3}$$

$$0.0058 - \omega_3 = 2\omega_3 - 0.037$$

$$\boxed{\omega_3 = 0.0142 \text{ kJ/kg}}$$

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Department of Mechanical Engineering

Lecture Notes

Subject Code : CE3391

Subject Name: FLUID MECHANICS AND MACHINERY

Sem/Year : 03/II

Regulation : 2021

FLUID PROPERTIES AND FLUID CHARECTERISTICS

static - fluid at rest

kinematics - fluid in motion & pressure force not considered

dynamics - fluid in motion & pressure force is considered.

Properties of fluid :

- Density (or) Mass density
- secific weight (or) weight density
- Specific volume
- Specific gravity
- Viscosity → dynamic viscosity, → Kinematic viscosity
- Compressibility & bulk modulus
- Surface tension & capillarity
- Vapour pressure
- Cavitation.

(i) Density (or) mass density: (ρ)

* Density is defined as ratio of mass of the fluid to volume of the fluid.

$$\rho = \frac{\text{mass of the fluid}}{\text{volume of the fluid}} = \frac{m}{v}$$

In SI units $\rho = kg/m^3$

(ii) Specific weight : (w)

* Specific weight is defined as ratio of weight of the fluid to volume of the fluid.

$$w = \frac{\text{weight of the fluid}}{\text{Volume of the fluid}}$$

$$= \frac{\text{mass} \times \text{acc}}{\text{volume}} = \frac{\text{mass} \times g}{\text{volume}}$$

$$w = \rho \times g$$

In SI units $w = N/m^3$

(iii) Specific volume:

Specific volume is defined as volume of the fluid to mass of the fluid is called specific volume.

$$= 1/\rho = \frac{\text{volume of fluid}}{\text{mass of fluid}}$$

In SI units $= m^3/Kg$

(iv) Specific gravity: (S)

Specific gravity is defined as density of unknown fluid to density of standard fluid.

$$S = \frac{\text{density of unknown fluid}}{\text{density of standard fluid}}$$

standard density of water $= 1000 \ kg/m^3$

for gases

$$S = \frac{\text{density of unknown gases}}{\text{density of standard gases}}$$

Standard density of air $= 1.18 \ kg/m^3$

4

Problem 1:

Calculate the specific weight, density and specific gravity of one litre of a liquid which weights 7 N.

Given:

Volume = one litre of liquid

weights = 7 N

To find:

Specific weight, density, specific gravity.

Sol:

Volume = 1 litre = 1000 cm^3 = 1000 $\times (10^{-2} m)^3$

$V = 10^3 \times 10^{-6}\ m^3 \Rightarrow V = 10^{-3}\ m^3$

(i) Specific weight (w) = $\frac{\text{weight}}{\text{Volume}}$

$= \frac{7}{10^{-3}} = 7000\ N/m^3$

(ii) density = $\frac{m}{V} \Rightarrow m = \frac{7}{9.81}$

$\rho = \frac{7 \times 1000}{9.81} = 713.5\ Kg/m^3$

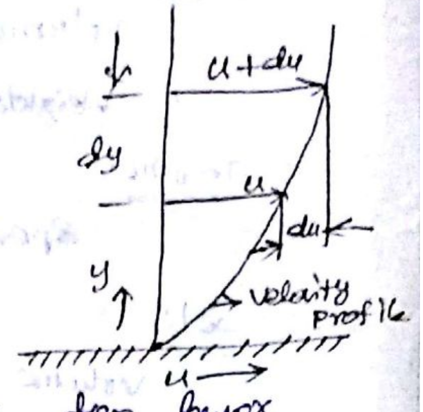
(iii) Specific gravity (S) = $\frac{\text{Density of fluid}}{\text{Density of water}}$

$= \frac{713.5}{1000}$

$S = 0.7135$

(v) <u>Viscosity</u>:

* Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



* The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

* This shear stress ~~on the adjacent~~ is proportional to the rate of change of velocity with respect to y.

* It is denoted by symbol τ (Tau)

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \rightarrow (1)$$

where

μ = dynamic viscosity

$\frac{du}{dy}$ = velocity gradient (or) rate of shear deformation.

from (1)

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

* Thus viscosity is defined as the shear stress required to produce unit rate of shear strain.

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units :

$$\mu = \frac{\text{shear force}}{\left(\frac{\text{change of velocity}}{\text{change of distance}}\right)} = \frac{\text{Force / Area}}{\left(\frac{\text{length}}{\text{time}}\right) \times \frac{1}{\text{length}}}$$

$$= \frac{\text{Force} / (\text{length})^2}{1/\text{Time}} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2}$$

SI unit

$$\mu = \frac{Ns}{m^2}$$

viscosity is also represent in CGS unit

one poise $= \frac{1}{10} \frac{Ns}{m^2}$

one centipoise $= \frac{1}{100}$ poise

Kinematic viscosity : (ν)

* It is defined as the ratio b/w the dynamic viscosity and density of fluid.

$$\nu = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho}$$

units : $\nu = \frac{\text{Force} \times \text{Time}}{(\text{length})^2 \times \frac{\text{mass}}{(\text{length})^3}} = \frac{(\text{mass} \times \text{acc}) \times \text{Time}}{\frac{\text{mass}}{\text{length}}}$

$$= \frac{\text{mass} \times \frac{\text{length}}{(\text{Time})^2} \times \text{Time}}{\frac{\text{mass}}{\text{length}}}$$

$$\nu = \frac{\text{length}^2}{\text{Time}} = m^2/s \quad (\text{SI units})$$

one stoke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s$

$= 10^{-4}\ m^2/s$

centistoke $= \frac{1}{100}$ stoke

2 mark

Newton's law of viscosity:

* It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau = \mu \frac{du}{dy}$$

* The constant of proportionality is called as co-efficient of viscosity (μ).

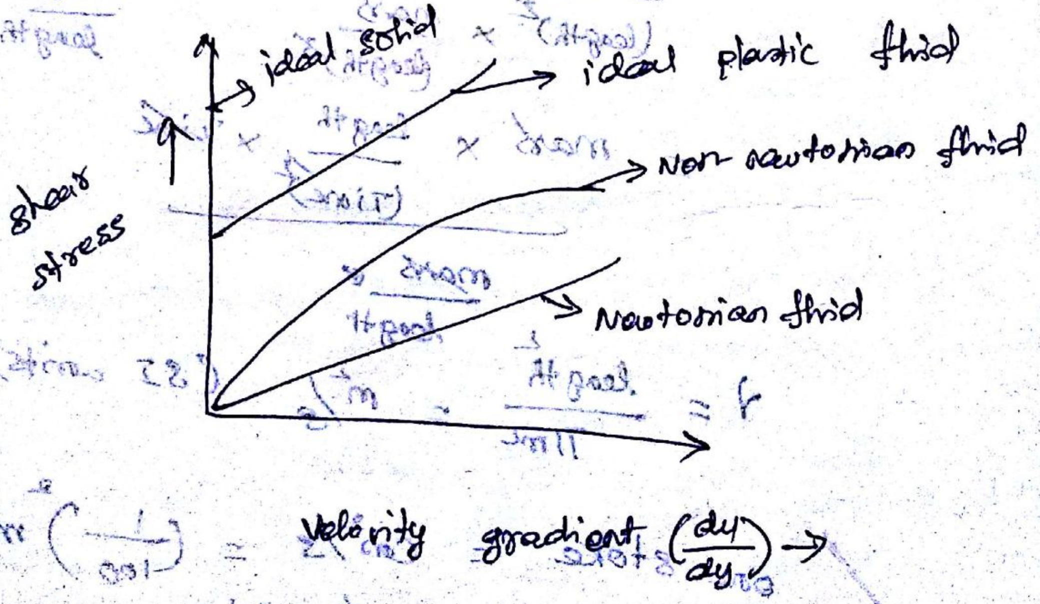
Newtonian fluids:-

* Fluids which obey the above relation are known as Newtonian fluids. Ex: Real fluid (water)

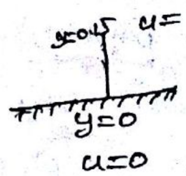
Non-newtonian fluids:-

* fluids which does not obey the above relation are non-newtonian fluids

Ex: Grease, tomato jam



B2: If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in metre per second, at a distance y metre above the plate, determine the shear stress at $y=0$ and $y=0.15$ m. Take dynamic viscosity of fluid as 8.63 poises. (8)



Given:

$u = \frac{2}{3}y - y^2$, F = 2 N/m²

$\text{viscosity} = 8.63 \text{ poises} = \frac{8.63}{10} \text{ NS/m}^2$

$(1 \text{ poise} = \frac{1}{10} \text{ NS/m}^2)$

To find:

~~viscosity~~ shear force at $y=0$ & $y=0.15$ m

Sol:

$u = \frac{2}{3}y - y^2 \longrightarrow ①$

$\text{shear force } (\tau) = \mu \frac{du}{dy}$

diff ① w.r.to y

$\frac{du}{dy} = \frac{2}{3} - 2y \Rightarrow \left.\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = 0.66$

$\left.\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2(0.15)$

$= 0.367$

shear force ~~viscosity~~ at $y=0$

$\tau)_{y=0} = \frac{8.63}{10} \times 0.66 = 0.5756 \text{ N/m}^2$

shear force at $y=0.15$

$\tau)_{y=0.15} = \frac{8.63}{10} \times 0.367 = 0.3167 \text{ N/m}^2$

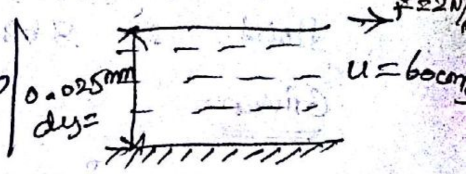
Result:

$\tau)_{y=0} = 0.5756 \text{ N/m}^2$

$\tau)_{y=0.15} = 0.3167 \text{ N/m}^2$

P3: A plate 0.025mm distant from a fixed plate, moves at 60cm/s and require a force of 2N per unit area i.e, 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.

Given:



$dy = 0.025 mm = 0.025 \times 10^{-3} m$

$F = 2N/m^2$

$u = 60 cm/s = 60 \times 10^{-2} m/s$

To find:

viscosity

Sol:

shear stress $\tau = \mu \frac{du}{dy}$

where change of velocity

$du = u - 0 = u = 0.60 m/s$

$dy = 0.02 mm = 0.025 \times 10^{-3} m$

$2 = \mu \frac{0.60}{0.025 \times 10^{-3}}$

$\mu = \frac{2 \times 0.025 \times 10^{-3}}{0.60}$

$\mu = 8.33 \times 10^{-5} Ns/m^2$

$\mu = 8.33 \times 10^{-5} \times 10$ poise) (1 poise $= 1/10 Ns/m^2$)

$\therefore \mu = 8.33 \times 10^{-4}$ poise.

Result:

$\mu = 8.33 \times 10^{-4}$ poise.

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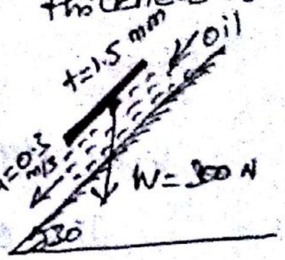
P4: Calculate the dynamic viscosity of an oil, which is used for lubrication between square plate of size 0.8m × 0.8m and an inclined plane with angle of inclination 30° as shown in figure. The weight of the square plate is 300N & it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5mm.



Given:

area = 0.8m × 0.8m

$\theta = 30°$

W = 300N

u = 0.3 m/s

To find:

viscosity.

$t = dy = 1.5mm = 1.5 \times 10^{-3} m$

Sol:

resolve the W along the plane

$F = W \cos 60° = 300 \times \cos 60 = 150N$

$\tau = \mu \frac{du}{dy}$

stress = F/A

$= \frac{150}{0.8 \times 0.8}$ N/m²

$\frac{150}{0.8 \times 0.8} = \mu \frac{0.3}{1.5 \times 10^{-3} m}$

(1 poise = 1/10 Ns/m²)

$\mu = 1.17$ Ns/m²

$\mu = 1.17 \times 10$

$\mu = 11.7$ poise

Result:

$\mu = 11.7$ poise

P5. The dynamic viscosity of an oil, used for lubrication b/w a shaft & sleeve is 6 poise the shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearings for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Given:

viscosity = 6 poise = $\frac{6}{10}$ = 0.6 Ns/m^2

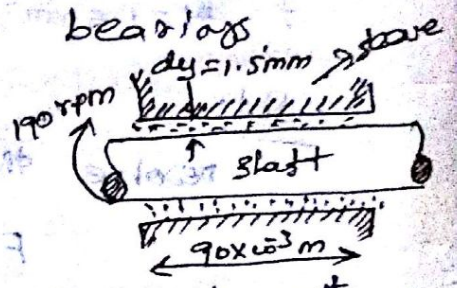
D = 0.4 m N = 190 rpm

length of sleeve L = 90 mm = 90×10^{-3} m

dy = 1.5 mm = 1.5×10^{-3} m

To find:

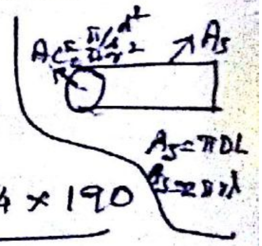
power lost in the bearings



Sol:

shear stress $\tau = \mu \frac{du}{dy}$

du = ?



W.K.T velocity of shaft $U = \frac{\pi D N}{60}$

$= \frac{\pi \times 0.4 \times 190}{60}$

U = 3.97 m/s

$\therefore du = U - 0 = 3.97$ m/s

$dy = 1.5 \times 10^{-3}$ m

$\therefore \tau = 0.6 \times \frac{3.97}{1.5 \times 10^{-3}} = 1588$ N/m^2

W.K. that
Shear stress = F/Area

F = $\tau \times A$

$$F = \frac{\overset{1588}{\cancel{1.58\times10^{-3}}}}{} \times \pi D L$$

$$= \overset{1588}{\cancel{1.58\times10^{-3}}} \times \pi \times 0.4 \times 0.09$$

$$F = \cancel{1.78\times10^{-4}\ N}\ 179.598\ N$$

Torque on the shaft

$$T = F \times r$$

$$= F \times D/2 = \overset{179.598}{\cancel{1.78\times10^{-4}}} \times 0.2$$

$$T = \cancel{3.573\times10^{-5}\ Nm}\ 35.919\ Nm$$

∴ power loss

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\times\pi\times190\times \overset{35.919}{\cancel{3.573\times10^{-5}}}}{60}$$

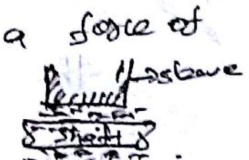
$$P = \cancel{7.109\times10^{-4}\ W}$$

$$P = 714.68\ W$$

Result:

power loss $P = \cancel{7.19\times10^{-4}\ W}\ 714.68\ W$

P6: A Newtonian fluid is filled for the clearance b/w a shaft and a concentric sleeve. The sleeve attains a speed of 50cm/s, when a force of 40 N is applied to the sleeve parallel to the shaf. determine. the speed if a force of 200 N is applied.



Given:

velocity(u_1) = 50 cm/s = 50×10^{-2} m/s

$F_1 = 40$ N

$F_2 = 200$ N

To find:

u_2 = ? when force $F_2 = 200$ N

Sol:

shear stress $\tau = \mu \frac{du}{dy}$

where $\tau = \frac{F}{A}$ $du = u - 0$

$dy = y$

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$$\therefore F_1 = \mu A \frac{du_1}{dy} \quad \{\mu, A, y \text{ are constant}\}$$

so $F_2 = \mu A \frac{du_2}{dy}$

$\div \frac{F_1}{F_2}$ ~~$F \propto u$~~

$\therefore$ ~~$\frac{F_1}{u_1} = \frac{F_2}{u_2}$~~

$$\frac{F_1}{F_2} = \frac{du_1}{du_2}$$

$$du_2 = u_2 = \frac{du_1 \times F_2}{F_1}$$

$$= \frac{0.5 \times 200}{40}$$

$$u_2 = 2.5 \text{ m/s}$$

Result:

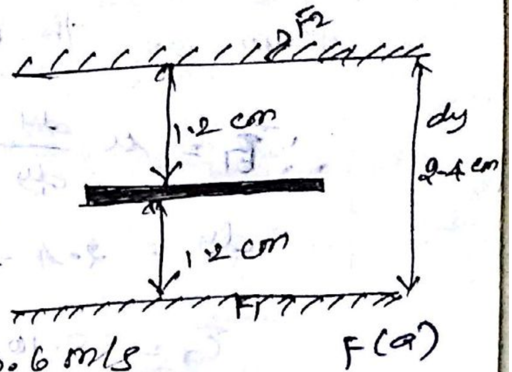
when $F_2 = 200\,N$ means $u_2 = 2.5$ m/s

P7: Two large plane surfaces are 2.4 cm apart the space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if:

(i) The thin plate is in the middle of the two plane surfaces.

(ii) The thin plate is at a distance of 0.8 cm from one of the plane surfaces? Take the dynamic viscosity of glycerine = 8.1×10^{-1} Ns/m^2 (16)

Given:



Distance b/w two large surfaces } = 2.4 cm
$= 2.4 \times 10^{-2}$ m

Area of thin plate (A) = 0.5 m^2

Velocity of thin plate $u = 0.6$ m/s

Viscosity of glycerine $\mu = 8.10 \times 10^{-1}$ Ns/m^2

Sol:

Case 1: F(a)

when thin plate is placed at middle of the surface what force is required?

F_1 = shear force on the upper side plate

F_2 = shear force on the lower side plate

$\therefore F = F_1 + F_2$

$\therefore \tau_1 = \mu \frac{du}{dy}$ (du = u - 0 = 0.6 velocity is both middle and both side of the plate)

dy = 1.2×10^{-2} m

$\tau_1 = 8.10 \times 10^{-1} \times \frac{0.6}{1.2 \times 10^{-2}} = 40.5$ N/m^2

$F_1 = \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly

$\tau_2 = 8.10 \times 10^{-1} \times \frac{0.6}{1.2 \times 10^{-2}} = 40.5$ N/m^2

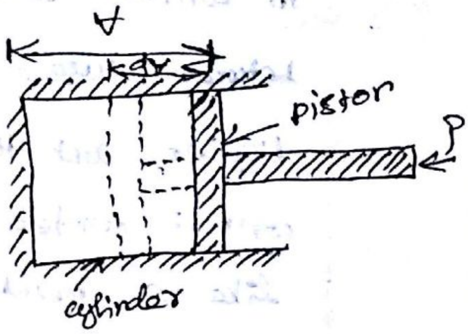
$F_2 = 20.25$ N

$\therefore F = F_1 + F_2 = 20.25 + 20.25$

$F = 40.50$ N //

(vi) Compressibility and bulk modulus:

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.



∴ Bulk modulus $K = \dfrac{\text{Increase of pressure}}{\text{volumetric strain}}$

$$= \frac{dp}{\frac{-d\forall}{\forall}}$$ In SI units N/m^2

$$K = \frac{-dp}{d\forall}\forall$$

$$\text{Compressibility}(C) = \frac{1}{K}$$

P1: What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of $0.0125\ m^3$ at $80\ N/cm^2$ pressure to a volume of $0.0124\ m^3$ at $150\ N/cm^2$ pressure?

Given:

Initial volume $\forall = 0.0125\ m^3$ — Initial pressure $= 80\ N/cm^2$

final volume $\forall = 0.0124\ m^3$ — final pressure $= 150\ N/cm^2$

To find: bulk modulus.

Sol:

Bulk modulus $K = \dfrac{dp}{\frac{-d\forall}{\forall}} = \dfrac{dp\ \forall}{-d\forall}$

Decrease in volume $d\forall = 0.0125 - 0.0124$

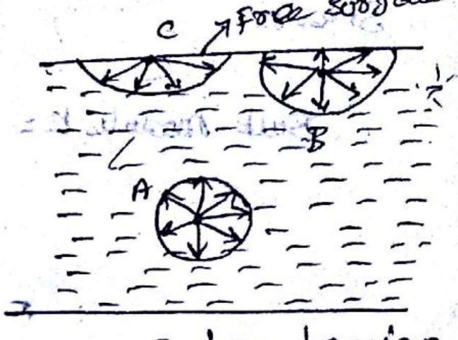
$d\forall = 0.0001\ m^3$

Increase in pressure $dp = 150 - 80 = 70\ N/m^2$ $\frac{70}{10^4}$

$$K = \frac{70 \times 0.0125}{0.0001} = 8750\ N/cm^2$$

(vii) Surface Tension & Capillarity:

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.



Surface tension.

Units: It is denoted by σ.

In SI units N/m

Surface tension on liquid droplet:

let, r - radius one bubble cut into two half surfaces,

σ - surface tension

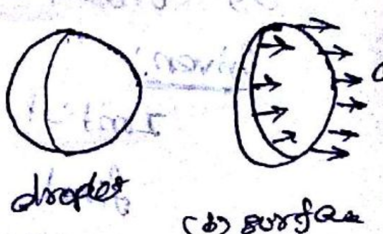
p - Intensity of pressure inside droplet

d - diameter of the droplet

i) tensile force due to surface tension

$= \sigma \times$ circumference

$= \sigma \times \pi d$ (or) $\sigma \times (2\pi r)$



ii) pressure force on the area

$= p \times \frac{\pi}{4} d^2$ (cross sectional area)

In equilibrium condition these two forces are equal and opposite.

$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$

$$\therefore p = \frac{4\sigma}{d}$$

Surface tension on hollow bubble:

* A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside.

* Thus two surfaces are subjected into surface tension.

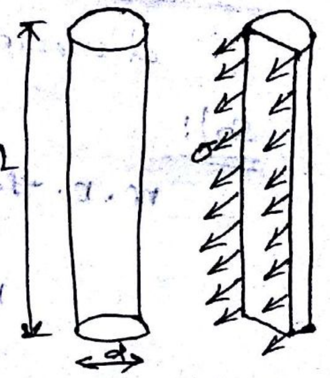
$$\therefore\ p \times \frac{\pi}{4} d^2 = 2(\sigma \times \pi d)$$

$$\boxed{P = \frac{8\sigma}{d}}$$

Surface tension in a liquid jet:

σ - surface tension

P - pressure intensity inside the liquid jet.



Consider equilibrium of the semi jet, we have,

Force due to pressure = P × area of semi jet

$= P \times L \times d$

Force due to surface tension $= \sigma \times 2L$

$$\therefore\ P \times L \times d = \sigma \times 2L$$

$$P = \frac{2\sigma}{d}$$

$$\boxed{P = \frac{2\sigma}{d}}$$

Sp1: The surface tension of water in contact with air at 20°c is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure, calculate the diameter of the droplet of water.

Given:

surface tension $\sigma = 0.0725\ N/m$

$P = 0.02\ N/cm^2 = 0.02 \times 10^4\ N/m^2$

To find:

diameter of the droplet of water.

Sol:

w.r.t that $p = \frac{4\sigma}{d}$

$10^4 \times 0.02 = \frac{4 \times 0.0725}{d}$

$\boxed{d = 0.00145\ m}$

Result:

$d = 0.00145\,m$

Note:

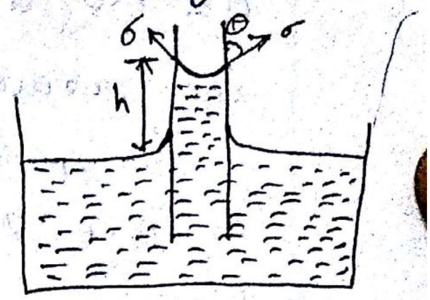
In case whether they ask pressure inside the droplet means formula

$\therefore$ pressure inside the droplet $= p +$ pressure outside the droplet.

(It is given in problem)

(viii) Capillarity :

* Capillarity is defined as a phenomenon of rise (or) fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.



* The rise of the liquid is known as capillarity rise while the fall of the liquid surface is known as capillary depression.

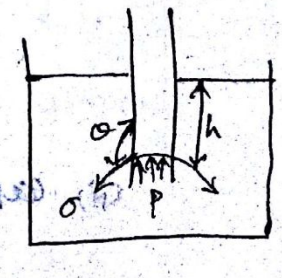
* units cm (or) mm of liquid.

* Its value depends upon the specific weight of the liquid
- diameter of the tube
- surface tension of the liquid.

Expression for capillary rise

$$h = \frac{4\sigma}{\rho \times g \times d}$$

Expression for capillary fall



$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

capillary rise for water θ = 0°
capillary rise for mercury θ = 128°

Pb: Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water & (b) mercury. Take surface tension $\sigma = 0.0725\ N/m$ for water and $\sigma = 0.52\ N/m$ for mercury in contact with air. The specific gravity for mercury is given as 13.6 & angle of contact $= 130°$.

Given:

$d = 2.5\ mm = 2.5 \times 10^{-3}\ m$

$\sigma_w = 0.0725\ N/m$

$\sigma_m = 0.52\ N/m$

$S_m = 13.6$

$\theta = 130°$

To find: capillary rise for water and mercury.

Sol:

(i) capillary rise for water ($\theta = 0°$)

$$S_m = \frac{\text{density of mercury}}{\text{density of standard fluid}}$$

$13.6 \times 1000 =$ density of mercury.

$\rho_m = 13.6 \times 10^3\ kg/m^3$

$\rho_w = 1000\ kg/m^3$

$$h_w = \frac{4\sigma\cos\theta}{\rho \times g \times d} = \frac{4 \times 0.0725 \times \cos 0°}{1000 \times 9.81 \times 0.0025}$$

$h_w = 0.0118\ m = 1.18\ cm.$

(ii) capillary rise for mercury ($\theta = 130°$)

$$h_m = \frac{4\sigma\cos\theta}{\rho g \times d} = \frac{4 \times 0.52 \times \cos 130°}{13.6 \times 10^3 \times 9.81 \times 0.0025}$$

$h_m = -0.004\ m = -0.4\ cm$

Capillary depression

P3: Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m. (22)

Given:

Capillary rise $h_w = 2mm = 2 \times 10^{-3}\ m$

$\sigma = 0.073575\ N/m$

$\theta_w = 0^\circ$ (assume)

d = diameter of the glass tube.

To find:

$d = ?$

Sol:

Capillary rise for water

$$h_w = \frac{4\sigma \cos\theta}{\rho g d}$$

$$2 \times 10^{-3} = \frac{4 \times 0.073575 \times \cos 0}{1000 \times 9.81 \times d}$$

$$d = 0.015\ m = 1.5\ cm.$$

Result:

The diameter of the glass tube should be 0.015 m (or) 1.5 cm.

(55)

(ix) Vapour pressure:

* A change from the liquid state to the gaseous state is known as vapourization. The vapourization occurs because of continuous escaping of the molecules through the free liquid surface. These accumulated vapour molecules exert a pressure on the liquid surface. This pressure is known as vapour pressure.

* Vapour pressure is the pressure at which the liquid is converted into vapours.

(x) Cavitation:

* The vapour molecules in flowing liquid develope the region of high pressure where they collapse, giving rise to high impact pressure.

* The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded & cavities are formed on them. This phenomenon is known as cavitation.

Flow Characteristics

Type of flow:

(i) steady & unsteady flows

(ii) uniform & non-uniform flows

(iii) laminar & turbulent flows.

(iv) compressible & incompressible flows.

(v) Rotational & irrotational flows.

(vi) one, two & three dimensional flow.

(i) Steady & unsteady flows:

* In steady flow, fluid characteristics like u, P, ρ etc, do not change at any with time. $\frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \partial \rho / \partial t = 0$

* unsteady flow, u, ρ, P may change $\frac{\partial v}{\partial t} \neq 0, \frac{\partial \rho}{\partial t} \neq 0, \partial \rho / \partial t \neq 0$

(ii) uniform & non-uniform flows:

* In uniform flow fluid velocity does not change with respect to space. $\frac{\partial v}{\partial s} = 0$

* non-uniform flow may change. $\frac{\partial v}{\partial s} \neq 0$

(iii) laminar & turbulent flow:

* Laminar flow is defined as type of flow in which the fluid particles moves along well defined paths.

* In turbulent flow fluid moves like zig-zag way.

(iv) compressible & Incompressible flow:

* In compressible flow, fluid density is not constant. ($\rho \neq$ constant)

* In compressible, flow fluid density do not constant. ρ = constant

(v) Rotational & irrotational flow:

* In rotational flow, fluid flow at along straight line, and also rotates about their own axis.

* It fluid flow at straight line but do not rotates at their own axis.

(vi) One, Two & Three dimensional flows:

* In one dimensional flow, fluid velocity is a function of one-co-ordinate only.

* In two dimensional flow velocity is a function of two co-ordinate space ie x,y

* In 3-D x, y, z co-ordinates.

The concept of control volume:

* "It is a volume fixed in a space (or) moving with constant velocity through which the fluid (gas (or) liquid) flows".

Rate of flow (or) Discharge (Q)

It is defined as quantity of fluid flowing per second through a section of a pipe (or) a channel.

$Q = A \times V$

for liquids Q is m^3/s (or) litre/s

for gases Q is N/s (or) kgf/s

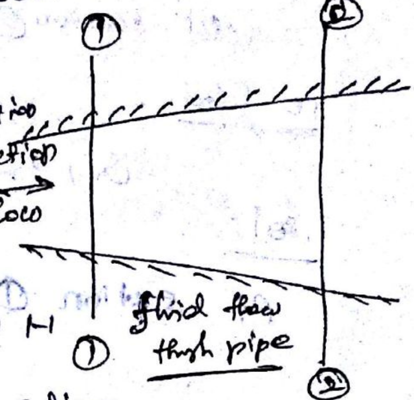
continuity equation:

* The equation based on the principle of conservation of mass is called continuity equation. 26

// conservation of mass ⇒ mass is constant in and out of the system

* Thus for fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

consider two cross-sections of a pipe as shown in figure,



let V_1 = Average velocity at 1-1

ρ_1 = density at section 1-1

A_1 = area of pipe at section 1-1

and V_2, ρ_2, A_2 are corresponding values at section 2

The rate of flow at section 1-1 $Q_1 = \rho_1 A_1 V_1$

" " " " 2-2 $Q_2 = \rho_2 A_2 V_2$

According the law of conversation of mass

$$Q_1 = Q_2$$

$$\therefore \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

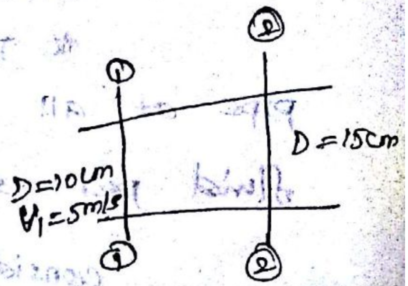
* The above equation is called continuity equation. It is applicable for compressible and incompressible fluid flow.

* If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation reduces to

$$A_1 V_1 = A_2 V_2$$

3) Pr. The diameters of a pipe at the sections 1 and 2 are 10 cm & 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.

Given:



At section ① $D_1 = 10cm$

$V_1 = 5 m/s$

At section ② $D_2 = 15 cm$

To find:

Q, V_2

Sol:

At section ① $D_1 = 10cm = 10 \times 10^{-2} m$

$A_1 = \pi/4 \times d^2 = 7.853 \times 10^{-3} m^2$

$V_1 = 5 m/s$

At section ② $D_2 = 15 cm = 15 \times 10^{-2} m$

$A_2 = \pi/4 \times (15 \times 10^{-2})^2$

$A_2 = 0.0176 m^2$

∴ Discharge

$Q = A_1 \times V_1$

$= 7.853 \times 10^{-3} \times 5$

$Q = 0.03926 m^3/s$

From the continuity equation

$A_1 V_1 = A_2 V_2$

$0.03926 = 0.0176 \times V_2$

$V_2 = 2.23 m/s$

Result:

$Q = 0.03926 m^3/s$

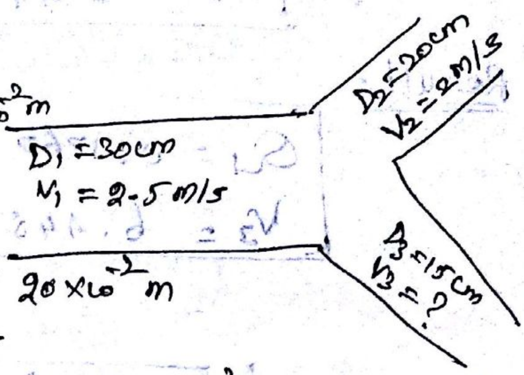
$V_2 = 2.23 m/s$

P2: A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm & 15 cm respectively. If the average velocity in the 30cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s. (28)

<u>Given:</u>

Pipe ① → $D_1 = 30cm = 30\times10^{-2} m$

$v_1 = 2.5 m/s$



pipe ② → $D_2 = 20cm = 20\times10^{-2} m$

$v_2 = 2 m/s$

pipe ③ → $D_3 = 15 cm = 15\times10^{-2} m$

<u>To find:</u> $V_3 = ?$

$Q_1 = ?$

<u>Sol:</u>

$D_1 = 0.3 m$

$A_1 = \pi/4 \times D_1^2 = \pi/4 \times (0.3)^2 = 0.07068 m^2$

∴ Discharge in pipe 1, $(Q_1) = A_1 \times V_1$

$= 0.07068 \times 2.5$

$Q_1 = 0.1767 m^3/s$

$D_2 = 20\times10^{-2} m$

$A_2 = \pi/4 \times (0.2)^2 = 0.0314 m^2$

$V_2 = 2 m/s$

∴ $Q_2 = A_2 \times V_2 = 0.0314 \times 2$

$Q_2 = 0.0628 m^3/s$

<u>To find V_3:</u>

$Q_1 = Q_2 + Q_3$

$0.1767 = 0.0628 + (A_3 V_3)$

(88)

$A_3 = \pi/4 (D_3^2) = \pi/4 \times (0.15)^2$

$\therefore A_3 = 0.01767 \ m^2$

$\therefore 0.1767 = 0.0628 + 0.01767 V_3$

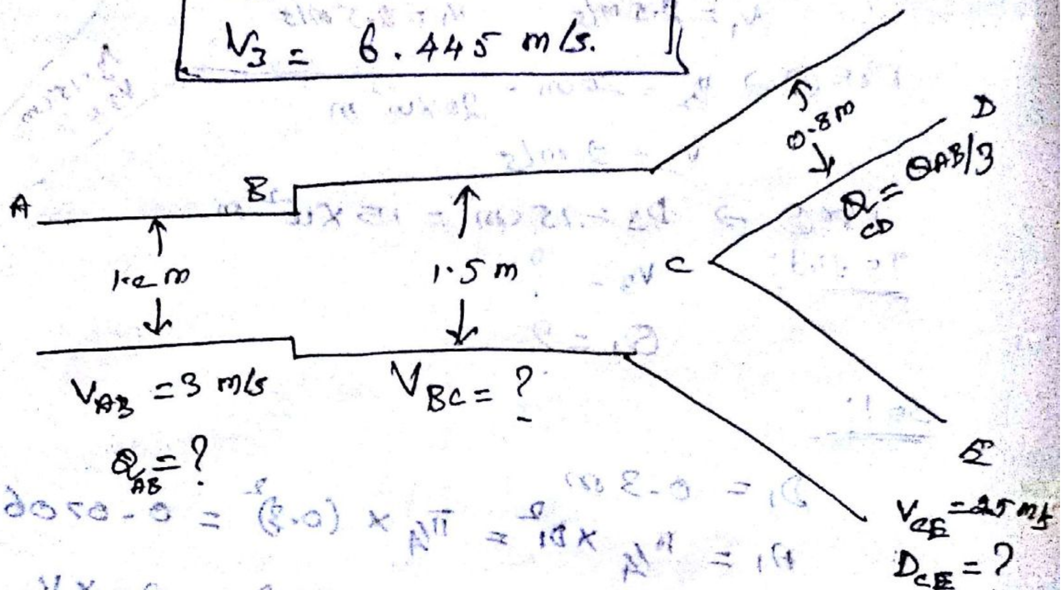
$$V_3 = \frac{0.1767 - 0.0628}{0.01767}$$

$$\boxed{V_3 = 6.445 \ m/s}$$

Result:

$Q_1 = 0.1767 \ m^3/s$
$V_3 = 6.445 \ m/s.$

P3:



P4: A jet of water from a 25mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular & neglecting any loss of energy, that will be the diameter at a point 4.5 m above the nozzle, If the velocity with which the jet leaves the nozzle is 12 m/s.

Given:

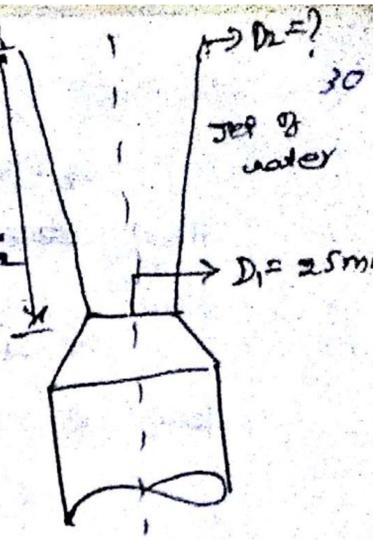
$D_1 = 25 \ mm = 0.025 \ m$

$v_1 = 12 \ m/s.$

$h = 4.5 m$

TO find:

$V_2 = ?$ & $D_2 = ?$



Sol:

let the velocity at height 4.5m = V_2

(consider that jet outlet remains ~~out~~ circular & neglecting any loss of energy).

Initial velocity $V_1 = 12\ m/s$

final velocity $V_2 = ?$

The value $g = +9.81\ m/s^2$ $h = 4.5m$

W.k.that

$$V_1^2 - V_2^2 = 2gh$$

$$(12)^2 - V_2^2 = 2 \times 9.81 \times 4.5$$

$\times -$

$$\Rightarrow (-12)^2 + V_2^2 = -2 \times 9.81 \times 4.5$$

$$V_2^2 = -88.29 + 144$$

$$V_2 = \sqrt{144 - 88.29}$$

$$V_2 = \sqrt{55.71}$$

$$\boxed{V_2 = 7.46\ m/s}$$

From continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\pi/4 \times (0.025)^2 \times 12 = \pi/4 \times (D_2)^2 \times 7.46$$

$$\boxed{D_2 = 0.0317\ m}$$

Result:

$D_2 = 0.0317\ m$

$V_2 = 7.46\ m/s$

Dynamics of fluid flow :

* Dynamics of fluid flow is the study of fluid ~~flow is~~ motion with the forces causing flow.

* It is analysed by newton's second law of motion,

$$F = m \times a$$

m → mass of the fluid; a → acc of the fluid

Equation of motion:

$$F = ma$$

In the fluid flow following forces are present.

(i) F_g - gravity force

(ii) F_p - pressure force

(iii) F_v - force due to velocity

(iv) F_t - " " turbulance

(v) F_c - " " compressibility.

∴ net force

$$F = F_g + F_p + F_v + F_t + F_c$$

neglect F_c means it is called Reynold's equation

F_t is negligible, is called navier - stokes' equation.

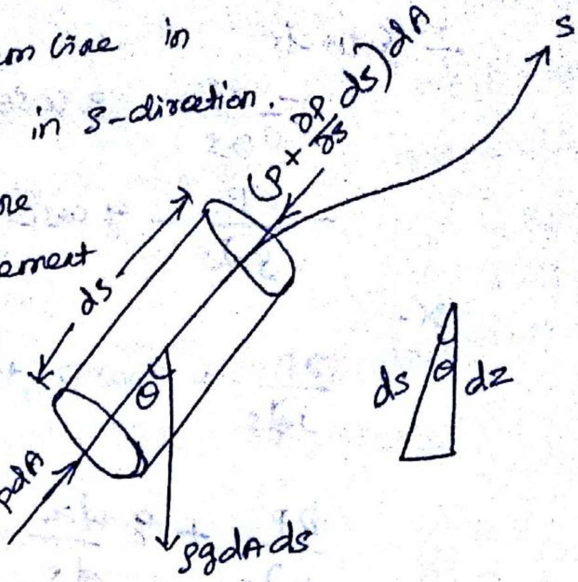
If the flow is assumed to be ideal viscous force F_v is zero and equation is Euler's equation

(X) Euler's equation of motion :

* This equation of motion force due to gravity and pressure are taken into consideration.

* consider a stream line in which fluid is flowing in s-direction.

* On a stream line Consider a cylindrical element of cross-section dA and length ds.



Force acting on the cylinder are,

(i) pressure force pdA in the direction of flow.

(ii) pressure force $\left(p+\frac{\partial p}{\partial s}ds\right)dA$ is opposite direction

(iii) weight of element $= (\rho \times g \times) \underbrace{dA \times ds}_{volume}$ (S.W)

$$W = \rho g\, dA\, ds$$

θ is angle b/w fluid stream line and element plane.

* The resultant force on the fluid element in the direction of fluid must be equal to the mass (density × volume) of the fluid × acceleration in the s direction.

$$p\,dA - \left(p+\frac{\partial p}{\partial s}ds\right)dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s \rightarrow (1)$$

Now $a_s = \frac{dv}{dt}$, where v is a function of s & t

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} \times \frac{dt}{dt} \quad \left(\because v = \frac{ds}{dt}\right)$$

$$a_s = \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

∴ In steady flow $\frac{\partial v}{\partial t} = 0 \Rightarrow a_s = \frac{v\,\partial v}{\partial s} \rightarrow (2)$

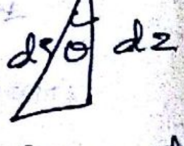
sub ② in ① we get

$$p\,dA - p\,dA - \frac{\partial p}{\partial s} ds\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times v\frac{\partial v}{\partial s}$$

$\div \rho\,dA\,ds$

$$-\frac{\partial p}{\rho\,\partial s} - g\cos\theta = v\frac{\partial v}{\partial s}$$

$$-\frac{\partial p}{\rho\,ds} - g\cos\theta - v\frac{\partial v}{\partial s} = 0$$



$\times -$

$$\frac{\partial p}{\rho\,ds} + g\cos\theta + v\frac{\partial v}{\partial s} = 0 \qquad \cos\theta = \frac{dz}{ds}$$

$$\frac{\partial p}{\rho\,ds} + g\frac{dz}{ds} + v\frac{\partial v}{\partial s} = 0$$

$$\frac{dp}{\rho\,ds} + g\frac{dz}{ds} + v\frac{dv}{ds} = 0$$

$$\frac{1}{ds}\left(\frac{dp}{\rho} + g\,dz + v\,dv\right) = 0$$

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = 0}$$

The above equation is called as Euler's equation of motion.

Bernoulli's equation from Euler's equation:

By integrating the euler's equation we get bernoulli's equation.

$$\int\frac{dp}{\rho} + \int g\,dz + \int v\,dv = 0 \qquad \rho = \text{constant}$$

∴ If flow is incompressible $\rho = C$

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$\div g$

$$\boxed{\frac{p}{\rho g} + z + \frac{v^2}{2g} = C \;(\text{or})\; \frac{p}{\rho g} + \frac{v^2}{2g} + z = C}$$

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The above equation is called bernoulli's equation, in which,

$\frac{P}{\rho g}$ = pressure energy per unit weight of fluid (or) P.H pressure head.

$\frac{v^2}{2g}$ – Kinetic energy per unit weight (or) Kinetic head

Z – potential energy per unit weight (or) potential head.

Bernoulli's equation assumption:

(i) The fluid is ideal i.e viscosity is zero.

(ii) The flow is steady.

(iii) flow is incompressible.

(iv) The flow is irrotational.

P: Water is flowing through a pipe of 5cm diameter under a presure of 29.43 N/cm² and with mean velocity of 2.0 m/s. Find the total head (or) total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Given:

diameter of pipe = 5cm = 0.05 m

pressure (p) = 29.43 N/cm² = 29.43×10^{4} N/m²

v = 2.0 m/s

datum head (z) = 5 m

To find:

Total head (or) Total Energy.

Sol:

Total head = pressure head + kinetic head + datum head

$P.H = P/\rho g = \frac{29.43 \times 10^{4}}{1000 \times 9.81} = 30\ m$

$K.H = v^2/2g = 2 \times 2/(2 \times 9.81) = 0.203\ m$

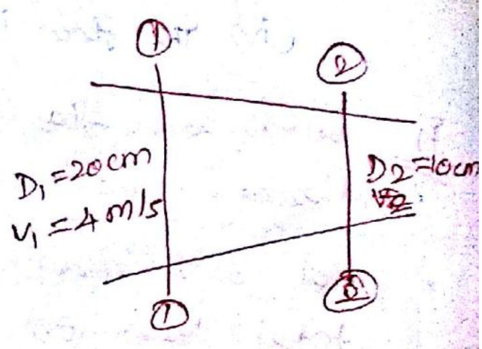
$\therefore TH = 30 + 0.204 + 5 = 35.204\ m.$

Result :

Total head $= 35.204\ m.$

P2: A pipe through which water is flowing is having diameters, 20 cm & 10 cm at the cross-sections ① & ② respectively. The velocity of water at section ① is given 4.0 m/s. Find the velocity head at sections ① & ② & also rate of discharge.

Given :



$D_1 = 20cm = 0.2m$

$D_2 = 10\ cm = 0.1m$

$V_1 = 4m/s$

To find :

(i) Velocity head ① @ ②

(ii) Q

Sol:

$D_1 = 20cm = 0.2\ m$

$A_1 = \pi/4\ D_1^2 = \pi/4 \times (0.2)^2 = 0.0314\ m^2$

$D_2 = 0.1m$

$A_2 = \pi/4 \times (D_2)^2 = \pi/4 \times (0.1)^2 = 7.85 \times 10^{-3}\ m^2$

(i) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4 \times 4}{2 \times 9.81} = 0.815\ m$$

(i) velocity head @ section ② $= \frac{v_2^2}{2g}$

From continuity equation

$A_1 V_1 = A_2 V_2$

$0.0314 \times 4 = 0.00785 \times V_2$

$\therefore V_2 = \frac{0.0314 \times 4}{0.00785}$

$V_2 = 4 \times 4 \, m/s = 16 \, m/s$

$\therefore$ velocity head @ section ② $= \frac{v_2^2}{2g}$

$= \frac{16 \times 16}{2 \times 9.81}$

$= 13.04 \, m$

(iii) Rate of discharge (Q)

$Q = A_1 V_1$ (or) $A_2 V_2$

$= 0.0314 \times 4$

$Q = 0.1256 \, m^3/s$

Result:

V.H @ ① = 0.815 m $Q = 0.1256 \, m^3/s$

② = 13.04 m

2M Statement of Bernoulli's theorem:

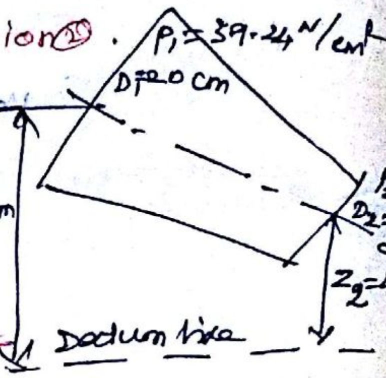
It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.

The total energy consist of pressure energy, kinetic energy and potential energy (or) datum energy.

$K.E = v^2/2g$ $P.E = Z$

$P.E = P/\rho g$ $\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{Constant}.$

P3: The water is flowing through a pipe having diameters 20 cm & 10 cm at section ① & ② respectively. The rate of flow through pipe is 35 litres/s. The section ① is 6m above datum and section ② is 4m above datum. If the pressure at section ① is 39.24 N/cm², find the intensity of pressure at section ②.



Given:

$D_1 = 20\,cm = 0.2\,m$

$D_2 = 10\,cm = 0.1\,m$

$Q = 35$ litre/s ~~= 35 cm³/s~~

$= 35 \times 1000\ cm^3/s = 35 \times 10^3 \times (10^{-2})^3\ m^3/s$

$= 35 \times 10^3 \times 10^{-6}\ m^3/s$

$\therefore Q = 35/1000\ m^3/s$

$P_1 = 39.24\ N/cm^2 = 39.24 \times 10^4\ N/m^2$

$z_1 = 6m \quad z_2 = 4m$

To find:

$P_2 = ?$

Sol:

$D_1 = 0.2\,m$

$\therefore A_1 = \pi/4\ D_1^2 = \pi/4 \times (0.2)^2 = 0.0314\ m^2$

$D_2 = 0.1\,m$

$A_2 = \pi/4 \times D_2^2 = \pi/4 \times (0.1)^2 = 0.00785\ m^2$

From continuity equation.

$A_1 V_1 = A_2 V_2$

$0.0314 \times V_1 = 0.00785\ V_2$

But we know $Q = 35/1000\ m^3/s$

$\therefore Q = V_1 A_1 = 0.0314 \times V_1 = 0.035$

$V_1 = 1.114 \text{ m/s}$

similarly

$0.00785 \times V_2 = 0.035$

$V_2 = 4.458 \text{ m/s}$

Apply Bernoulli's equation @ ① & ② we get P_2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{39.24 \times 10^4}{1000 \times 9.81}\right) + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{(1000 \times 9.81)} + \frac{(4.458)^2}{(2 \times 9.81)} + 4$$

$$40 + 0.0632 + 6 = \frac{P_2}{9810} + 1.0129 + 4$$

$$46.0632 = P_2/9810 + 5.0127$$

$$\therefore \frac{P_2}{9810} = 46.0632 - 5.0127$$

$$\boxed{P_2 = 40.27 \times 10^4 \text{ N/m}^2}$$

Result:

$P_2 = 40.27 \times 10^4 \text{ N/m}^2$

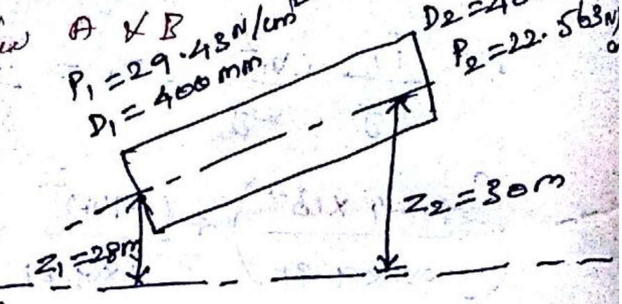
Bernoulli's equation for real fluid:

* Bernoulli equation derived on the assumption that fluid is non-viscous & therefore frictionless.

* But all the real fluids are viscous & hence offer resistance to flow. Thus there are always some losses are in fluid flow, and hence in the applications of bernoulli's equations these losses are taken into consideration.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$ → head loss @ ① & ②

P1: A pipe of diameter 400mm carries water at a velocity of 25 m/s. The pressure at the points A & B are given as 29.43 N/cm² & 22.563 N/cm² respectively while the datum head at A & B are 28m & 30m. Find the loss of head b/w A & B



Given:

$D_1 = 400m = 0.4 m$

$P_1 = 29.43 \times 10^4 \ N/m^2$

$P_2 = 22.563 \times 10^4 \ N/m^2$

$z_1 = 28m \qquad z_2 = 30m$

$V_2 = V_1 = 25 m/s$

To find:

loss of head ① - ②

Total energy at ①

$$T.E)_1 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 28$$

$$= 89.85 m.$$

Total energy @ ②

$$T.E)_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 30$$

$$T.E)_2 = 84.85 m$$

head loss = ① - ② = 89.85 - 84.85 = **4.99 M**

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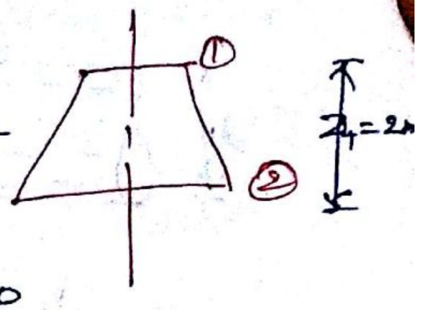
P2: A conical tube of length 2.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is $\frac{0.35(v_1-v_2)^2}{2g}$, where v_1 is the velocity at the smaller end and v_2 is velocity at lower end respectively. Det pressure head at the lower end. Flow takes place in the downward direction.

Given:



$L = 2m$

$v_1 = 5 m/s$ $\quad h_L = \frac{0.35(v_1-v_2)^2}{2g}$

$v_2 = 2 m/s$

$P_1/\rho g = 2.5m$ $\quad z_1 = 2 \quad z_2 = 0$

To find:

$P_2/\rho g = ?$

Sol:

head loss $h_L = \frac{0.35\,(5-2)^2}{2\times 9.81} = 0.16 m$

$h_L = 0.16 m$

Apply Bernoulli equation.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$2.5 + \frac{(5)^2}{2\times 9.81} + 2 = \frac{P_2}{\rho g} + \frac{2^2}{2\times 9.81} + 0.16$$

$$0.16 + 0.2038 + \frac{P_2}{\rho g} = 2.5 + 1.274 + 2$$

$\therefore P_2/\rho g = 5.410$ m of fluid

Practical Applications of Bernoulli's equation

(i) venturimeter

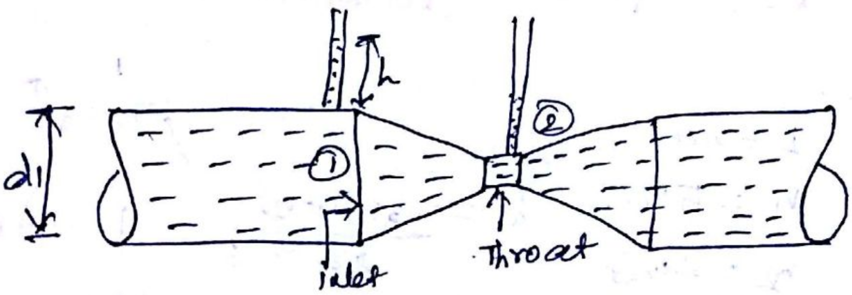
(ii) orifice meter

(iii) Pitot tube.

venturimeter: _used for measuring the rate of flow of a fluid flowing through a pipe

consist of three parts

- A short converging part
- Throat
- Diverging part



P_1 - pressure at section ①

d_1 - diameter at section ①

v_1 - velocity at section ①

a_1 - area of the section ① $= \pi/4 \, d_1^2$

Similarly for section ② P_2, d_2, v_2, a_2

Apply Bernoulli's equation ① & ② we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} \rightarrow ①$$ (for horizontal pipe $z_1 = z_2$)

$\frac{P_1 - P_2}{\rho g}$ is the pressure head (h) at section ① & ②

$$\therefore h = \frac{P_1 - P_2}{\rho g}$$

sub $h = \frac{P_1 - P_2}{\rho g}$ in ①

we get
$$h = \frac{V_2^2 - V_1^2}{2g} \rightarrow (2)$$

From continuity equation

$$a_1 V_1 = a_2 V_2 \Rightarrow V_1 = \frac{a_2 V_2}{a_1}$$

Sub V_1 in (2)

$$\therefore h = \frac{V_2^2 - \left(a_2^2 V_2^2 / a_1^2\right)}{2g}$$

$$h = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\therefore V_2^2 = 2gh \times \frac{a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \sqrt{2gh \times \frac{a_1^2}{a_1^2 - a_2^2}}$$

$$V_2 = a_1 \times \frac{\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$\therefore$ Discharge $Q = V_2 a_2$

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

* The above equation is called theoretical discharge.

* The actual discharge will be less than theoretical discharge.

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d - co-efficient of ~~discharge~~ venturimeter, that is less than 1.

(c) Value of 'h' is given by differential U-tube manometer:

S_h - Sp. gravity of the heavier liquid

S_o - Sp. gravity of the liquid flowing through pipe

x - difference of the heavier liquid column in U-tube

Case (i):

horizontal ventorimeter

manometer fluid > liquid flowing through pipe

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

Case (ii)

manometer fluid < liquid flow through pipe

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

Case (iii): Inclined ventorimeter with Differential U-tube manometer.

If differential manometer contains heavier liquid h is given as

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Case (iv):

If differential manometer contains lighter liquid, h is given as.

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

P: An oil of sp. gr. 0.8 is flowing thrgh a venturimeter having inlet diameter 20 cm & throat diameter 10 cm. The oil mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil thrgh the horizontal venturimeter. Take $C_d = 0.98$ (44)

Given:

sp. gravity of oil = 0.8 (pipe)

" " mercury = 13.6 (manometer)

$d_1 = 20\text{ cm} = 0.2\text{ m}$

$d_2 = 10\text{ cm} = 0.1\text{ m}$

$x = 25\text{ cm} = 0.25\text{ m}$

$C_d = 0.98$

To find:

$Q = ?$

Sol:

W.K. that discharge of venturimeter

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 0.25\left[\frac{13.6}{0.8} - 1\right]$$

$h = 2.95\text{ m}$ of oil

$a_1 = \pi/4 \times d_1^2 = \pi/4 \times (0.2)^2 = 0.0314\text{ m}^2$

$a_2 = \pi/4 \times d_2^2 = \pi/4 \times (0.1)^2 = 0.00785\text{ m}^2$

1 m = 100 cm
1 litre = 1000 cm³

$$\therefore Q = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.95}$$

$Q =$ ~~0.32517 m³/s~~ $= 0.32517 \times 10^6\text{ cm}^3/\text{s} = 325170\text{ cm}^3/\text{s}$

0.07018 m³/s $\quad Q = 325170/1000$ litre/s $= 325$ litre/s

B: The inlet & throat diameters of a horizontal venturimeter are 30cm & 10cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm² while the vaccum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 14% of the differential head is lost between the inlet & throat. Find also the value of Cd for the venturimeter.

Given:

1m = 100cm

$d_1 = 30cm = 0.3 m$

$d_2 = 10cm = 0.1 m$

$P_1 = 13.734 \ N/cm^2 = 13.734 \times 10^4 \ N/m^2$

vaccum pressure head at throat } $\frac{P_2}{\rho g} = -37$ cm of mercury

$= \frac{-37}{100} \times 13.6$

Sol:

$P_2/\rho g = -5.032$ m of water

Differential head

$h = P_1/\rho g - P_2/\rho g$

$= \frac{13.734 \times 10^4}{1000 \times 9.81} + 5.032$

$h = 19.032 \ m$

head loss $h_f = 4\%.h$

$= 4/100 \times 19.032$

$h_f = 0.7612 \ m^2$

$a_1 = \pi/4 \ d_1^2 = \pi/4 \times (0.3)^2$

$= 0.0706 \ m^2$

$a_2 = \pi/4 \ d_2^2 = \pi/4 \times (0.1)^2$

$= 0.00785 \ m^2$

$$C_d = \sqrt{\frac{h - h_f}{h}}$$

$$= \sqrt{\frac{19.032 - 0.7612}{19.032}}$$

$$C_d = 0.979 = 0.98$$

Discharge $Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{0.0706 \times 0.00785}{\sqrt{(0.0706)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 19.032}$$

$$Q = 0.1495 \ m^3/s$$

Problems on inclined venturimeter:

P: Find the discharge of water flowing thrgh a pipe 30 cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and throat is measured by a liquid of s.p gra. 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head between main and throat is 0.2 times the kinetic head of the pipe.

Given:

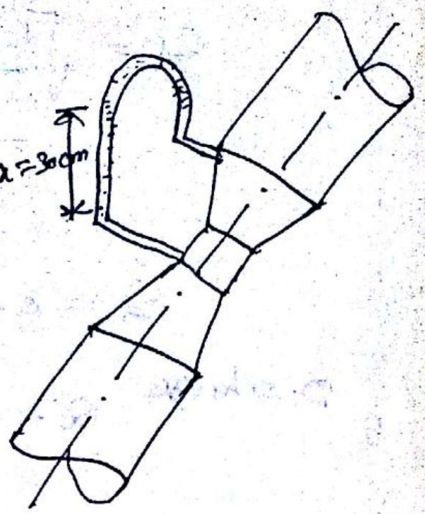
$d_1 = 30 cm = 0.3 m$

$d_2 = 15 cm = 0.15 m$

$S_0 = 0.6$

The differnce of ~~pressure~~ head (heavier liquid manometer tube) $x = 30 cm = 0.3 m$

$h_L = 0.2 \times V_1^2/2g$



To find:

$Q = ?$

Sol:

$d_1 = 0.3\,m$

$a_1 = \pi/4\, d_1^2$

$a_1 = \pi/4 \times (0.3)^2 = 0.0706\,m^2$

$a_2 = \pi/4 \times (0.15)^2 = 0.0176\,m^2$

If the head loss is given means the bernoulli's equation is written by

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$$

$$h + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$$

$\therefore$ ~~$h = x\left[\frac{\rho g}{\rho_0}\right]$~~

$$h = x\left[1 - \frac{S_1}{S_0}\right]$$ (because the take u manometer having lighter liquid)

$$= 0.3\left[1 - \frac{0.6}{1.0}\right]$$ (S_0 is not given so take 1.0)

$h = 0.12\,m$

$$\therefore 0.12 + \frac{V_1^2 - V_2^2}{2g} = 0.2\frac{V_1^2}{2g}$$

$$\frac{(V_1^2 - 0.2V_1^2) - V_2^2}{2g} = -0.12$$

$$\frac{0.8 v_1^2 - v_2^2}{2g} = -0.12 \rightarrow (1)$$ (48)

From continuity equation

$$a_1 v_1 = a_2 v_2$$

$$0.0706 \times v_1 = 0.0176 \times v_2$$

$$v_1 = \frac{0.0176 \times v_2}{0.0706} = 0.24\, v_2$$

$$v_1 = 0.24\, v_2$$

sub v_1 in (1)

$$\frac{0.8 \times (0.24\, v_2)^2 - v_2^2}{2g} = -0.12$$

$$0.046 v_2^2 - v_2^2 = -0.12 \times 2 \times 9.81$$

~~$+v_2^2 = +\frac{2.3544}{0.046}$~~

~~$v_2 =$~~

$$+0.954\, v_2^2 = +2.3544$$

$$v_2 = \frac{2.3544}{0.954}$$

~~$v_2 = 2.46$ m/s~~

$v_2 = 1.55$ m/s

∴ Discharge $Q = a_2 v_2$

$= 0.0176 \times$ ~~2.46~~ 1.55

$Q =$ ~~0.0434 m³/s~~

$=$ ~~43400 cm³/s~~

$Q =$ ~~43.4 litre/s~~

$Q = 0.026\ m^3/s$

$Q = 26$ litre/s

P2: A 30cm X 15cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards, the difference in elevation of the throat section and entrance section of the venturimeter is 30cm. The differential U-tube mercury manometer shows a gauge deflection of 25cm. Calculate:

(i) the discharge.

(ii) The pressure difference b/w the entrance section & the throat section. Take the co-efficient of discharge as 0.98 & specific gravity of mercury as 13.6.

Sol:

The differential head $h = x\left[\frac{S_g}{S_o} - 1\right]$

Then $Q =$ ___

The pressure diff b/w entrance & throat

w.k. that

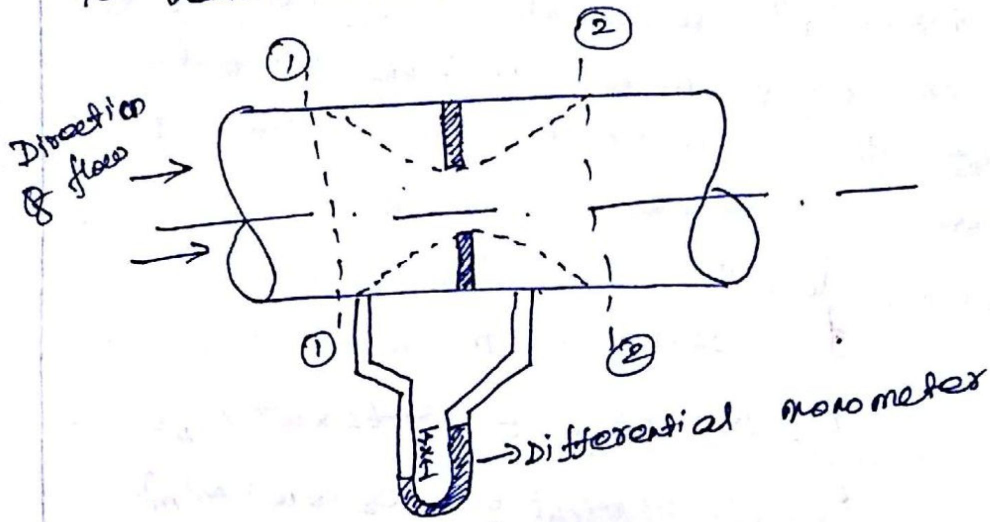
$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h$$

$$z_1 - z_2 = 30cm$$

$$(P_1 - P_2) = ?$$

(ii) Orifice meter:

* It is a device used for measuring the rate of flow of a fluid thro' the pipe.
* It is a cheaper device as compared to venturi meter.



Orifice meter discharge

$$Q = \frac{C_d \times a_0 \times a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

where,

C_d - co-efficient of discharge for orifice meter.

The co-efficient of discharge for the orifice meter is much smaller than that for a venturimeter.

a_1 - pipe area

a_0 - orifice area.

Q: An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. the pressure gauges fitted upstream & downstream of the orifice meter gives readings of 19.62 N/cm² & 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Given:

$d_0 = 10cm = 0.1m$

$d_1 = 20cm = 0.2m$

$P_1 = 19.62 N/cm^2 = 19.62\times10^4 N/m^2$

$P_2 = 9.81 N/cm^2 = 9.81\times10^4 N/m^2$

$C_d = 0.6$

To find:

$Q = ?$

Sol:

$$\text{discharge } Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$a_0 = \pi/4 \times (0.1)^2 = 0.00785 m^2$

$a_1 = \pi/4 \times (0.2)^2 = 0.0314.16 m^2$

we know that h is the differentia mono meter head. (assume not given) $(z_1 = z_2 = 0)$

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

$\frac{P_1}{\rho g} = \frac{19.62\times10^4}{1000\times9.81} = 20m$ of water

$\frac{P_2}{\rho g} = \frac{9.81\times10^4}{1000\times9.81} = 10m$ of water

$\therefore h = 20 - 10 = 10$ m of water.

$$\therefore Q = 0.6 \times \frac{0.00785 \times 0.0314}{\sqrt{(0.00785)^2 - (0.0314)^2}} \times \sqrt{2 \times 9.81 \times 10}$$

$$\boxed{Q = 68.21 \text{ litre/s}}$$

B: An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of the orifice meter = 0.64.

Given:

$d_0 = 15$ cm $= 0.15$ m

$d_1 = 30$ cm $= 0.3$ m

$x = 50$ cm $= 0.5$ m

Manometer mercury $S_h = 13.6$

pipe oil $S_o = 0.9$

$C_d = 0.64$

To find:

$Q = ?$

Sol.

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

$$= 0.5\left[\frac{13.6}{0.9} - 1\right]$$

$h = 7.05.5$ m of oil

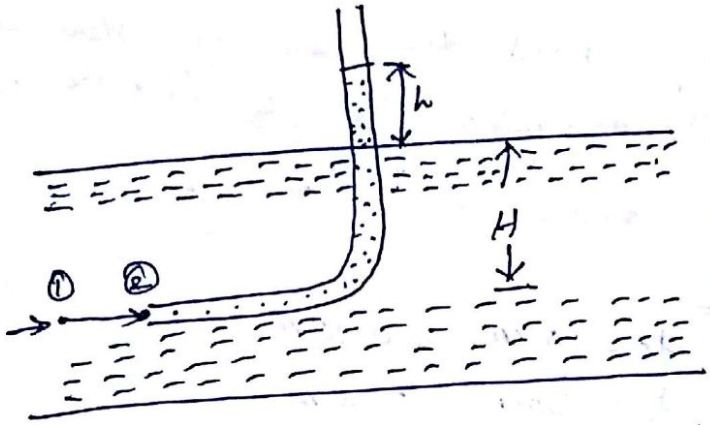
$$\therefore Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh} = 137.414 \text{ litre/s}$$

sub all value

(iii) Pitot tube:

* It is a device used to measure the velocity of flow at any point in a pipe (or) a channel.

* It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conservation of the kinetic energy into pressure energy.



It consist of glass tube, bent at right angles as shown in figure.

P_1 - intensity of pressure at point (1)

v_1 - velocity of flow at (1)

P_2 - pressure at point (2)

v_2 - velocity at point (2)

H - depth of tub in the liquid.

h - rise of liquid in the tube above the free surface.

$V_{th} = \sqrt{2gh}$ $\quad h = v^2/2g$

$V_{act} = C_v \sqrt{2gh}$

C_v - coefficient of pitot tube

Here also monometer liquid mercury mean difference of head is obtained by $h = x\left[\frac{S_h}{S_0} - 1\right]$

Q: Find the velocity of the flow of an oil thrgh a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100 mm. Take Co-efficient of pitot tube 0.98 & s.p. gr. of oil = 0.8 (4)

Given:

$C_v = 0.98$ $S_g = 13.6$

$S_o = 0.8$ $x = 100mm = 0.1m.$

To find:

velocity of flow $= C_v \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times h}$

Diff of pressure head $h = x\left[\frac{S_g}{S_o} - 1\right]$

$= 0.1\left[\frac{13.6}{0.8} - 1\right]$

$h = 1.6$

$V = 0.98 \times \sqrt{2 \times 9.81 \times 1.6}$

$V = 5.49$ m/s

The momentum equation:

According to Newton's 2nd law of motion

$F = m \times a$

$a = \frac{dv}{dt}$

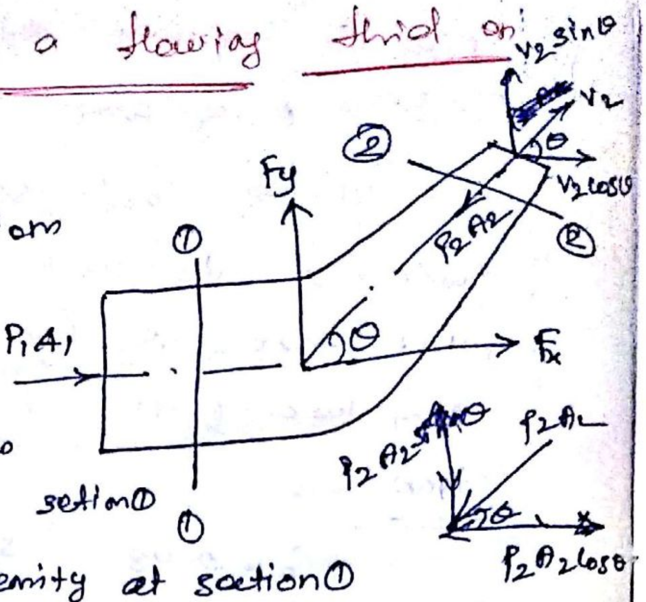
$F = m\frac{dv}{dt}$

$F = \frac{d(mv)}{dt}$

$F.dt = d(mv)$

The above equation is known as momentum principle.

Force exerted by a flowing fluid on a pipe end:



consider two sections ① & ② as shown in figure.

V_1 – velocity of flow at section ①

P_1 – pressure intensity at section ①

A_1 – area of cross-section of pipe at section ①

V_2, A_2, P_2 – corresponding at section ②

F_x & F_y – are the force exerted by the flowing fluid on the bend in the directions of x & y respectively

These forces are considered in the opposite direction.

∴ in x-direction $= -F_x$

y-direction $= -F_y$

Other external forces acting on the fluid are P_1A_1 & P_2A_2 in section ① & ② respectively

∴ The momentum equation in direction is given by

Net force acting on the x-direction = Rate of change of momentum

$P_1A_1 - P_2A_2\cos\theta - F_x$ = (mass per sec) × (Change of velocity)

$= \rho Q \times$ (Final velocity − Initial velocity in x)

$= \rho Q \times (V_2 \cos\theta - V_1)$

$$-F_x = \rho Q \times (V_2\cos\theta - V_1) - P_1A_1 + P_2A_2\cos\theta$$

$$F_x = \rho Q\,(V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta$$

Similarly the momentum equation in y-direction

$$-F_y - P_2A_2\sin\theta = \rho Q\,(V_2\sin\theta - 0)$$

$$-F_y = \rho Q\,(V_2\sin\theta) + P_2A_2\sin\theta$$

$$F_y = \rho Q\,(-V_2\sin\theta) - P_2A_2\sin\theta$$

the resultant force acting on the bend and

$$= \sqrt{F_x^2 + F_y^2}$$

Angle made by the resultant force

$$\tan\theta = \frac{F_y}{F_x}$$

P1: A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600mm & 300mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² & rate of flow of water is 600 litre/s.

Given:

$D_1 = 600mm = 0.6m$

$D_2 = 300mm = 0.3m$

$P_1 = 8.829\ N/cm^2 = 8.829 \times 10^4\ N/m^2$

$Q = 600\ litre/s = 600 \times 1000\ cm^3/s$

$= 6 \times 10^5 \times 10^{-6}\ m^3/s$

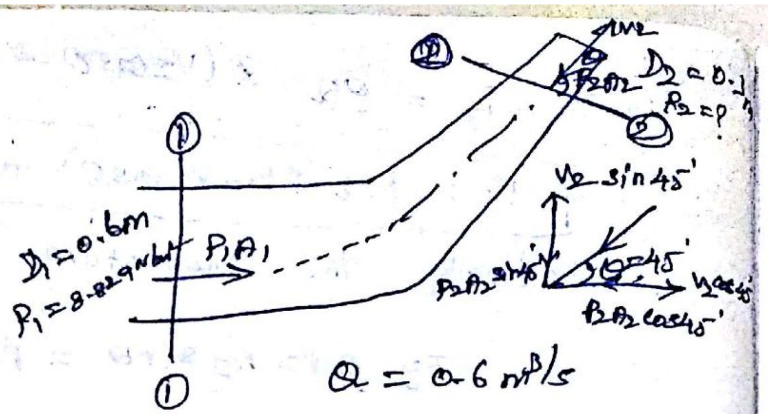
$Q = 0.6\ m^3/s$

To find:

(i) net force

(ii) θ

Sol:



$D_1 = 0.6 m$

$A_1 = \pi/4 \times d_1^2 = \pi/4 (0.6)^2 = 0.2827 m^2$

$D_2 = 0.3 m$

$A_2 = \pi/4 \times d_2^2 = \pi/4 \times (0.3)^2 = 0.0706 m^2$

$Q = 0.6 m^3/s$

$\therefore Q = A_1 V_1$

$V_1 = 0.6 / 0.2827 = 2.122 m/s$

$V_2 = Q/A_2 = 0.6/0.0706 = 8.498 m/s$

Apply Bernoulli equation.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{8.829 \times 10^4}{1000 \times 9.81}\right) + \frac{(2.122)^2}{2 \times 9.81} + 0 = \frac{P_2}{(1000 \times 9.81)} + \frac{(8.498)^2}{2 \times 9.81} + 0$$

$$9 + 0.2295 = \frac{P_2}{9810} + 3.680$$

$$9.2295 - 3.680 = \frac{P_2}{9810}$$

$$\therefore P_2 = 5.44 \times 10^4 \, N/cm^2$$

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Force on the x & y direction

$$F_x = \rho Q (V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$$

$$= (1000 \times 0.6) \times (2.122 - 8.49 \times \cos 45^\circ) + (8.829 \times 10^4 \times 0.2827) - (5.44 \times 10^4 \times 0.0706 \times \cos 45^\circ)$$

$$= 600 (2.122 - 6.00) + (24959.583) - 2715.74$$

$$\boxed{F_x = 19917.043\ N}$$

$$F_y = \rho Q (-V_2 \sin\theta) - P_2 A_2 \sin\theta$$

$$= (1000 \times 0.6)(-8.49 \sin 45^\circ) - (5.44 \times 10^4 \times 0.0706 \times \sin 45^\circ)$$

$$\boxed{F_y = -6317.74\ N}$$

(-) sign indicate Fy is acting on downward direction.

∴ Resultant force $F_R = \sqrt{F_x^2 + F_y^2}$

$$F_R = \sqrt{(19917.043)^2 + (-6317.74)^2}$$

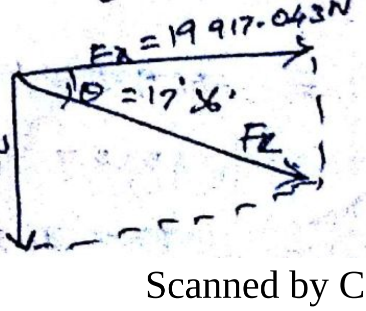
$$\boxed{F_R = 20895.035\ N}$$

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{+6317.74}{19917.043}\right)$$

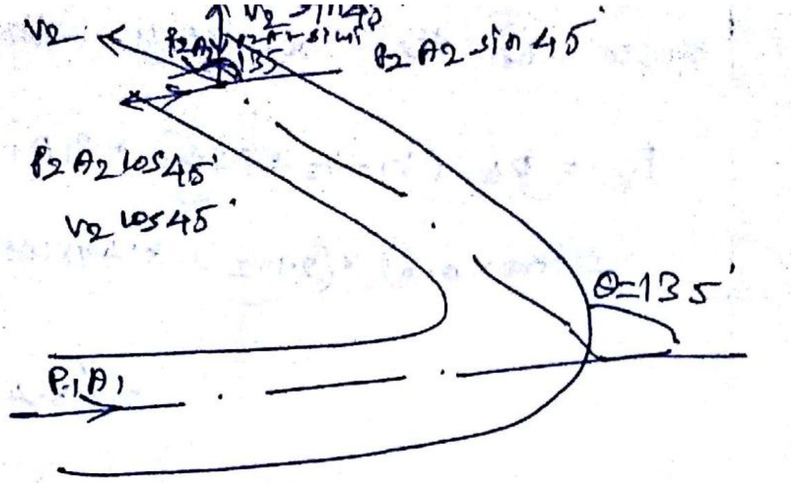
$$\boxed{\theta = 17^\circ 36'}$$

Result:



(8) P2; Only one diagram

Given:

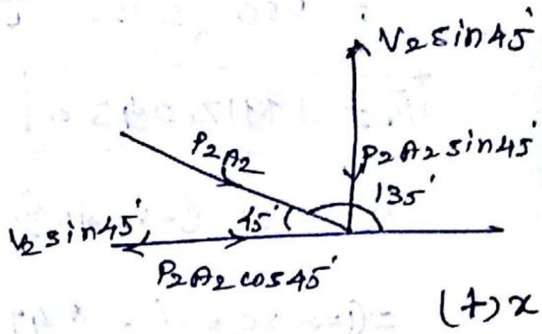


$P_1 = P_2 = 39.24 \times 10^4\ N/m^2$

$D_1 = D_2 = 300m = 0.3m$

$A_1 = A_2 = \pi/4 \times D^2 = \pi/4\,(0.3)^2$

$A_1 = A_2 = 0.0706\ m^2$



$V_1 = V_2 = \dfrac{Q}{A} = \dfrac{0.25}{0.0706} = 3.54 m/s$

Q:

250 litres/s of water is flowing in a pipe having a diameter of 300mm. If the pipe is bent by 135' (that is change from initial to final direction is 135'), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².

Sol:

V_2 in negative x-direction $= -V_2\cos\theta$

$P_2A_2\cos\theta$ in positive x-direction $= +P_2A_2\cos\theta$

$F_x = \rho Q(V_1 - V_2\cos\theta) + P_1A_1 + P_2A_2\cos\theta$

$= 1000 \times 0.25\,(3.54 - (3.54\cos 45')) + (39.24 \times 10^4 \times 0.0706)$
$+ (39.24 \times 10^4 \times 0.0706 \times \cos 45')$

$=$ ~~259.2~~04 $+$ ~~27703.44~~ (6.043) $+ 19589.290$

1510.7   27703.44

$F_x =$ ~~9624.93~~ N 48803.43 N

$F_y = \rho Q \left[-V_2 \sin\theta \right] - P_2 A_2 \sin\theta$

$= 1000 \times 0.25 \left(-3.54 \sin 45^\circ \right) - \left(39.24 \times 10^4 \times 0.0706 \right) \sin 45$

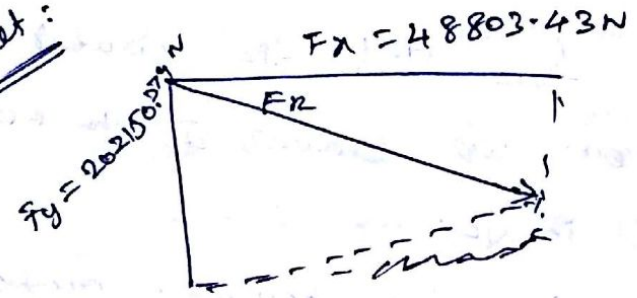
$= -625.789 - 19589.29$

$F_y = -20215.079\ N$

$\tan\theta = \dfrac{20215.079}{48803.43}$

$\theta = \tan^{-1}(0.4142) = 22^\circ 30'$

Result:



UNIT-4

PUMPS

- → centrifugal pump
- → Reciprocating pump } (X)
- → Rotary pump.

pump:

* hydraulic machine which convert the mechanical energy into hydraulic energy are called pumps.

* Hydraulic energy in the form of pressure energy.

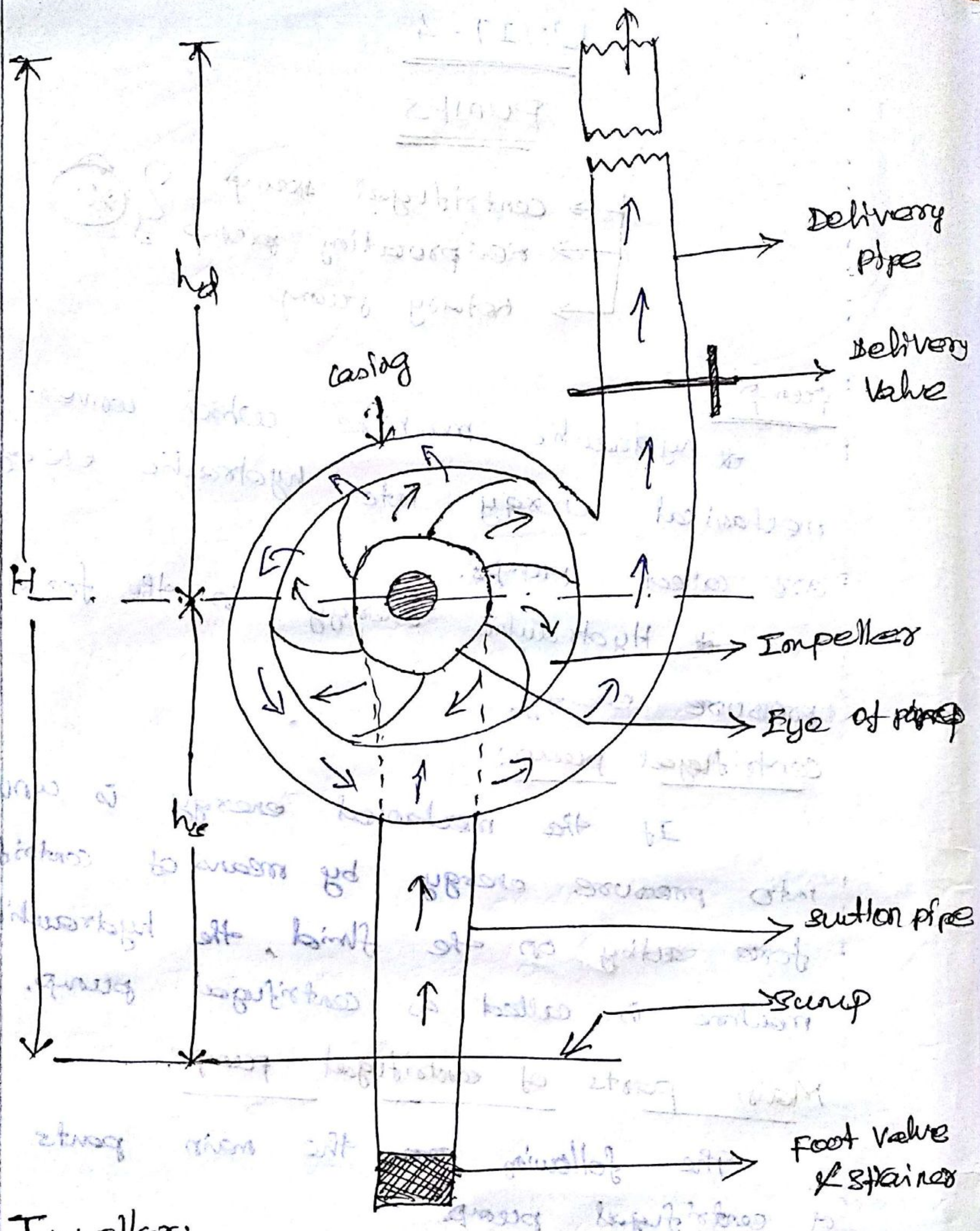
centrifugal pump:

If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called as centrifugal pump.

Main parts of centrifugal pump:

The following are the main parts of a centrifugal pump.

(i) Impeller
(ii) casing
(iii) suction pipe with a foot valve and a strainer.
(iv) Delivery pipe.



<u>Impeller:</u>

* The rotating part of a centrifugal pump is called "Impeller".

* It consists of a series of backward curved vanes.

* The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

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Casing:

* It is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing & enters the delivery pipe.

Three types,

(i) Volute casing

(ii) Vortex casing

(iii) Casing with guide blades.

(i) Volute casing:

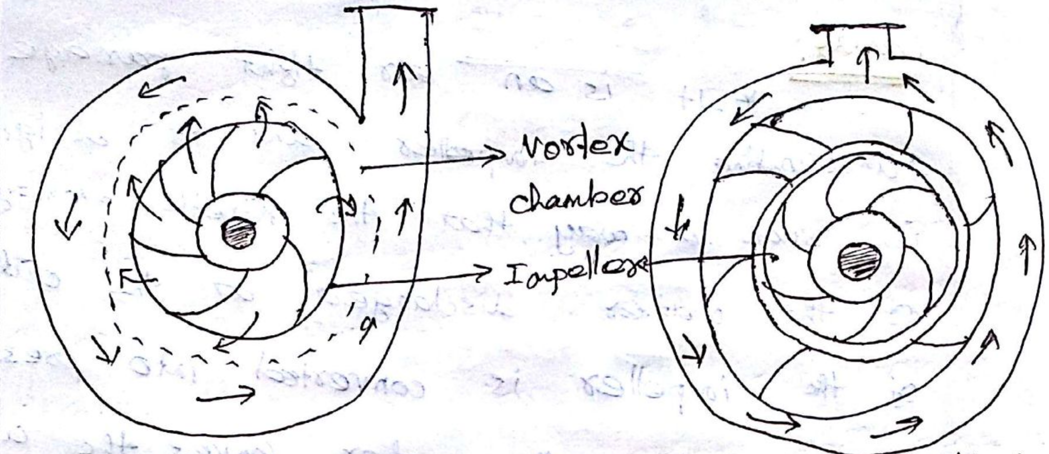
* It is normally used casing as shown in figure. It is of spiral type in which area of flow increases gradually.

* The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing thrgh the casing.

* Due to this efficiency of the pump increased.

(ii) Vortex Casing:

* If a circular chamber introduced between the casing and the impeller as shown in figure, the casing is known as vortex casing. Here the efficiency of the pump increased larger than the volute casing.



(iii) Casing with Guide Blades:

* In which Impeller is surrounded by series of guide blades mounted on a ring which is known as diffuser.

* The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without stock. Also the area of the guide vanes increases, thus reducing the velocity of flow thrh the guide vanes and consequently increasing the pressure of water.

Suction pipe with foot valve and strainer:

* A pipe whose one end is connected to the inlet of the pump & other end dips into water in a sump is known as suction pipe.

* A foot valve which is a non-return valve (or) one-way type of valve is fitted at the lower end of the suction pipe.

* The foot valve opens only in the upward direction.

* A strainer is also fitted at the lower end of the suction pipe.

Delivery pipe:

* A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height is known as delivery pipe.

Working principle of centrifugal pump:

step1:

The delivery valve is closed and the suction pipe, casing and portion of the delivery pipe up to the delivery valve is completely filled with the liquid so that no air pocket is left. This process is called priming.

step2:

* The electric motor is started to rotate the impeller by keeping the delivery valve still closed.

* The rotation of the impeller causes strong suction cor vaccum just at the eye of the casing.

step3:

* The speed of the impeller is gradually increased till the impeller rotates at its normal speed and developes normal

hydraulic energy required for pumping the liquid.

Step4:

* The delivery valve is opened after the impeller attains the normal speed. Now the liquid is continuously sucked by the suction pipe and passes through the eye of casing. then its enters the impeller at its centre i.e., at their inlet tips.

* This liquid is impelled out by the rotating vanes and it comes out at the outlet tips of the vanes into the casing.

* During this process, the pressure head as well as velocity head of the liquid are increased.

Step5:

the liquid is now entered into vortex/ volute chamber of casing where some of the velocity head is converted into pressure head in the casing.

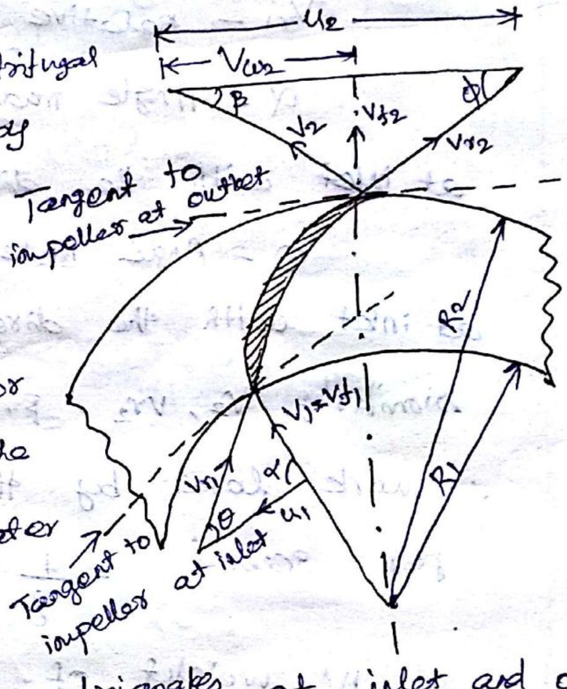
Step6:

From casing the liquid passes into the delivery pipe and is lifted to the required height.

Step 7:

When pump is stopped, the delivery valve should be closed. otherwise, there may be some back flow from the reservoir. If there is a foot valve, no need to close the delivery valve because it is a one-way non-return valve. It arrests the back flow.

Work done by the centrifugal pump (or) (By impeller) on water



* In case of centrifugal pump, work is done by the impeller on the water.

* The expression for the workdone by the impeller on the water is obtained by using drawing velocity triangles at inlet and outlet of the impeller.

* The water enters the impeller radially at inlet for best effiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet.

$$\therefore \alpha = 90° \text{ and } V_{w1} = 0$$

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let, N - Speed of the impeller in rpm

D_1 - Diameter of impeller at inlet

u_1 - Tangential velocity of impeller at inlet

$= \frac{\pi D_1 N}{60}$

D_2 - Diameter of impeller at outlet

u_2 - Tangential velocity of impeller at outlet

$= \frac{\pi D_2 N}{60}$

V_1 - Absolute velocity of water at inlet

V_{r1} - Relative velocity of water at inlet

α - Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane

θ - Angle made by relative velocity (V_{r1}) at inlet with the direction of motion of vane.

Similarly V_2, V_{r2}, β, ϕ at outlet.

∴ work done by the impeller on the water per second $= \frac{1}{g} V_{w_2} u_2 = \frac{W}{g} \cdot V_{w_2} \cdot u_2$

W = weight of water $= \rho \times g \times Q$

~~Q - volume of water~~

where Q - volume of water

Q = Area × velocity of flow $= \pi D_1 B_1 \times V_{f_1}$

$= \pi D_2 B_2 \times V_{f_2}$

where B_1, B_2 - are impeller width at inlet and outlet

V_{f_1}, V_{f_2} - velocity of flow at inlet & outlet

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Definition of head and Efficiencies of Centrifugal pump:

(i) **Suction head (h_s)** - vertical height of the centre line of the centrifugal pump above the water surface in the tank. It is denoted by (h_s).

(ii) **Delivery head (h_d)** - Distance b/w centre line of the pump and the water surface in the tank. It is denoted by (h_d)

(iii) **Static head (H_s)** → $H_s = h_s + h_d$

(iv) Manometric head (H_m)

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

h_s - suction head

h_d - delivery head

h_{fs} - suction pipe friction

h_{fd} - delivery pipe friction

V_d - velocity of water in delivery pipe.

Efficiency of a centrifugal pump:

(i) Manometric efficiency: (η_{man})

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$\eta_{man} = \frac{g H_m}{V_{w2} u_2}$$

The power at the impeller = $\frac{\text{Workdone by impeller per second}}{1000}$ KW

$$= \frac{W}{g} \frac{V_{w2} u_2}{1000} \text{ KW}$$

(ii) mechanical Efficiency : (η_m)

$$\eta_m = \frac{\text{power at the impeller}}{\text{power at the shaft}}$$

$$= \left(\frac{W}{g} \frac{V_{w_2} u_2}{1000}\right) / S.P.$$

S.P - shaft power.

(iii) Overall efficiency (η_o):

It is defined as power output of the pump to power input to the pump in (kW)

$$\left.\begin{array}{l}\text{output power of} \\ \text{the pump}\end{array}\right\} = \frac{\left(\begin{array}{c}\text{Weight of water} \\ \text{lifted}\end{array}\right) \times H_m}{1000}$$

$$= \frac{W \times H_m}{1000}$$

$$\left.\begin{array}{l}\text{power input to the} \\ \text{pump}\end{array}\right\} = \begin{array}{l}\text{power supplied by} \\ \text{electric motor}\end{array}$$

$= S.P$ of the pump

Also $\boxed{\eta_o = \eta_{man} \times \eta_m}$

From the velocity diagram:

From inlet velocity triangle $\tan\theta = \frac{V_{f_1}}{u_1}$

" outlet " " $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$

P1. The internal and external diameters of the impeller of a centrifugal pump are 200mm and 400 mm respectively. The pump is running at 1200 rpm. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the workdone by the impeller per unit weight of water.

Given:

Internal diameter of impeller $D_1 = 200\,mm = 0.2\,m$

External diameter of impeller $D_2 = 400\,mm = 0.4\,m$

Speed $N = 1200\,rpm$

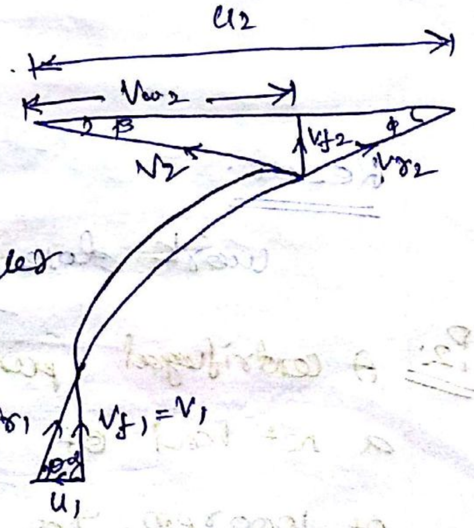
Vane angle inlet $\theta = 20°$ outlet $\phi = 30°$ $V_{f1} = V_{f2}$

$\alpha = 90°$ $V_{w1} = 0$

To find:

workdone by the impeller.

Sol:



work done by the impeller

$$= \frac{1}{g} V_{w_2} u_2$$

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_1 = \frac{\pi \times 0.2 \times 1200}{60} = 12.56\ m/s$$

$$u_2 = \frac{\pi \times D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13\ m/s$$

From inlet velocity triangle $\tan\theta = \frac{(V_1 = V_{f1})}{u_1}$

$$V_{f1} = \tan\theta \times u_1$$

$$= \tan 20^\circ \times 12.56$$

$$\boxed{V_{f1} = 4.57 \text{ m/s} = V_{f2}}$$

∴ From outlet velocity triangle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{4.57}{25.13 - V_{w2}}$$

$$25.13 - V_{w2} = \frac{4.57}{\tan 30^\circ}$$

$$-V_{w2} = 7.915 - 25.13$$

$$\boxed{V_{w2} = 17.21 \text{ m/s}}$$

∴ work done by impeller per unit weight of water $= \frac{1}{g} V_{w2} U_2$

$$= \frac{17.21 \times 25.13}{9.81}$$

$$\boxed{\text{work done} = 44.086 \text{ Nm/N}}$$

Result:

work done = 44.086 Nm/N

P2: A centrifugal pump delivers water against a net head of 14.5 meters and a design speed of 1000 rpm. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if monometric efficiency is 95%

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Given:

$H_m = 14.5 m$ $D_2 = 300 mm = 0.3 m$

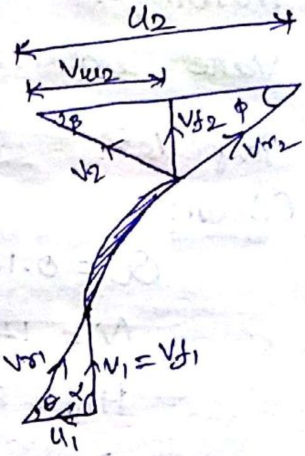
$N = 1000 rpm$ $B_2 = 50 mm = 0.05 m$

(back/outlet angle) $\phi = 30°$ $\eta_{man} = 95\%$.

To find:

Discharge Q.

Sol:



Tangential outlet velocity of impeller

$$u_2 = \frac{\pi D_2 N}{60}$$

$$= \frac{\pi \times 0.3 \times 1000}{60}$$

$$u_2 = 15.70 \ m/s$$

Manometric efficiency $\eta_{man} = 0.95$

$$\frac{g H_m}{V_{w2} u_2} = 0.95$$

$$\frac{9.81 \times 14.5}{V_{w2} \times 15.7} = 0.95$$

$$\frac{9.81 \times 14.5}{15.7 \times 0.95} = V_{w2}$$

$$V_{w2} = 9.54 \ m/s$$

From outlet velocity Δle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\tan 30° = \frac{V_{f2}}{15.7 - 9.54}$$

$$V_{f2} = 3.556 \ m/s$$

∴ Discharge $Q = \pi D_2 B_2 \times V_{f2}$

$$= \pi \times 0.3 \times 0.05 \times 3.556$$

$$Q = 0.1675 \ m^3/s$$

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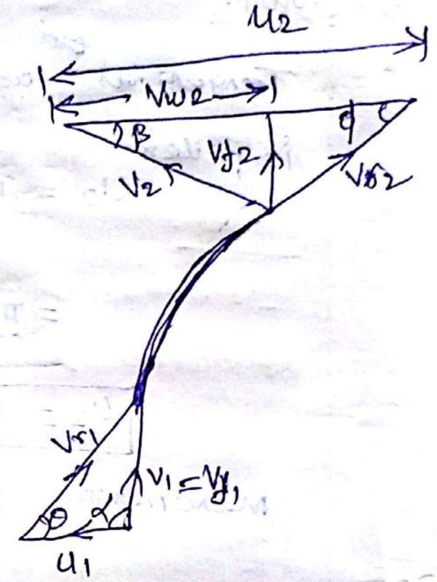
P3: A centrifugal pump is to discharge 0.188 m³/s at a speed of 1450 rpm. against a head of 25m. the impeller diameter is 250mm, its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the outlet periphery of the impeller.



Given:

$Q = 0.188\ m^3/s$

$N = 1450\ rpm$

$H_m = 25m$

$D_2 = 250mm = 0.25m$

$B_2 = 50mm = 0.05m$

$\eta_{man} = 75\% = 0.75$

To find:

outlet vane angle

Sol:

Tangential velocity of impeller at outlet

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60}$$

$$\boxed{u_2 = 18.98\ m/s}$$

Discharge $Q = \pi D_2 B_2 \times V_{f2}$

$$0.188 = \pi \times 0.25 \times 0.05 \times V_{f2}$$

$$\boxed{V_{f2} = 4.787\ m/s}$$

Manometric effiency $\eta_{man} = \dfrac{g H_m}{V_{w2} u_2}$

$$0.75 = \frac{9.81 \times 25}{V_{w2} \times 18.98}$$

$$V_{w2} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.22 \text{ m/s}$$

$$\boxed{V_{w2} = 17.22 \text{ m/s}}$$

From outlet velocity Δle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{4.787}{18.98 - 17.22}$$

$$\tan\phi = 2.7198$$

$$\boxed{\therefore \phi = 69^\circ\ 48'}$$

P4: A centrifugal pump having outer diameter equal to two times the inner diameter & ~~constant and~~ running at 1000 rpm works against a total head of 40m. The velocity of flow thro' the impeller constant & equal to 2.5 m/s. The vanes are set back, at an angle of 40° at outlet. If the outer diameter of the impeller is 500mm and width at outlet is 50 mm, determine.

(i) Vane angle at inlet (ii) work done by impeller per second

(iii) Manometric efficiency ~~(iv) mechanical efficiency~~

~~(v) overall efficiency. if power required to drive the pump is 16.186 kW~~

Given:

$N = 1000$ rpm

$D_2 = 2D_1$

$H_m = 40$ m

$V_{f1} = V_{f2} = 2.5$ m/s

$\phi = 40^\circ$

$D_2 = 500$ mm $= 0.5$ m

$B_2 = 50$ mm $= 0.05$ m

Power reqd (S.P) $= 16.186$ kW

To find:

(i) θ

(ii) Work done by impeller per sec

(iii) η_{man}

~~(iv) η_m~~

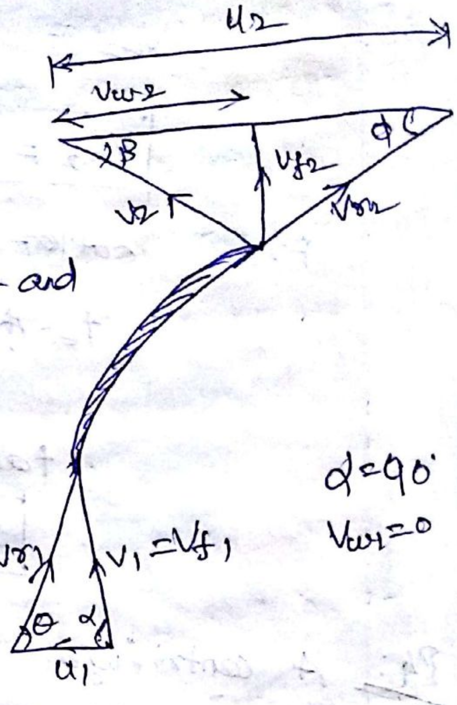
~~(v) η_o~~

Sol:

~~Absolute~~ internal diameter

$D_1 = \frac{0.5}{2} = 0.25\,m$

Tangential velocity at inlet and outlet are



$u_1 = \frac{\pi D_1 N}{60}$

$= \frac{\pi \times 0.25 \times 1000}{60}$

$u_1 = 13.08\ m/s$

$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .5 \times 1000}{60}$

$u_2 = 26.17\ m/s$

(i) vane angle at inlet

From inlet velocity Δle

$\tan\theta = \frac{V_{f1}}{u_1} = \frac{2.5}{13.08} = 0.191$

$\theta = \tan^{-1}(0.191) = 10°\,49'$ or 10.81

(ii) Work done by impeller per second:

$= \frac{W}{g} V_{w2} \cdot u_2$

weight of water } $W = \rho \times g \times Q$

Discharge $= \pi \times D_2 \times B_2 \times V_{f2}$

$Q = \pi \times 0.5 \times 0.05 \times 2.5 = 0.1963\ m^3/s$

$\therefore W = 1000 \times 9.81 \times 0.1963$

$W = 1926.188\ N$ ~~N/s~~

from outlet velocity Δle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{2.5}{26.17 - V_{w2}}$$

$$\tan 40^\circ = \frac{2.5}{26.17 - V_{w2}}$$

$$26.17 - V_{w2} = \frac{2.5}{\tan 40^\circ} = 2.979$$

$$\therefore V_{w2} = 23.19 \text{ m/s}$$

$\therefore$ workdone per second $= \frac{1926.188}{9.81} \times 23.19 \times 26.17$

$= 119160.99$ Nm/s

(iii) manometric efficiency (η_{man}):

$$\eta_{man} = \frac{g H_m}{V_{w2} u_2} = \frac{9.81 \times 40}{23.19 \times 26.17}$$

$\eta_{man} = 0.64$ | 64%.

(iv) ~~mechanical efficiency (η_m)~~

~~η_m = workdone power at impeller / s.p~~

~~power at impeller = workdone by impeller / 1000 kW~~

~~$= \frac{119160.99}{1000} = 119.16$ kW~~

~~$\therefore \eta_m = \frac{119.16}{16.186} =$~~

~~overall efficiency (η_o) = power at outlet of pump / input power to pump~~

~~$= \frac{W \times H_m}{1000 \times s.p}$~~

~~$= \frac{1926.188 \times 40}{1000 \times 16.186}$~~

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P5: The outlet diameter of an impeller of a centrifugal pump is 400 mm & outlet width is 50 mm. The pump is running at 800 rpm. and is working against a total head of 15 m. The vanes angle at outlet is 40° & manometric efficiency is 75%. Determine.

(i) velocity of flow at outlet

(ii) velocity of water leaving the vane.

(iii) angle made by the absolute velocity at outlet with the direction of motion at outlet.

(iv) Discharge.

Given:

$D_2 = 400\,mm = 0.4\,m$

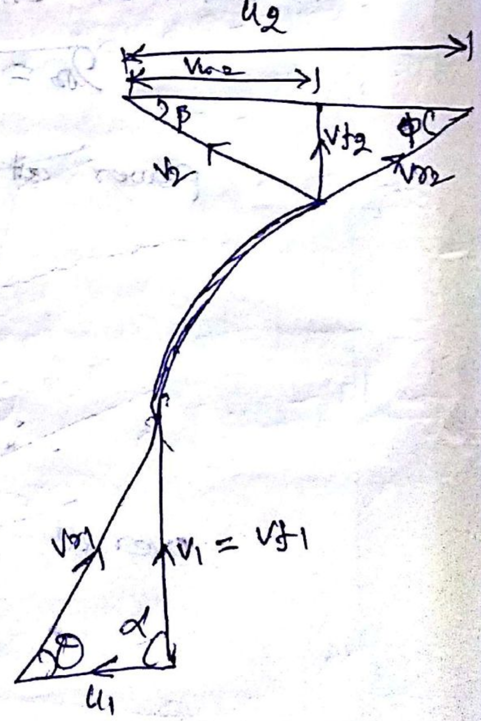
$B_2 = 50\,mm = 0.05\,m$

$N = 800$ rpm

$H_m = 15\,m$

$\phi = 40°$

$\eta_{man} = 75\% = 0.75$



To find:

(i) V_{f_2}

(ii) V_2

(iii) β

(iv) Q

(iv) Discharge $(Q) = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.4 \times 0.05 \times 2.908$

$Q = 0.1827\ m^3/s$

Sol:

(i) Velocity of flow at outlet (V_{f2})

Tangential velocity at outlet $u_2 = \frac{\pi \times D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60}$

$$u_2 = 16.75 \text{ m/s}$$

From outlet velocity Δle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

manometric efficiency $\eta_{man} = \frac{g H_m}{V_{w2} u_2}$

$$0.75 = \frac{9.81 \times 15}{V_{w2} \times 16.75}$$

$$\therefore V_{w2} = \frac{9.81 \times 15}{0.75 \times 16.75}$$

$$V_{w2} = 11.713 \text{ m/s}$$

$$\therefore \tan 40^\circ = \frac{V_{f2}}{16.758 - 11.715}$$

$$\therefore V_{f2} = \tan 40^\circ \times 5.087$$

$$V_{f2} = 2.908 \text{ m/s}$$

(ii) velocity of water leaving the vane: (V_2)

From outlet velocity Δle

$$V_2^2 = V_{w2}^2 + V_{f2}^2$$

$$\therefore V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(11.713)^2 + (2.908)^2}$$

$$\therefore V_2 = 12.08 \text{ m/s}$$

(iii) angle made by absolute velocity @ outlet (β)

$$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{2.908}{11.713}$$

$$\beta = \tan^{-1}(0.2482) = 13^\circ 56'$$

$$\beta = 13^\circ 56'$$

characteristic curves of centrifugal pump:

* Characteristic curves of centrifugal pumps are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump.

* These curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rate, head and speed.

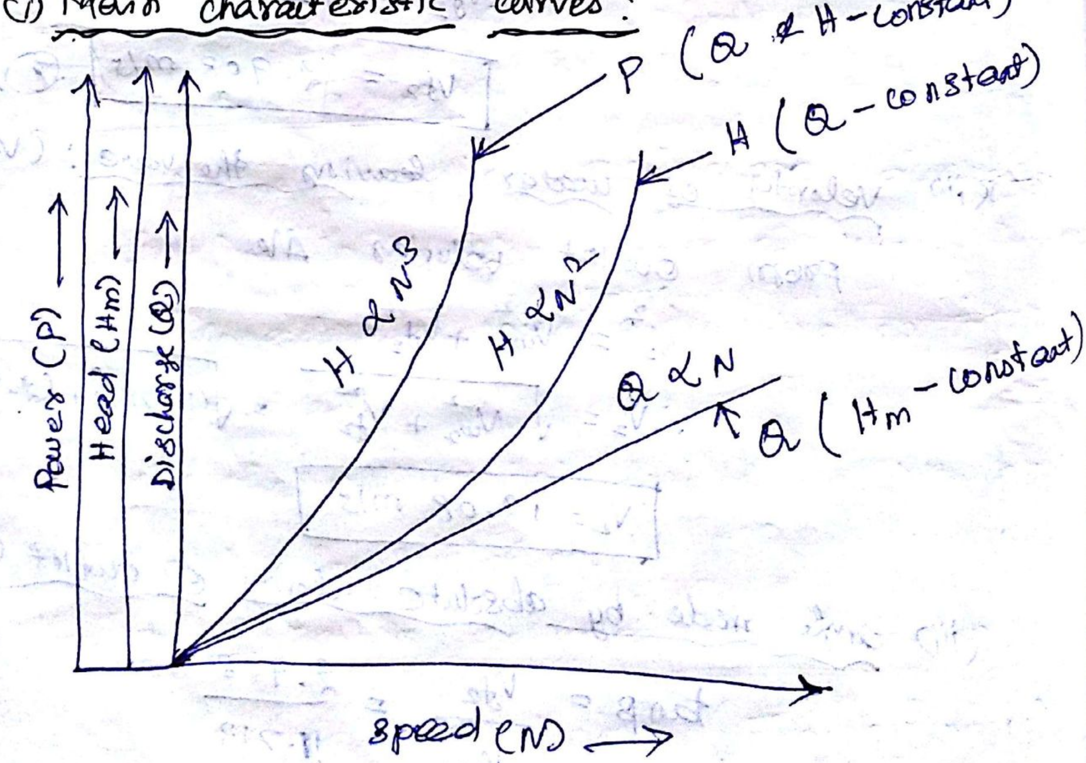
The following are important characteristics curves for pumps,

(i) Main characteristic curves

(ii) operating characteristic curves

(iii) constant efficiency or Muschel curves.

(i) Main characteristic curves:



* The characteristic curves of a centrifugal pump consists of power, head, discharge & speed.

* It was drawn b/w Power, Head & discharge to the speed of the pump.

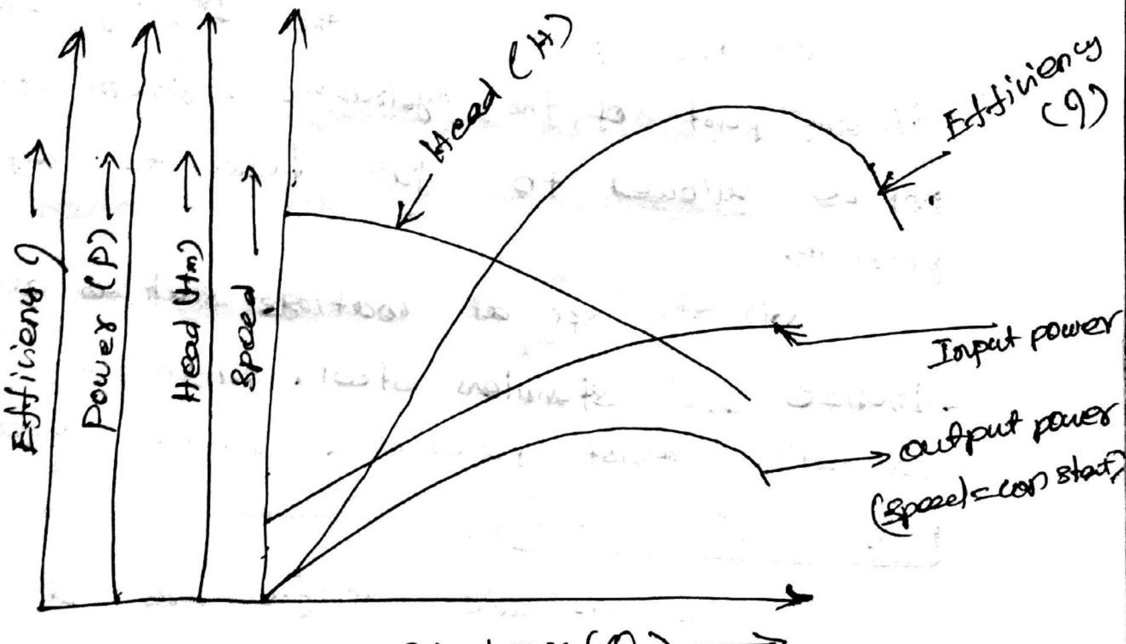
* For plotting the curves of discharge (Q) means we should maintain Hm is constant parameter.

* For plotting " " (H) means

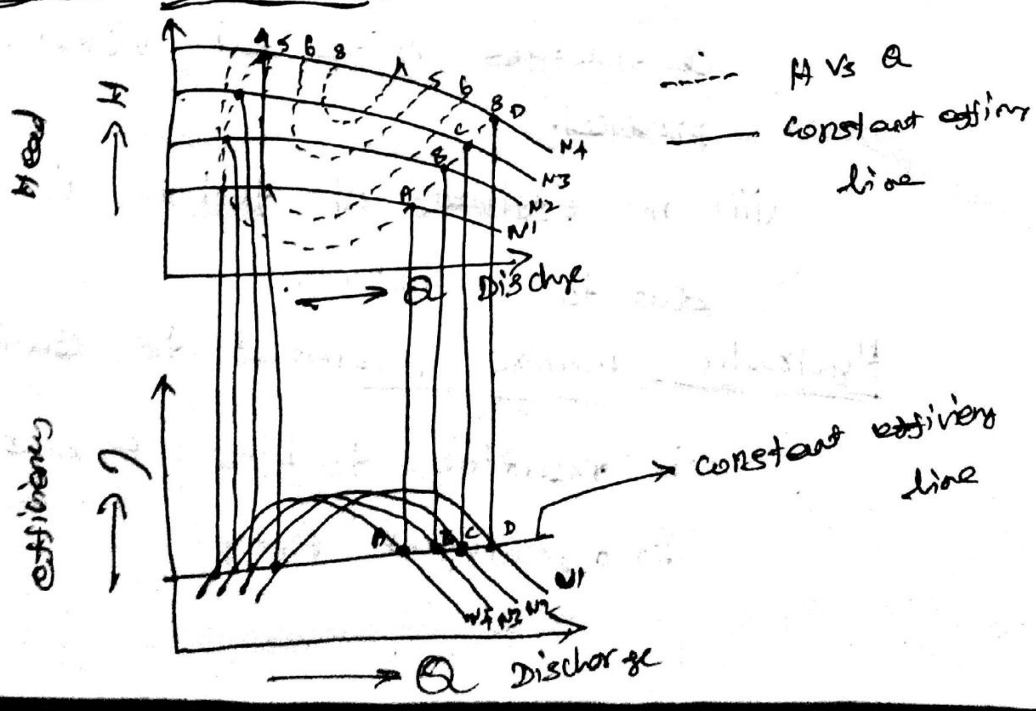
" " " (Q) is " "

* For plotting curves for power means we should maintain discharge & H is constant param

(ii) operating characteristics curves:



constant efficiency curves:



CAVITATION:

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, very high pressure is created.

Precaution Against cavitation:

(i) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure.

(ii) The special coatings such as aluminium bronze and stainless steel, which are cavitation resist material, should be used.

Effects of Cavitation:

(i) The metallic surface are damaged.

(ii) Due to sudden collapse of vapour bubble, considerable noise and vibration is produced.

(iii) The efficiency of turbine is reduce due to cavitation.

Hydraulic machine subjected to cavitation:

(i) reaction turbine (Francis turbine)

(ii) centrifugal pumps

Cavitation in Turbines:

* In turbines only reaction turbines are subjected to cavitation.

* In reaction turbines the cavitation may occur at the outlet of the runner (or) at the inlet of the draft tube where the pressure is considerably reduced. (below the vapour pressure).

Cavitation in centrifugal pumps:

* In centrifugal pump the cavitation may occur at the inlet of the impeller of the pump (or) at the suction side of the pump, where the pressure is considerably reduced. (below the vapour pressure).

problem based on friction in a pump:

1. Find the power required to drive a centrifugal pump which delivers 0.04 m^3/s of water of a height of 20m thro a 15 cm diameter pipe & 100m long. The overall efficiency of the pump is 70%. & coefficient of friction $f = 0.15$ in the formula $h_f = 4fLV^2/2gd$.

Given:

Discharge $(Q) = 0.04\ m^3/s$

height $H = 20m$

diameter of pipe $d_s = d_d = 15\ cm = 0.15m$

length of pipe $L = L_s = L_d = 100m$

overall efficiency $\eta_o = 70\% = 0.7$

$f = 0.15$ $h_f = \frac{4fLV^2}{2gd}$

To find: power required to drive the pump.

Sol.

Over all efficiency $\eta_o = \frac{\text{output power by the impeller}}{\text{Input power} = S.P = E.P}$

$$\eta_o = \frac{\left(\frac{W H_m}{1000}\right)}{S.P}$$

W - weight of water

$W = \rho g Q$

$= 1000 \times 9.81 \times 0.04$

$\boxed{W = 392.4\ N}$

H_m - manometric head

$= (H_s + H_d) + h_{fd} + h_{fs} + \frac{V_d^2}{2g}$

$H_m = H + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$

Discharge $Q = A \times V_d$

$0.04 = \pi/4 \times d^2 \times (V_d = V_s)$

$\therefore V_d = V_s = 0.04 / 0.0176 = 2.26\ m/s$

$\therefore h_{fd} = \frac{4 f L_d V_d^2}{2 g d_d} = \frac{4 \times 0.015 \times 100 \times (2.26)^2}{2 \times 9.81 \times 0.15}$

$h_{fd} = 10.445\ m = h_{fs}$ because dia & L are same

$\therefore H_m = 20 + 10.445 + 10.445 + \frac{2.26^2}{2 \times 9.81}$

$\boxed{H_m = 41.150\ m}$

$\therefore$ output power $= \frac{392.4 \times 41.150}{1000}$ kW

$O/P = 16.147$ kW

$\therefore \eta_o = O/P / S.P$

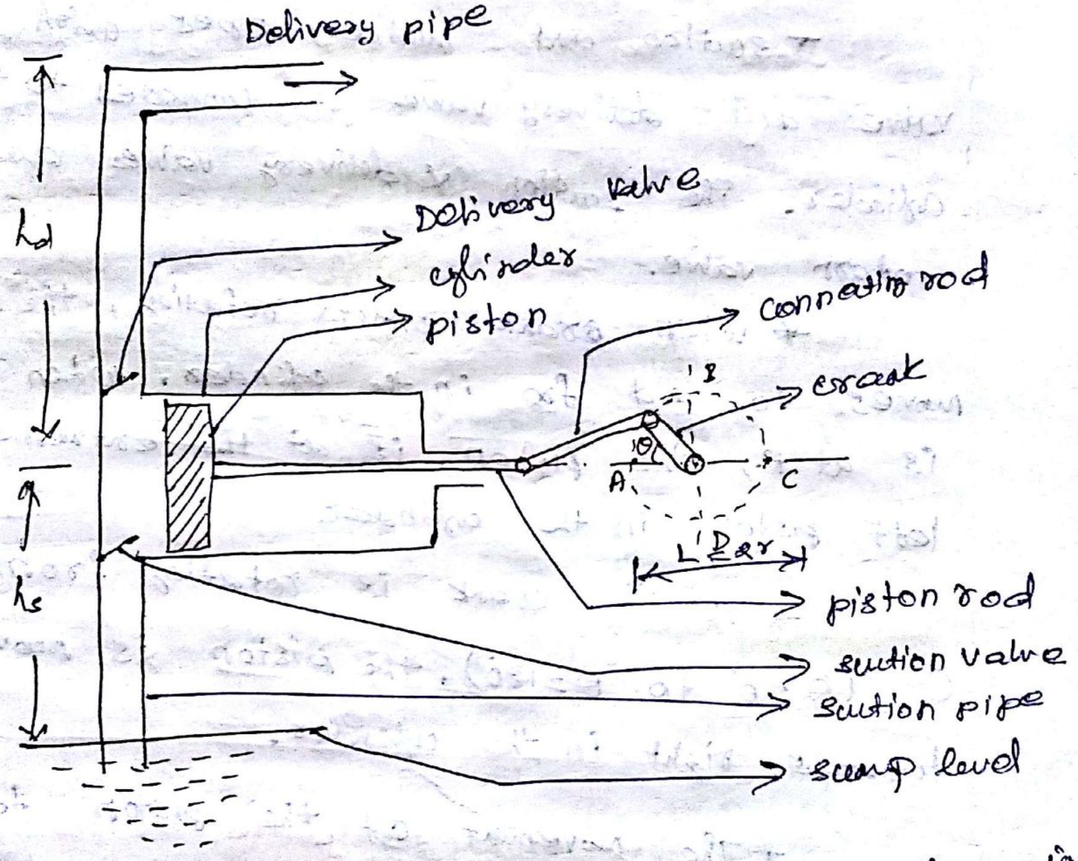
$S.P = \frac{16.147}{0.7} = 23$ kW

S.P is the power required to drive the C.P.

Reciprocating pump:

If the mechanical Energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a piston is reciprocating which exerts the thrust on the liquid and increases its hydraulic energy, the pump is known as reciprocating pump.

Main parts of a reciprocating pump:



1. A cylinder with a piston, piston rod, connecting rod
2. suction pipe
3. delivery pipe
4. suction valve
5. Delivery valve.

Working of a reciprocating pump:

* A single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder.

* The movement of the piston is obtained by connecting the piston rod to crank by means of connecting rod. The crank is rotated by means of an electric motor.

* Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction & delivery valves are non-return valve.

* When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at A, the piston is at the extreme left piston in the cylinder.

* As the crank is rotating from A to C ($\theta = 0^\circ$ to $\theta = 180^\circ$), the piston is moving towards right in the cylinder.

* The movement of the piston towards right creates a partial vaccum in the cylinder. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder.

* Thus, the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

* when crank is rotating from C to A

i.e, ($\theta = 180°$ to $\theta = 360°$), the piston from its extreme right position starts moving towards left in the cylinder.

* The movement of the piston towards left increase the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

For Single acting Reciprocating pump

formulas as follows

let D - Diameter of the cylinder

A - cross sectional area of the piston

$= \pi/4 \, D^2$

r - radius of crank

N - r.p.m of the crank

L - length of the stroke $= 2 \times r$

h_s - Height of the axis of the cylinder from water surface in sump.

h_d - Height of delivery outlet above the cylinder axis.

Volume of water delivered in one revolution

= Area × length of stroke

$= A \times L$

Number of revolution per second $\} = \frac{N}{60}$

∴ Discharge of the pump per second

Q = Discharge in one revolution × No. of revolution per sec

$= A \times L \times \frac{N}{60}$

$$Q = \frac{ALN}{60}$$

weight of the water $W = \rho \times g \times Q$

Workdone by reciprocating pump

work done per sec = {weight of water lifted per sec} × {Total height through which water is lifted}

$= W \times (h_s + h_d)$

where $(h_s + h_d)$ = Total height through which water is lifted

$W = \frac{\rho g ALN}{60} = \frac{\rho g ALN}{60}$

∴ work done per second

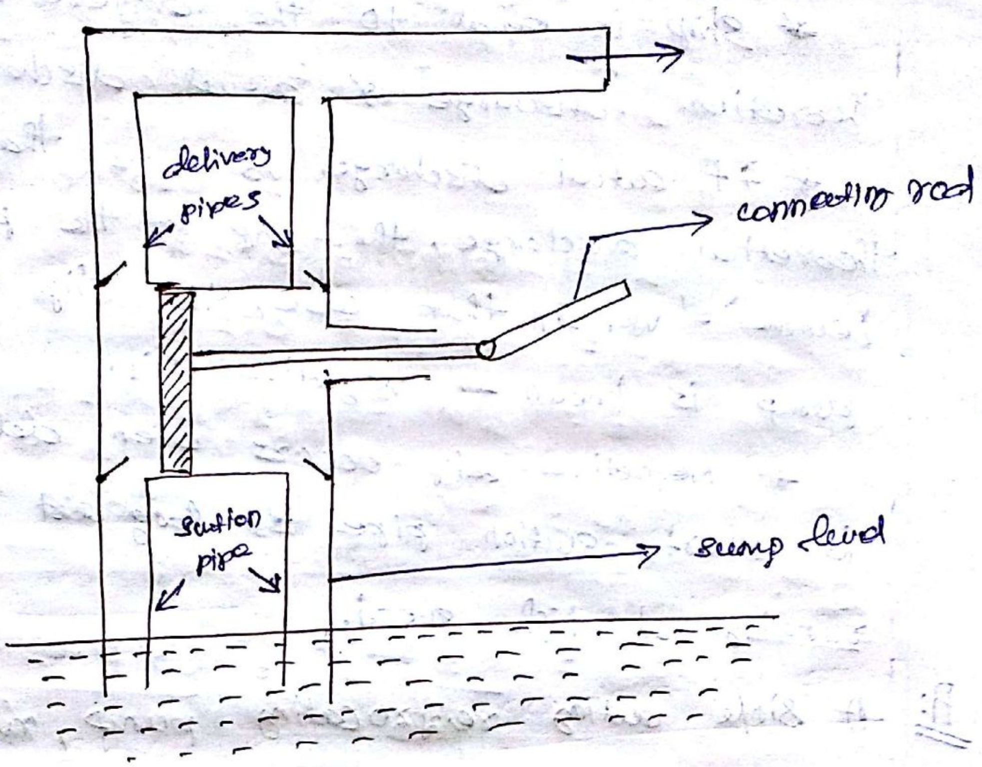
$$= \frac{\rho \times g \times A \times L \times N}{60} \times (h_s + h_d)$$

∴ power required to drive the pump in kw

$$P = \frac{\text{workdone per second}}{1000}$$

$$P = \frac{\rho g ALN \times (h_s + h_d)}{60 \times 1000}$$

Double acting reciprocating pump:



Discharge $Q = \frac{2ALN}{60}$

Work done per second $= 2\rho g \frac{ALN}{60} \times (h_s + h_d)$

power required $P = \frac{2\rho g ALN \times (h_s + h_d)}{60,000}$

Slip of Reciprocating pump:

Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump.

$$Slip = Q_{th} - Q_{act}$$

$$\% \text{ slip} = \left(\frac{Q_{th} - Q_{act}}{Q_{th}}\right) \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100$$

where C_d – co-efficient of discharge $= \frac{Q_{act}}{Q_{th}}$

Negative Slip of the reciprocating pump:

* Slip is equal to the difference of theoretical discharge & actual discharge.

* If actual discharge is more than the theoretical discharge, the slip of the pump will become '-' ve. In that case, the slip of the pump is known as negative slip.

* Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

P: A single acting reciprocating pump, running at 50 rpm., delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm & stroke length 400mm. Determine:

(i) The theoretical discharge of the pump.

(ii) Co-efficient of discharge

(iii) Slip and the percentage slip of the pump.

Given:

$N = 50$ rpm

$Q = 0.01\ m^3/s$

$D = 200\ mm = 0.2\ m$

$L = 400\ mm = 0.4\ m$

To find:

(i) Q_{th}

(ii) C_d

(iii) % slip.

Sol:

(i) Theoretical dischge.

$$Q_{th} = \frac{ALN}{60}$$

$A = \pi/4 \times d^2$

$= \pi/4 \times 0.2^2$

$A = 0.0314\ m^2$

$$= \frac{0.0314 \times 0.4 \times 50}{60}$$

$Q_{th} = 0.01047\ m^3/s$

(ii) Co-efficient of dischge

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.954$$

(iii) % slip.

$slip = Q_{th} - Q_{act}$

$= 0.01047 - 0.01$

$slip = 4.7 \times 10^{-4}\ m^3/s$

$$\% \ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} = \frac{4.7 \times 10^{-4}}{0.01047} \times 100$$

$\% \ slip = 4.48\%$.

Result:

(i) $Q_{th} = 0.01047\ m^3/s$

(ii) $C_d = 0.954$

(iii) % slip = 4.48%

P2: A double-acting reciprocating pump, running at 40 rpm, is discharging 1.0 m^3 of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery & suction head are 20 m & 5 m respectively. Find the slip of the pump & power.

Given:

$N = 40$ rpm

$Q_{act} = 1\ m^3/min = 1/60\ m^3/s$

$= 0.0166\ m^3/s$

$L = 400\ mm = 0.4\ m$

$D = 200\ mm = 0.2\ m$

$A = \pi/4\ d^2 = \pi/4\ (.2)^2 = 0.0314\ m^2$

$h_s = 5\ m \quad h_d = 20\ m$

Sol:

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times 0.0314 \times 0.4 \times 40}{60}$$

$$Q_{th} = 0.01675\ m^3/s$$

$$\text{Slip} = Q_{th} - Q_{act} = 0.01675 - 0.166$$

$$\text{Slip} = 0.0009\ m^3/s$$

$$\text{power} = \frac{2\rho g ALN}{60000} \times (h_s + h_d)$$

$$= \frac{2 \times 1000 \times 9.81 \times 0.0314 \times 0.4 \times 40 \times}{60000}$$

$$P = 4.109\ kW$$

Result:

$Q_{th} = 0.01675\ m^3/s$

Slip $= 0.0009\ m^3/s$

$P = 4.109\ kW$

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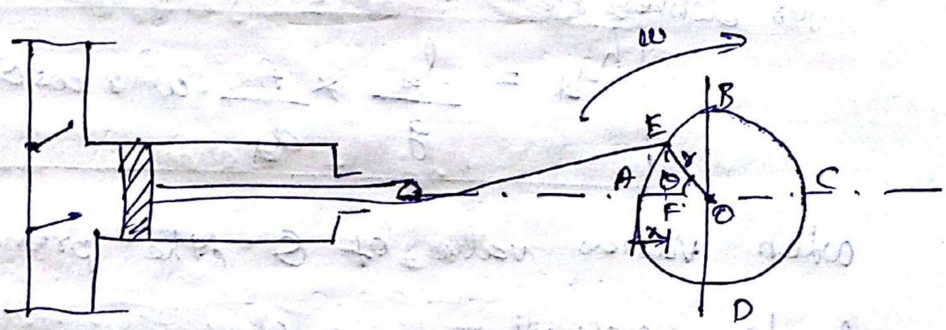
Variation of velocity and acceleration in the sution and delivery pipes due to acceleration of the piston.

let
- ω - angular speed of the crank
- A - area of the cylinder
- a - area of the pipe (sution or delivery)
- l - length of the pipe. (sution or delivery)
- r - radius of the crank.



Velocity of water in pipe

$$v = \frac{A}{a}\,\omega r \sin\omega t$$

Acceleration of water in pipe

(5+20)

$$a_c = \frac{A}{a}\,\omega^2 r \cos\omega t$$

Force required to accelerate the water in the pipe = mass of water in pipe × Acceleration of water

$$F = \rho a l \times \frac{A}{a}\,\omega^2 r \cos\omega t = \rho l A\,\omega^2 r \cos\omega t$$

∴ Intensity of pressure due to acceleration

$$= \frac{\text{Force required to accelerate the water}}{\text{Area of pipe}}$$

$$= \rho l \frac{A}{a}\,\omega^2 r \cos\theta$$

pressure head due to acceleration.

$$h_a = \frac{\text{Intensity of pressure due to acc}}{\text{weight density of liquid}}$$

$$h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r \cos\theta$$

for suction pipe

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$$

for delivery pipe

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta$$

when various value of θ the pressure head (h_a) due to acceleration may change.

when (i) $\theta = 0°$ $h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r$

(ii) $\theta = 90°$ $h_a = 0$

(iii) $\theta = 180°$ $h_a = -\frac{l}{g} \times \frac{A}{a} \omega^2 r$

$\therefore$ maximum pressure head due to acceleration ($h_{a\,max}$) $= \frac{l}{g} \frac{A}{a} \omega^2 r$

Effect of variation of velocity on Friction in the sution & delivery pipes.

$$h_f = \frac{4fLV^2}{2gd} \qquad V = \frac{A}{a}\omega r \sin\theta$$

$$\therefore h_f = \frac{4fL}{2gd}\left[\frac{A}{a}\omega r \sin\theta\right]^2$$

for sution pipe

$$h_{f_s} = \frac{4fL_s}{2gd_s} \times \left[\frac{A}{a_s}\omega r \sin\theta\right]^2$$

for delivery pipe

$$h_{fd} = \frac{4fL_d}{2gd_d} \times \left[\frac{A}{a_d} \times \omega r \sin\theta\right]^2$$

when.

(i) $\theta = 0°$ $\quad h_f = 0$

(ii) $\theta = 90°$ $\quad h_f = \frac{4fL}{2gd}\left[\frac{A}{a}\omega r\right]^2$

(iii) $\theta = 180°$ $\quad h_f = 0$

$\therefore$ maximum value of loss of head due to friction

$$(h_{f_{max}}) = \frac{4fL}{2gd}\left[\frac{A}{a}\omega r\right]^2$$

P1: The cylinder bore diameter of a single-acting reciprocating pump is 150mm and its stroke is 300mm. The pump runs at 50 rpm & lifts water thrh a height of 25m. The delivery pipe is 22m long and 100mm in diameter. Find the theoretical discharge & the theoretical power required to run the pump. If the actual discharge is 4.2 litres/s, find the % slip. Also determine the acceleration head at the beginning and middle of the delivery stroke.

Given:

Dia of cylinder $D = 150\,mm = 0.15\,m$

stroke $L = 300\,mm = 0.3\,m$

speed $N = 50\,rpm$

$H = 25\,m$

$h_d = 22\,m$

$d_d = 100\,mm = 0.1\,m$

actual discharge $= 4.2$ litres/s $= 0.0042\,m^3/s$

To find:

(i) Q_{th} (ii) P_{th} (iii) % slip

(iv) acc) beginning of delivery pipe.

(v) acc) middle of delivery stroke

Sol:

$$Area = \pi/4 \times D^2 = \pi/4 \times 0.15^2$$

$$A = 0.0176\,m^2$$

(i) Theoretical discharge Q_{th}

$$Q_{th} = \frac{ALN}{60}$$

$$= \frac{0.0176 \times 0.3 \times 50}{60} = 4.417 \times 10^{-3}$$

$$Q_{th} = 4.417 \text{ litre/s}$$

(ii) Theoretical power (P_{th})

$$= \frac{\rho \times g \times Q_{th} \times (h_s + h_d)}{1000}$$ [$h_s + h_d = 17$]

$$= \frac{1000 \times 9.81 \times 4.417 \times 10^{-3} \times 25}{1000}$$

$P_{th} = 1.083$ KW

(iii) % Slip:

$$\% \text{ slip} = \frac{Q_{th} - Q_{act}}{Q_{th}}$$

$$= \frac{4.417 - 4.2}{4.417} \times 100$$

% slip = 4.912

(iv) Acceleration head at the begining of delivery stroke

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 \times r \times \cos\theta$$

$$a_d = \pi/4 \times d_d^2 = \pi/4 \times (0.1)^2 = 0.00785 \ m^2$$

$$L = 2r \Rightarrow \frac{0.3}{2} = r \Rightarrow r = 0.15 \ m$$

at begining $\theta = 0°$ $\therefore \cos 0° = 1$ $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 50}{60}$

$\omega = 5.23$ rad/s

$$\therefore h_{ad} = \frac{22}{9.81} \times \frac{0.0176}{0.00785} \times (5.23)^2 \times 0.15$$

$h_{ad} = 20.629$ m

(v) Acceleration head at middle of delivery pipe.

@ middle $\theta = 90°$ $\cos 90° = 0$

$\therefore h_{ad} = 0$

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UNIT - V

Hydraulic Machines - Turbines

Turbines:

* Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy.

* This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus mechanical energy is converted into electric energy.

Definition of heads and efficiencies of a turbine:

(i) **Gross head:**

- The difference b/w the head race level and tail race level when no water is flowing is known as Gross head.
It denoted by H_g.

(ii) **Net head:**

It is also called effective head and is defined as the head available ~~that~~ at the inlet of the turbine.

$$H = H_g - h_f$$ where H_g - Gross head

$$h_f = \frac{4fLV^2}{2gd}$$

V - Velocity of flow in penstock
L - length of penstock
D - Diameter of penstock.

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(iii) Efficiency of a Turbine :

(a) Hydraulic efficiency η_h

(b) Mechanical efficiency η_m

(c) Volumetric efficiency η_v

(d) overall efficiency η_o

(a) Hydraulic efficiency η_h :

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$$

$$= \frac{R.P}{W.P}$$

R.P - power delivered to runner,

$$(R.P)_{\text{pelton turbine}} = \frac{W}{g} \frac{[V_{w_1} \pm V_{w_2}] \times u}{1000} \text{ KW}$$

$$(R.P)_{\text{radial flow turbine}} = \frac{W}{g} \frac{[V_{w_1} u_1 \pm V_{w_2} u_2]}{1000} \text{ KW}$$

W.P - Power supplied at inlet of turbine

$$= \frac{W \times H}{1000} \text{ KW}$$

where,

W - weight of the water striking the vanes of the turbine per second.

$W = \rho \times g \times Q$

V_{w_1} - velocity of whirl at inlet

V_{w_2} - velocity of whirl at outlet

u - Tangential velocity of vane.

u_1 - Tangential velocity of vane @ inlet

u_2 - Tangential velocity of vane @ outlet

H - Net head on the turbine.

power supplied at inlet

$$W.P = \frac{\rho \times g \times Q \times H}{1000} \quad KW$$

(b.) Mechanical efficiency (η_m)

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner.}}$$

$$\eta_m = \frac{S.P}{R.P}$$

(c.) Volumetric efficiency (η_v)

$$\eta_v = \frac{\text{volume of water actually striking the runner}}{\text{volume of water supplied to the turbine}}$$

(d.) Overall efficiency (η_o)

$$\eta_o = \frac{\text{volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}}$$

$$\eta_o = \frac{\text{shaft power}}{\text{water power}}$$

$$\eta_o = \eta_m \times \eta_h$$

Water power in KW $= \dfrac{\rho \times g \times Q \times H}{1000}$

shaft power is represented by 'P'.

P - shaft power.

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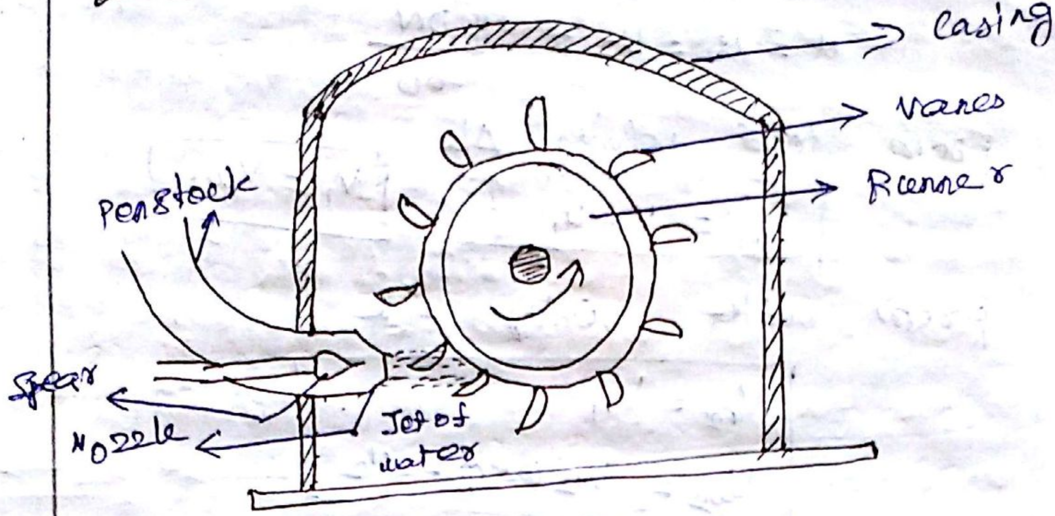
Pelton wheel (or) Turbine

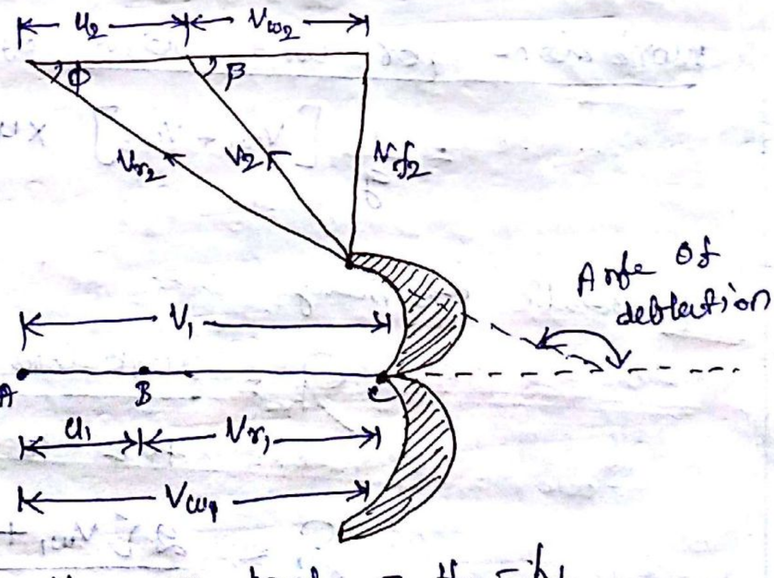


Main parts

(i) Nozzle (ii) Runner & buckets

(iii) casing (iv) Breaking Jet.

Workdone For pelton wheel turbine and velocity Triangles:



where H - net head = $H_g - h_f$

where D^* - Dia of penstock

D - Dia of wheel

d - diameter of Jet

N - speed of the wheel. rpm

V_1 – velocity of jet at inlet $= \sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}$$

From inlet velocity Δle

$$V_{r_1} = V_1 - u_1$$

$$V_1 = V_{w_1}$$

From outlet velocity Δle

$$V_{r_2} = V_{r_1}$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2$$

Workdone by the jet on the runner per second

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \times u \quad Nm/s$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{1000} \; kW$$

Work done per unit weight of water striking

$$= \frac{1}{g}[V_{w_1} + V_{w_2}] \times u$$

Hydraulic efficiency

$$\eta_h = \frac{\text{Workdone per second}}{\text{K.E of jet per second}}$$

$$\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1^2}$$

Points to be Remembered for pelton wheel:

(i) The velocity of Jet @ Inlet given by

$$V_1 = C_v \sqrt{2gH}$$

C_v - co-efficient of velocity = 0.98 (or) 0.99

H - net head.

(ii) The velocity of wheel (u)

$$u = \phi \sqrt{2gH}$$

ϕ - speed ratio varies from 0.43 to 0.48

(iii) The angle of deflection of the jet is taken as 165° if no angle of deflection is given.

(iv) mean diameter (or) the pitch diameter D of the pelton wheel

$$u = \frac{\pi DN}{60} \text{ (or) } D = \frac{u \times 60}{\pi \times N}$$

(v) Jet Ratio

$$m = \frac{D}{d}$$

ratio of pitch diameter of the wheel to the diameter of the jet.

for most cases m=12

(vi) Number of buckets on a runner is given by,

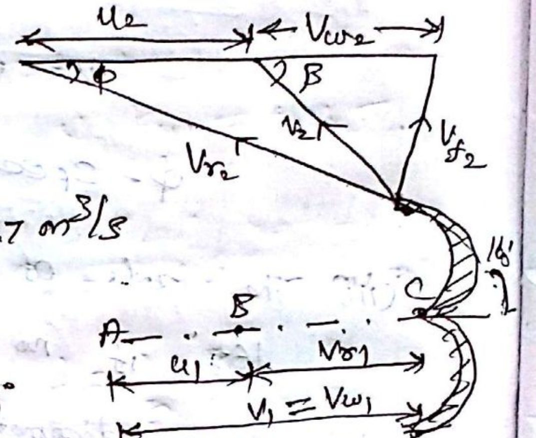
$$Z = 15 + \frac{D}{2d} = 15 + 0.5\,m$$

m - Jet ratio

(vii) Number of Jets.

It is obtained by dividing the total rate of flow thro' the turbine by the rate of flow of water thro' a single jet.

P1: A pelton wheel has a mean bucket speed of 10 meters per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 meters. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.



Given:

$u = u_1 = u_2 = 10 \ m/s$

$Q = 700 \ litre/s = 0.7 \ m^3/s$

$H = 30 m$

Angle of deflection $= 160°$

$\therefore \phi = 180° - 160° = 20°$

$C_v = 0.98$

To find:

(i) Power to runner

(ii) hydraulic efficiency.

Sol:

(i) Power to runner:

$$\text{Power} = \frac{\text{Work done per second}}{1000} \ kW$$

Work done per second on the runner $\} = \rho a V_1 [V_{w1} + V_{w2}] \times u$

$V_1 = V_{w1}$, $V_{w2} = V_{r2} \cos\phi - u_2$

$V_1 = C_v \sqrt{2gH}$

$\therefore V_1 = 0.98 \sqrt{2 \times 9.81 \times 30}$

$\boxed{V_1 = 23.7 \ m/s} = V_{w1}$

$V_{w_2} = V_{r_2} \cos\phi - u_2$

$V_{r_2} = V_{r_1}$

$V_{r_1} = V_1 - u_1$

$\therefore V_{r_1} = 23.7 - 10 = 13.7 \text{ m/s}$

$V_{r_1} = V_{r_2} = 13.7 \text{ m/s}$

$\therefore V_{w_2} = 13.7 \cos 20^\circ - 10$

$= 12.873 - 10$

$V_{w_2} = 2.87 \text{ m/s}$ $\qquad Q = aV_1$

$\therefore$ work done per second $= 1000 \times 0.7 (23.7 + 2.87) \times 10$

$= 186,016.52 \text{ Nm/s}$

$\therefore$ power $= \frac{186016.52}{1000}$ kW

power = 186.016 kW

(ii) hydraulic efficiency (η_h) $= \frac{2 [V_{w_1} + V_{w_2}] \times u}{V_1^2}$

$= \frac{2 [23.77 + 2.94] \times 10}{23.7^2}$

$\eta_h = 94.8\%$

Result :

(i) P = 186.016 kW

(ii) η_h = 94.8%

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P2: A pelton wheel is to be designed for the following specifications: shaft power = 11772 kW

Head = 380 meters, speed = 750 rpm;

overall efficiency = 86 %, Jet diameter is not to exceed one-sixth of the wheel diameter;

Determine:

(i) The wheel dia. (ii) Number of jets required

(iii) Diameter of Jet.

Take $K_{v_1} = 0.985$ & $K_{u_1} = 0.45$

Given:

$S.P = 11772$ KW

$\eta_0 = 86\% = 0.86$

$H = 380$ m

$N = 750$ rpm

$d = \frac{1}{6} D$

$K_{v_1} = C_v = 0.985$

$K_{u_1} = \phi = 0.45$

To find:

(i) D (ii) Number of jet

(iii) d.

Sol:

(i) Diameter of the wheel 'D'

$$u = \frac{\pi D N}{60}$$ u is unknown parameter

w.k.that

speed ratio $K_{u_1} = \phi = 0.45$

the velocity of wheel.

$$u = u_1 = u_2 = \phi \sqrt{2gH}$$

$$\therefore u = u_1 = u_2 = 0.45 \sqrt{2 \times 9.81 \times 380}$$

$$u = u_1 = u_2 = 38.85 \text{ m/s}$$

$$\therefore 38.85 = \frac{\pi \times D \times 750}{60}$$

$$\therefore D = \frac{38.85 \times 60}{\pi \times 750} = 0.989 \text{ m}$$

$$\boxed{D = 0.989 \text{ m}}$$

(ii) Diameter of Jet 'd'

$$d = \frac{1}{6} D$$

$$= \frac{1}{6} \times 0.989$$

$$\boxed{d = 0.164 \text{ m}}$$

(iii) Number of Jet required

$$= \frac{\text{Total Dischge}}{\text{dischge of one jet}} = \frac{Q}{q}$$

Dischge of one jet 'q' = $a \times V_1$

$$q = \pi/4 \, d^2 \times V_1$$

$$V_1 = C_v \sqrt{2gH} = 0.985 \sqrt{2 \times 9.81 \times 380} = 85.05 \text{ m/s}$$

$$a = \pi/4 \, d^2 = \pi/4 \times 0.164^2 = 0.0211 \text{ m}^2$$

$$\therefore q = V_1 \times a = 85.05 \times 0.0211 = 1.796 \text{ m}^3/\text{s}$$

Now $\eta_0 = \frac{S.P}{W.P} = \frac{11772}{\frac{\rho g Q \times H}{1000}} = 0.86$

$$\text{Total dischge } Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 0.86 \times 380}$$

$$Q = 3.671 \text{ m}^3/\text{s}$$

$$\therefore \text{Number of Jet} = \frac{3.671}{1.796} = 2.044$$

$$\simeq 2 \text{ Jets}$$

Q3: The penstock supplies the water from a reservoir to the pelton wheel with a gross head of 500m. One third of the gross head is lost in friction in the penstock. The rate of flow of water thru the nozzle fitted at the end of the penstock is 2.0 m^3/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic effi'ncy of the pelton wheel. Take speed ratio = 0.45 & $C_v = 1.0$

Given:

$H_g = 500m$

$h_f = \frac{H_g}{3} = \frac{500}{3} = 166.67m$

$H = H_g - h_f = 500 - 166.67 = 333.33m$

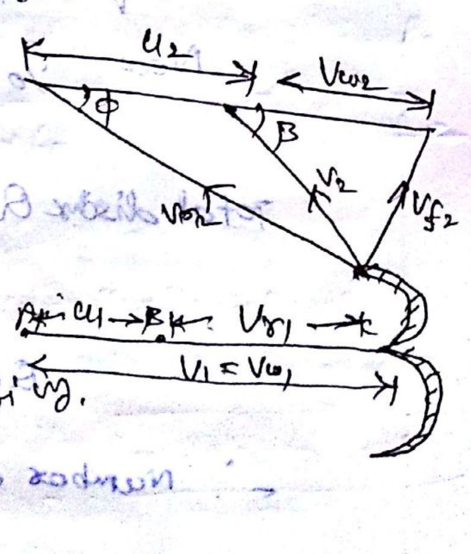
$Q = 2\ m^3/s = a \times V_1$

Angle of deflection = 165°

$\therefore \phi = 180° - 165° = 15°$

Speed ratio = 0.45

$C_v = 1.0$



To find:

(i) Power

(ii) hydraulic effi'ncy.

Sol:

(i) Power given by the water 'P'

$$\text{Power} = \frac{\text{work done per second}}{1000} \text{ kW}$$

work done per second $= \rho g Q H$ (struck through) $\rho a V_1 [V_{w_1} + V_{w_2}] \times u$

Velocity of Jet: $V_1 = C_v\sqrt{2gH} = 1.0\sqrt{2 \times 9.81 \times 333.33}$

$V_1 = 80.86$ m/s $= V_{w_1}$

Velocity of wheel $u = u_1 = u_2 = \text{speed ratio} \times \sqrt{2gH}$

$= 0.45 \times \sqrt{2 \times 9.81 \times 333.33}$

$u = u_2 = u_1 = 36.39$ m/s

$V_{w_2} = V_{r_2}\cos\phi - u_2$

$V_{r_1} = V_{r_2}$

Now $V_{r_1} = V_1 - u_1 = 80.86 - 36.39$

$V_{r_2} = V_{r_1} = 44.46$ m/s

$\therefore V_{w_2} = 44.46 \cos 15' - 36.39$

$= 42.95 - 36.39$

$V_{w_2} = 6.56$ m/s

$\therefore$ work done per second $= 1000 \times 2 \times [80.86 + 6.56] \times 36.39$

$= 6362670.142$ Nm/s

$\therefore$ power $= \frac{6362670.142}{1000}$ kW

power $= 6362.670$ kW

(ii) hydraulic effi. η_h $= \frac{2(V_{w_1} + V_{w_2}) \times u}{V_1^2} = \frac{2 \times (80.86 + 6.56) \times 36.39}{(80.86)^2} = 0.973$

$\eta_h = 97.3\%$

P4: A pelton wheel is to be designed for a head of 60 m when running at 200 rpm. The pelton wheel develops 95.6475 KW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 & co-efficient of the velocity is equal to 0.98.

Given:

$H = 60\,m$

$N = 200\,rpm$

$S.P = 95.6475\,KW$

$u = u_1 = u_2 = 0.45 \times$ Velocity of Jet (V_1)

$\eta_o = 0.85$

$C_v = 0.98$

To find:

(i) velocity of Jet (ii) Diameter of jet

(iii) size of Buckets (iv) Number of buckets on the wheel.

Sol:-

(i) velocity of Jet

$$V_1 = C_v \sqrt{2gH}$$

$$= 0.98 \sqrt{2 \times 9.81 \times 60}$$

$$V_1 = 33.624\ m/s$$

$\therefore$ velocity of buckets $u = u_1 = u_2 = 0.45 \times V_1$

$= 0.45 \times 33.624$

$$u = 15.13\ m/s$$

$$\therefore U = \frac{\pi D N}{60} = 15.13$$

$$\therefore D = \frac{15.13 \times 60}{\pi \times 200} = 1.44\ m$$

(ii) Diameter of Jet 'd'

$$Q = a \times V_1 = \pi/4 \, d^2 \times V_1$$

Now $\eta_o = \frac{S.P}{W.P} = \frac{95.6475}{\frac{1000 \times g \times Q \times H}{1000}}$

$$0.85 = \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$\therefore Q = 0.1911 \, m^3/s$

$\therefore 0.1911 = \pi/4 \times d^2 \times 33.624$

$\therefore d^2 = \frac{0.1911 \times 4}{\pi \times 33.624} = 7.239 \times 10^{-3}$

$\boxed{\therefore d = 0.08 \, m} = 80 \, mm$

(iii) Size of Buckets

width of Buckets $= 5 \times d$

$= 5 \times 80 = 400 \, mm$

Depth of Buckets $= 1.2 \times d$

$= 1.2 \times 80 = 96$

Depth $= 96 \, mm$

(iv) Number of Buckets on the wheel is given by

$$Z = 15 + \frac{D}{2d}$$

$$= 15 + \frac{1.44}{2 \times (80)} = 15.009$$

$\boxed{Z = 15}$

Result:

(i) $V_1 = 33.624 \, m/s$

(ii) $D = 1.44 \, m$

(iii) $d = 0.08 \, m = 80 \, mm$

(iv) width of Bucket $= 400 \, mm$

(v) depth of Bucket $= 96 \, mm$

(vi) Number of Buckets $Z = 15$

Radial flow reaction turbine

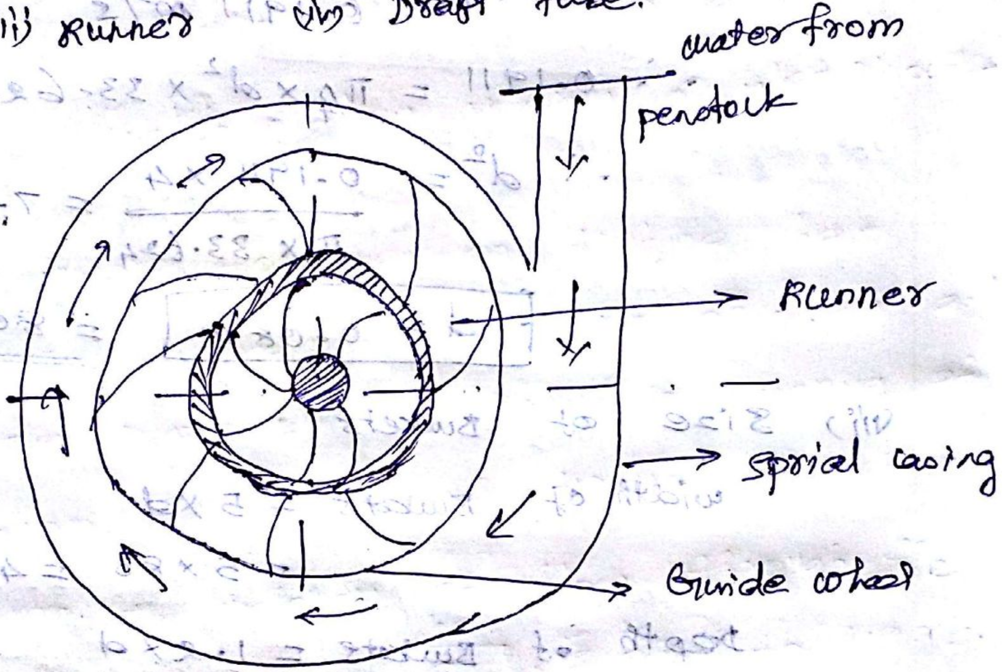
Francis turbine

– water flows in radial direction.

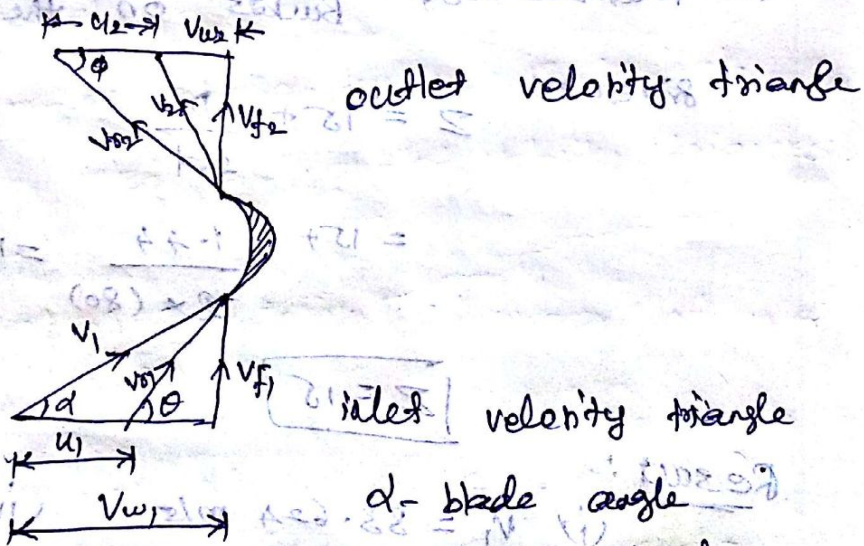
Main parts:

(i) casing (ii) Guide mechanism

(iii) Runner (iv) Draft tube.



Velocity diagrams:



α – blade angle

∴ Discharge radial

$\beta = 90°$ ∴ $V_{w2} = 0$

θ – runner blade angle

Workdone per second on the runner by water

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \because Q = aV_1$$

u_1, u_2 tangential velocity of wheel at inlet and outlet

$$u_1 = \frac{\pi D_1 N}{60}, \quad u_2 = \frac{\pi D_2 N}{60}$$

D_1, D_2 - diameter of runner

N - speed of the turbine in rpm.

Hydraulic efficiency:

$$\eta_h = \frac{R.P}{W.P}$$ R.P - runner power, W.P - water power

$$\boxed{\eta_h = \frac{V_{w_1} u_1 \pm V_{w_2} u_2}{gH}}$$

If the discharge is radial at outlet $V_{w_2} = 0$

$$\therefore \boxed{\eta_h = \frac{V_{w_1} u_1}{gH}}$$

(i) **Speed ratio:**

$$\phi = \frac{u_1}{\sqrt{2gH}}$$

$$\boxed{u_1 = \phi \sqrt{2gH}}$$

(ii) **flow ratio:**

$$V_{f_1} = \sqrt{2gH}$$

$$\therefore \text{flow ratio} = \frac{V_{f_1}}{\sqrt{2gH}}$$

(iv) **Head 'H'**

$$H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$\boxed{\therefore H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g}}$$

because $V_{w_2} = 0$

(iii) Discharge of the turbine : (Q)

$$Q = A \times V_f$$

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}$$

where D_1, D_2 = Diameter of runner @ inlet & outlet

B_1, B_2 = width of runner at inlet & outlet

V_{f_1}, V_{f_2} = flow velocity @ inlet & outlet

(1) An inward flow reaction turbine has external & internal diameters as 0.9m and 0.45m respectively. The turbine is running at 200 rpm. and width of turbine at inlet is 200mm. The velocity of flow through the runner is constant and is equal to 1.8m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles.

Determine:

(i) The absolute velocity of water at inlet of runner. (V_1)

(ii) The velocity of whirl at inlet. (V_{w_1})

(iii) The relative velocity at inlet. (V_{r_1})

(iv) The runner blade angles (θ & ϕ)

(v) width of the runner at outlet (B_2)

(vi) mass of water flowing thrh the runner per second.

(vii) Head at the inlet of the turbine.

(viii) power developed and hydraulic efficiency of the turbine.

Given:

External diameter $D_1 = 0.9m$

Internal " $D_2 = 0.45m$

Speed $N = 200 rpm$

width at inlet $B_1 = 200 mm = 0.2m$

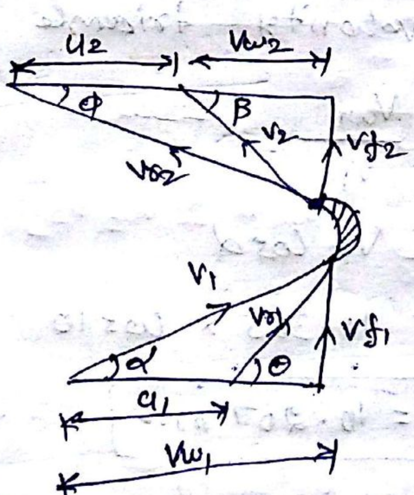
velocity of flow $V_{f_1} = V_{f_2} = 1.8 m/s$

Guide blade angle $\alpha = 10°$

Discharge at outlet = Radial

$\beta = 90°$ and $V_{w_2} = 0$

Sol:



tangential velocity of wheel at inlet & outlet

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.9 \times 200}{60}$$

$$\boxed{u_1 = 9.424 \ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.45 \times 200}{60}$$

$$\boxed{u_2 = 4.712 \ m/s}$$

(i) Absolute velocity of water at inlet (V_1)

From inlet velocity triangle

$$\sin \alpha = \frac{V_{f1}}{V_1}$$

$$V_1 = \frac{V_{f1}}{\sin \alpha}$$

$$V_1 = \frac{1.8}{\sin 10^\circ} = 10.365 \ m/s$$

$$\boxed{V_1 = 10.365 \ m/s}$$

(ii) Velocity of whirl @ inlet (V_{w1}) :

from inlet velocity triangle

$$\cos \alpha = \frac{V_{w1}}{V_1}$$

$$V_{w1} = V_1 . \cos \alpha$$

$$= 10.365 \times \cos 10$$

$$\boxed{V_{w1} = 10.207 \ m/s}$$

(iii) Relative velocity at inlet (V_{r1})

From inlet velocity triangle by using bithagoras theorem

$$V_{r1}^2 = V_{f1}^2 + (V_{w1} - u_1)^2$$

$$V_{r1} = \sqrt{V_{f1}^2 + (V_{w1} - u_1)^2}$$

$$V_{r1} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

$$\therefore V_{r1} = 1.963 \text{ m/s}$$

(iv) The runner blade angle means θ & ϕ

from inlet velocity triangle

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\tan\theta = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$$\theta = \tan^{-1}(2.298)$$

$$\theta = 66^\circ 29'$$

From outlet velocity triangle

$$\tan\phi = \frac{V_{f2}}{u_2}$$

$$\tan\phi = \frac{1.8}{4.712} = 2.$$

$$\phi = 20.9 \text{ or } 20^\circ 54'$$

(V) width of runner at outlet (B_2):

$$Q_1 = Q_2$$

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$$

$$D_1 B_1 = D_2 B_2$$

$$B_2 = \frac{D_1 B_1}{D_2} = \frac{0.9 \times 0.2}{0.45}$$

$$\boxed{B_2 = 0.4\, m}$$

(vi) mass of water flowing through the runner per second:

Discharge $Q = \pi D_1 B_1 V_{f_1}$

$$= \pi \times 0.9 \times 0.2 \times 1.8$$

$$\boxed{Q = 1.0178\, m^3/s}$$

$$mass = \rho \times Q$$

$$= 1000 \times 1.0178$$

$$\boxed{mass/s = 1017.8\, kg/s}$$

(vii) Head at inlet?: H

$$H - \frac{V_2^2}{2g} = \frac{1}{g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]$$

$$V_{w_2} = 0$$

$$\boxed{V_{f_1} = V_2}$$

$$\therefore H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g}$$

$$\therefore H = \frac{10.207 \times 9.424}{2 \times 9.81} + \frac{(1.8)^2}{2 \times 9.81}$$

$$= 9.805 + 0.165$$

$$\boxed{H = 9.97\,m}$$

(viii) <u>power developed:</u>

$$P = \frac{\text{work done per sec}}{1000}$$

work done per sec $= \dfrac{\rho Q\,[V_{w_1} u_1]}{1000}$ $\quad V_{w_2} = 0$

$$= \frac{1000 \times 1.0178 \times 10.207 \times 9.42}{1000}$$

$$\boxed{P = 97.9\,kW}$$

(ix) <u>Hydraulic efficiency: (η_h)</u>

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$= \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834$$

$$\boxed{\eta_h = 98.34\,\%}$$

MADHA
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ENGINEERING COLLEGE

Department of Mechanical Engineering

Lecture Notes

Subject Code : ME3392
Subject Name: ENGINEERING MATERIALS AND METALLURGY
Sem/Year : 03/II
Regulation : 2021

ME3392 ENGINEERING MATERIALS AND METALLURGY L T P C

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COURSE OBJECTIVES:

1 To learn the constructing the phase diagram and using of iron-iron carbide phase diagram for microstructure formation.
2 To learn selecting and applying various heat treatment processes and its microstructure formation.
3 To illustrate the different types of ferrous and non-ferrous alloys and their uses in engineering field.
4 To illustrate the different polymer, ceramics and composites and their uses in engineering field.
5 To learn the various testing procedures and failure mechanism in engineering field.

UNIT I CONSTITUTION OF ALLOYS AND PHASE DIAGRAMS 9

Constitution of alloys – Solid solutions, substitutional and interstitial – phase diagrams, Isomorphous, eutectic, eutectoid, peritectic, and peritectoid reactions, Iron – Iron carbide equilibrium diagram. Classification of steel and cast-Iron microstructure, properties and application.

UNIT II HEAT TREATMENT 9

Definition – Full annealing, stress relief, recrystallisation and spheroidising –normalizing, hardening and tempering of steel. Isothermal transformation diagrams – cooling curves superimposed on I.T. diagram – continuous cooling Transformation (CCT) diagram – Austempering, Martempering – Hardenability, Jominy end quench test -case hardening, carburizing, Nitriding, cyaniding, carbonitriding – Flame and Induction hardening – Vacuum and Plasma hardening – Thermo-mechanical treatments- elementary ideas on sintering.

UNIT III FERROUS AND NON-FERROUS METALS 9

Effect of alloying additions on steel (Mn, Si, Cr, Mo, Ni, V,Ti& W) – stainless and tool steels – HSLA - Maraging steels – Grey, white, malleable, spheroidal – alloy cast irons, Copper and its alloys – Brass, Bronze and Cupronickel – Aluminium and its alloys; Al-Cu – precipitation strengthening treatment – Titanium alloys, Mg-alloys, Ni-based super alloys – shape memory alloys- Properties and Applications-overview of materials standards

UNIT IV NON-METALLIC MATERIALS 9

Polymers – types of polymers, commodity and engineering polymers – Properties and applications of PE, PP, PS, PVC, PMMA, PET, PC, PA, ABS, PAI, PPO, PPS, PEEK, PTFE, Thermoset polymers – Urea and Phenol formaldehydes –Nylon, Engineering Ceramics – Properties and applications of Al_2O_3, SiC, Si_3N_4, PSZ and SIALON – intermetallics- Composites- Matrix and reinforcement Materials-applications of Composites - Nano composites.

UNIT V MECHANICAL PROPERTIES AND DEFORMATION MECHANISMS 9

Mechanisms of plastic deformation, slip and twinning – Types of fracture – fracture mechanics- Griffith’s theory- Testing of materials under tension, compression and shear loads – Hardness tests (Brinell, Vickers and Rockwell), Micro and nano-hardness tests, Impact test Izod and charpy, fatigue and creep failure mechanisms.

TOTAL: 45 PERIODS

UNIT-I

Materials Science – Investigating relationships that exist between the structure and properties of materials.

Materials Engineering – On the basis of these structure-property correlations, designing or engineering the structure of a material to produce a pre-determined set of properties.

STRUCTURE

Structure of a material usually relates to the arrangement of its internal components.

Subatomic - Structure involves electrons within the individual atoms and interactions with their nuclei.

Atomic level- structure encompasses the organization of atoms or molecules relative to one another.

Microscopic – Which contains large groups of atoms that are normally agglomerated together.

Macroscopic – Viewable with the naked eye.

SOLID SOLUTIONS

- Metals usually form homogenous liquid solutions in the liquid state
- Even after their transformation to a solid crystalline state, the metal retain their homogeneity and their solubility, a solid solution is said to have formed.
    - Cu atoms are solvent atoms and Zn atoms are solute.
    - They form individual crystal structure of their own and form substitutional solid solution

According to Hume- Rothery rule, a solid solution may form

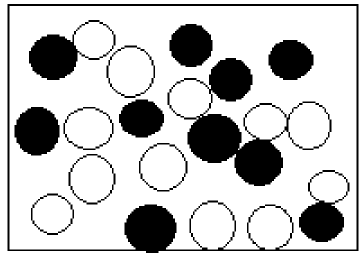
- Similar atomic radii (15% or less difference)
- Same crystal structure
- Similar electronegativities
- Similar valency

Types of solid solutions

a) Substitutional solid solution
    - Disordered
    - Ordered

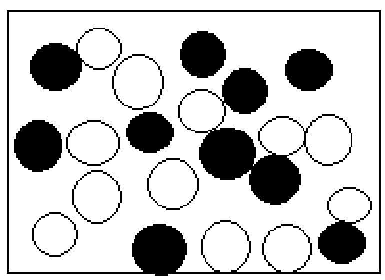
b) Interstitial solid solution

Substitutional solid solution:-



- There is a direct substitution of one type of atom for another so that solute atoms enter the crystal to take positions of solvent atoms
- It forms when solute and solvent atoms possess equal or approximately equal diameters. For eg: atomic diameter of cu = 2.551 Å and that of nickel is 2.487 Å and the two form substitutional solid solution

Disordered substitutional solid solution:-



- The solute atoms do not occupy any specific position but are distributed at random in the lattice structure of the solvent in the substitutional solid solution. This condition is called disordered.
- When this type of solution crystallizes, the core contains more atoms of higher melting point compared to outer fringes having more atoms of lower melting points

Ordered substitutional solid solution:-

- The alloy in the disordered condition when cooled slowly undergoes re arrangement of atoms
- It produces uniform distribution of solute and solvent atoms. They move to definte orderly positions in the lattice.

Interstitial Solid solutions:-

- It forms when solute atoms are very small as compared to solvent atoms and they are unable to substitute due to large diameter differences. They fit into interstices or spaces in the lattice

- Atomic size factor is not the only reason. Small interstitial solute atoms dissolve much more readily in transition metals (Fe, Ni,Mn,Mo,Cr,W,etc.,) than in other metals

What kinds of phases exist

Based on state → Gas, Liquid, Solid

Based on atomic order → Amorphous, Quasi-crystalline, Crystalline

Based on band structure → Insulating, Semi-conducting, Semi-metallic, Metallic Based on Property → Para-electric, Ferromagnetic, Superconducting

Based on stability → Stable, Metastable, Unstable

Also sometimes- Based on size/geometry of an entity → Nanocrystalline, mesoporous, layered.

Phase transformation

Phase transformation is the change of one phase into another. For example

- Water → Ice and α-Fe (BCC) → γ-Fe (FCC)

Grain

The single crystalline part of polycrystalline metal separated by similar entities by a grain boundary

Solute

The component of either a liquid or solid solution that is present to a lesser or minor extent; the component that is dissolved in the solvent.

Solvent

The component of either a liquid or solid solution that is present to a greater or major extent; the component that dissolves the solute.

System

System has two meanings. First, ‘‘system’’ may refer to a specific body of material or object. Or, it may relate to the series of possible alloys consisting of the same components, but without regard to alloy composition.

Solubility Limit

For many alloy systems and at some specific temperature, there is a maximum concentration of solute atoms that may dissolve in the solvent to form a solid solution; this is called a Solubility Limit.

Microstructure

(Phases + defects + residual stress) & their distributions

Structures requiring magnifications in the region of 100 to 1000 times. (or) The distribution of phases and defects in a material.

Phase diagram

Map that gives relationship between phases in equilibrium in a system as a function of T, P and composition. Map demarcating regions of stability of various phases

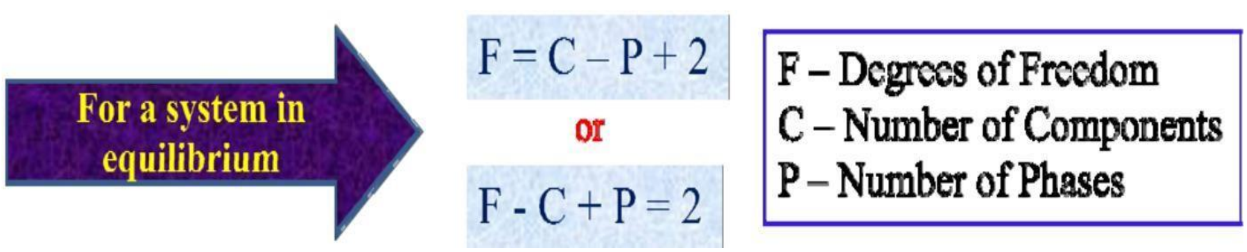
GIBBS PHASE RULE

The phase rule connects the Degrees of Freedom, the number of components in a system and the number of phases present in a system via a simple equation.

To understand the phase rule one must understand the variables in the system along with the degrees of freedom.

We start with a general definition of the phrase "degrees of freedom".

Degrees of Freedom: The degree of freedom, F, are those externally controllable conditions of temperature, pressure, and composition, which are independently variable and which must be specified in order to completely define the equilibrium state of the system



The degrees of freedom cannot be less than zero so that we have an upper limit to the number of phases that can exist in equilibrium for a given system.

Variables in a phase diagram

C – No. of components

P – No. of phases

F – No. of degrees of freedom

Variables in the system = Composition variables + Thermodynamic variables

Composition of a phase specified by (C – 1) variables (If the composition is expressed in %ages then the total is 100% → there is one equation connecting the composition variables and we need to specify only (C - 1) composition variables)

No. of variables required to specify the composition of all phases: P(C – 1) (as there are P phases and each phase needs the specification of (C – 1) variables)

Thermodynamic variables = P + T (usually considered) = 2 (at constant pressure (e.g. atmospheric pressure) the thermodynamic variable becomes 1)

Total no. of variables in the system = P (C – 1) + 2

F < no. of variables → F < P (C – 1) + 2For a system in equilibrium the chemical potential of each species is same in all the phases

If α, β, γ… are phases, then: $\mu A(\alpha) = \mu A(\beta) = \mu A(\gamma)$…..

Suppose there are 2 phases (α and β phases) and 3 components (A, B, C) in each phase then : $\mu A(\alpha) = \mu A(\beta)$, $\mu B(\alpha) = \mu B(\beta)$, $\mu C(\alpha) = \mu C(\beta)$ → i.e. there are three equations. For each component there are (P – 1) equations and for C components the total number of equations is C(P – 1). In the above example the number of equations is 3(2 – 1) = 3 equations.

F = (Total number of variables) – (number of relations between variables)

= P(C – 1) + 2] – [C(P – 1)] = C – P + 2

In a single phase system F = Number of variables

P↑ → F↓ (For a system with fixed number of components as the number phases increases the degrees of freedom decreases.

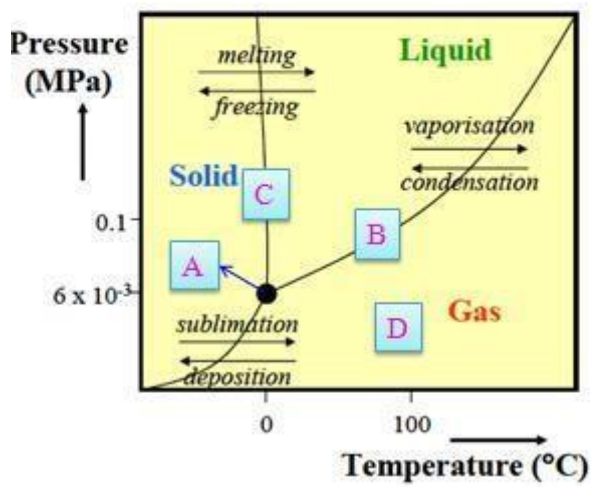
UNARY PHASE DIAGRAM

Let us start with the simplest system possible: the unary system wherein there is just one component.

Though there are many possibilities even in unary phase diagram (in terms of the axis and phases), we shall only consider a T-P unary phase diagram.

Let us consider the water (H2O) unary phase diagram

The Gibbs phase rule here is: F=C-P+2 (2 is for T&P) (no composition variables here)



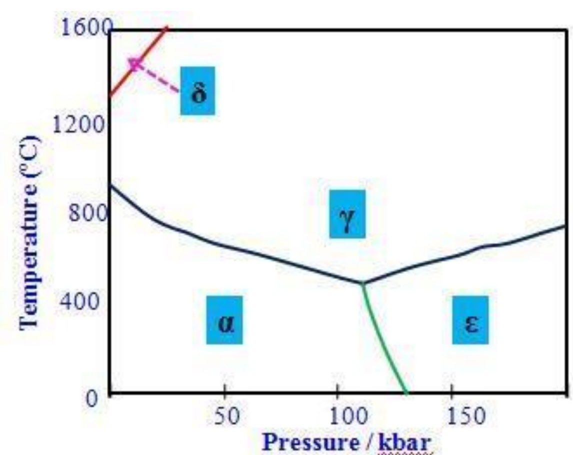
Along the 2 phase co-existence (at B & C) lines the degree of freedom (F) is 1 → i.e. we can chose either T or P and the other will be automatically fixed

The 3 phase co-existence points (at A) are invariant points with F=0. (Invariant point implies they are fixed for a given system).

The single phase region at point D, T and P can both be varied while still being in the single phase region with F = 2.

The above figure represents the phase diagram for pure iron. The triple point temperature and pressure are 490ºC and 110 kbars, respectively. α, γ and ε refer to

ferrite, austenite and ε-iron, respectively. δ is simply the higher temperature designation of α.



BINARY PHASE DIAGRAM

Binary implies that there are two components.

Pressure changes often have little effect on the equilibrium of solid phases (unless of course we apply 'huge' pressures).

Hence, binary phase diagrams are usually drawn at 1 atmosphere pressure. The Gibbs phase rule is reduced to:

Variables are reduced to : $F = C - P + 1$ (1 is for T).

T & Composition (these are the usual variables in materials phase diagrams)

Along the 2 phase co-existence (at B & C) lines the degree of freedom (F) is 1 → i.e. we can chose either T or P and the other will be automatically fixed

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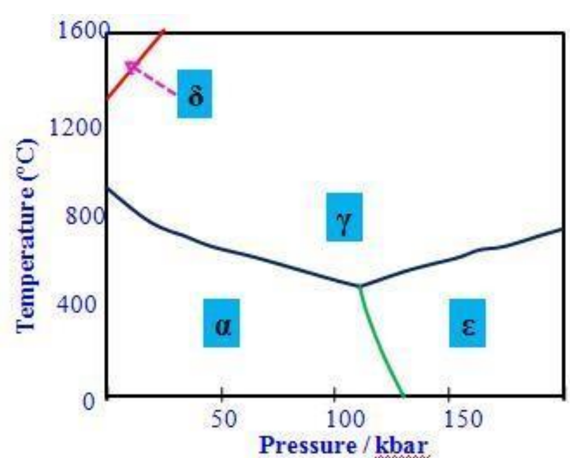

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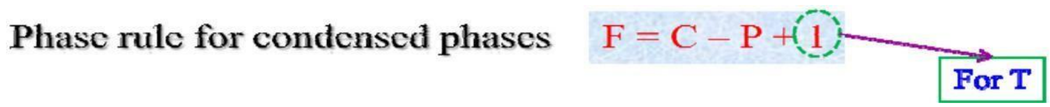
T & Composition (these are the usual variables in materials phase diagrams)

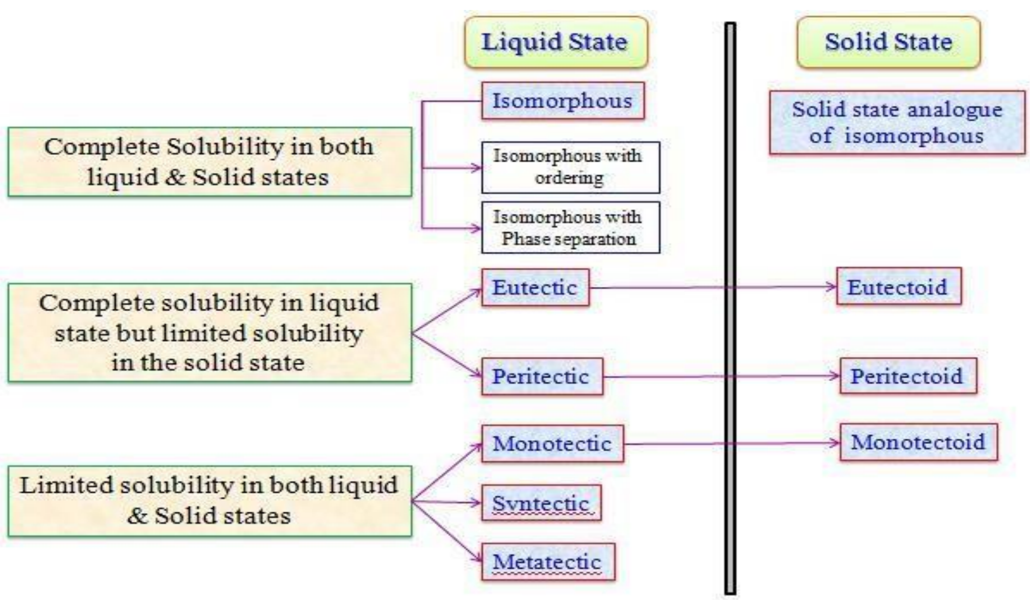


In the next page we consider the possible binary phase diagrams. These have been classified based on:

Complete solubility in both liquid & solid states

Complete solubility in both liquid state, but limited solubility in the solid state

Limited solubility in both liquid & solid states



Isomorphous Phase Diagram

Isomorphous phase diagrams form when there is complete solid and liquid solubility.

Complete solid solubility implies that the crystal structure of the two components have to be same and Hume-Rothery rules to be followed.

Examples of systems forming isomorphous systems: Cu-Ni, Ag-Au, Ge-Si, Al_2O_3-Cr_2O_3

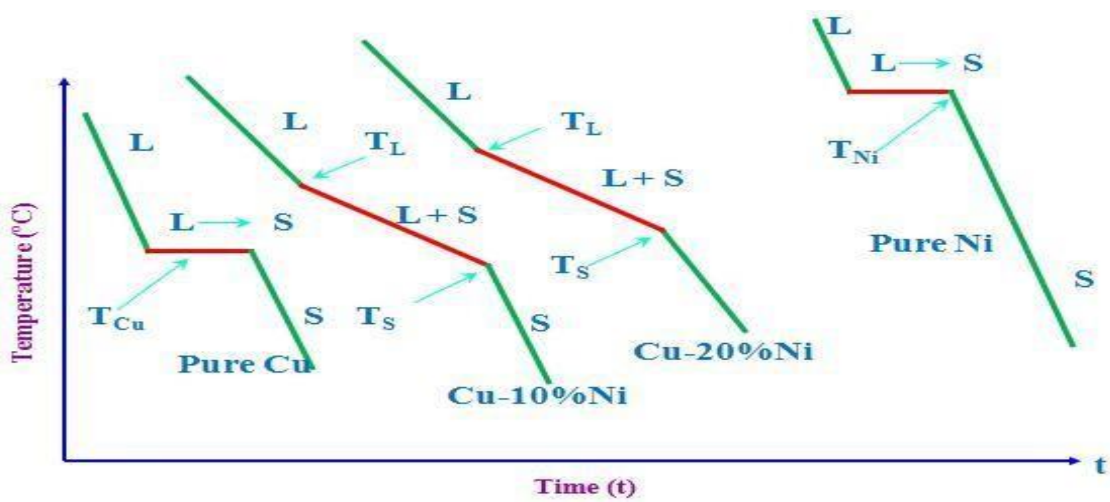
Both the liquid and solid contain the components A and B.

In binary phase diagrams between two single phase regions there will be a two phase region → In the isomorphous diagram between the liquid and solid state there is the (Liquid + Solid) state.

The Liquid + Solid state is NOT a semi-solid state → it is a solid of fixed composition and structure, in equilibrium with a liquid of fixed composition.

In some systems (e.g. Au-Ni system) there might be phase separation in the solid state (i.e.,the complete solid solubility criterion may not be followed) → these will be considered as a variation of the isomorphous system (with complete solubility in the solid and the liquid state.

Cooling curves: Isomorphous system



Isomorphous Phase Diagram

This very simple case is one complete liquid and solid solubility, an isomorphous system.

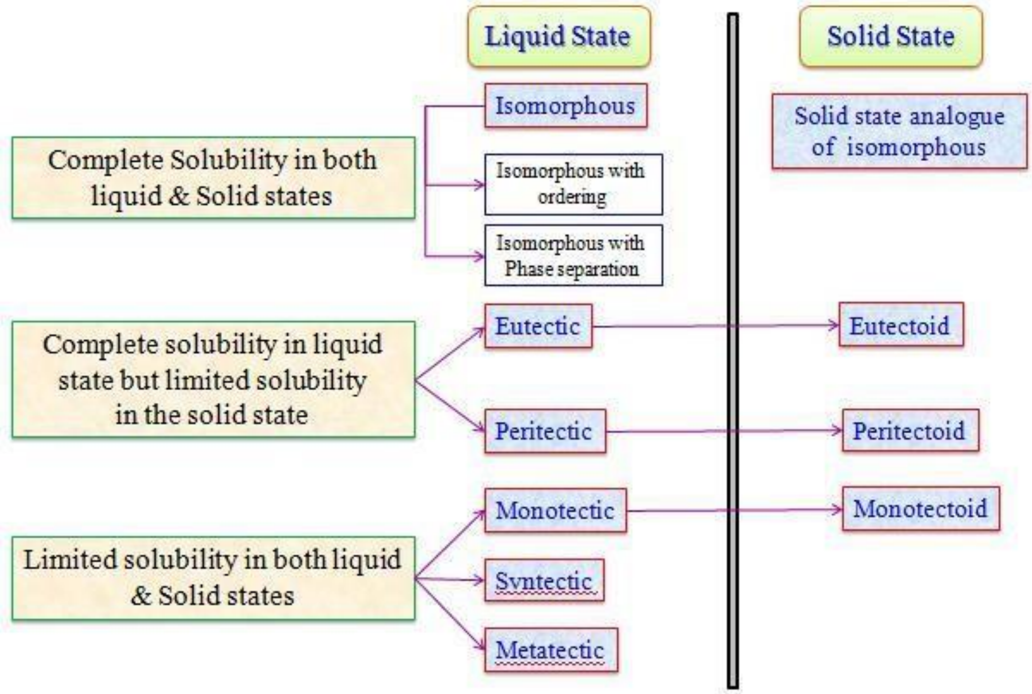
The example is the Cu-Ni alloy of Fig. 3.1.

The complete solubility occurs because both Cu and Ni have the same crystal structure (FCC), near the same radii, electronegativity and valence.The liquidus line separates the liquid phase from solid or solid + liquid phases. That is, the solution is liquid above the liquidus line.

Complete solubility in both liquid & solid states

Complete solubility in both liquid state, but limited solubility in the solid state

Limited solubility in both liquid & solid states



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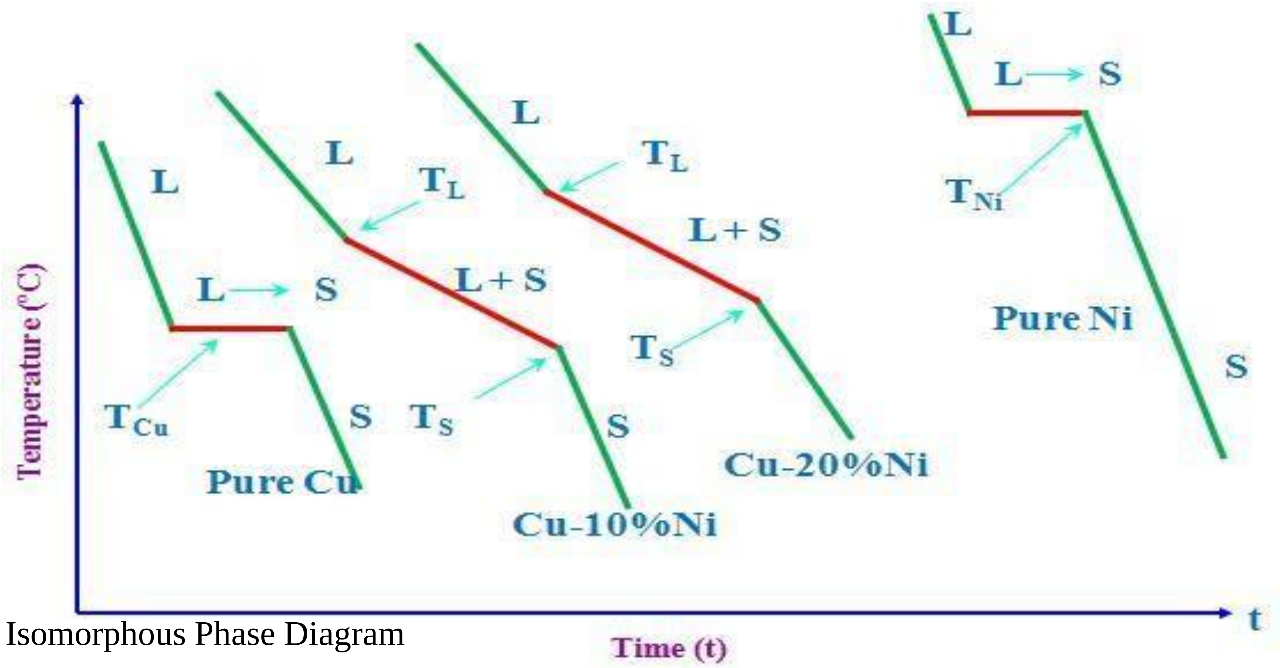

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The solidus line is that below which the solution is completely solid (does not contain a liquid phase

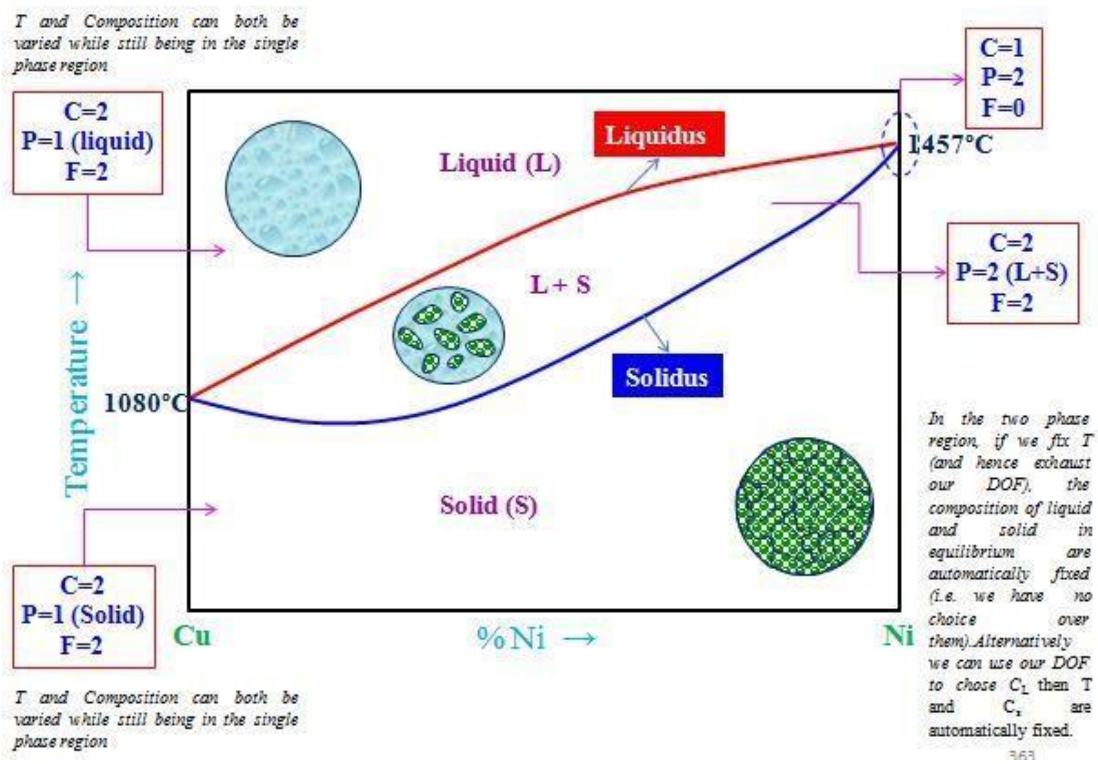


Figure 3.1: The copper–nickel phase diagram.

Tie line and Lever rule

Chemical Composition of Phases: To determine the actual chemical composition of the phases of an alloy, in equilibrium at any specified temperature in a two phase region, draw a horizontal temperature line, called a tie line, to the boundaries of the field . These points of intersection are dropped to the base line, and the composition is read directly.

Relative Amounts of Each Phase:

To determine the relative amounts of the two phases in equilibrium at any specified temperature in a two phase region, draw a vertical line representing the alloy and a horizontal temperature line to the boundaries of the field.

The vertical line will divide the horizontal line into two parts whose lengths are inversely proportional to the amount of the phases present. This is also known as Lever rule.

The point where the vertical line intersects the horizontal line may be considered as the fulcrum of a lever system.

The relative lengths of the lever arms multiplied by the amounts of the phases present must balance.

Tie line and Lever rule

We draw a horizontal line (called the Tie Line)at the temperature of interest (say T0). Let Tie line is XY.

Solid (crystal) of composition C 1 coexists with liquid of composition C 2

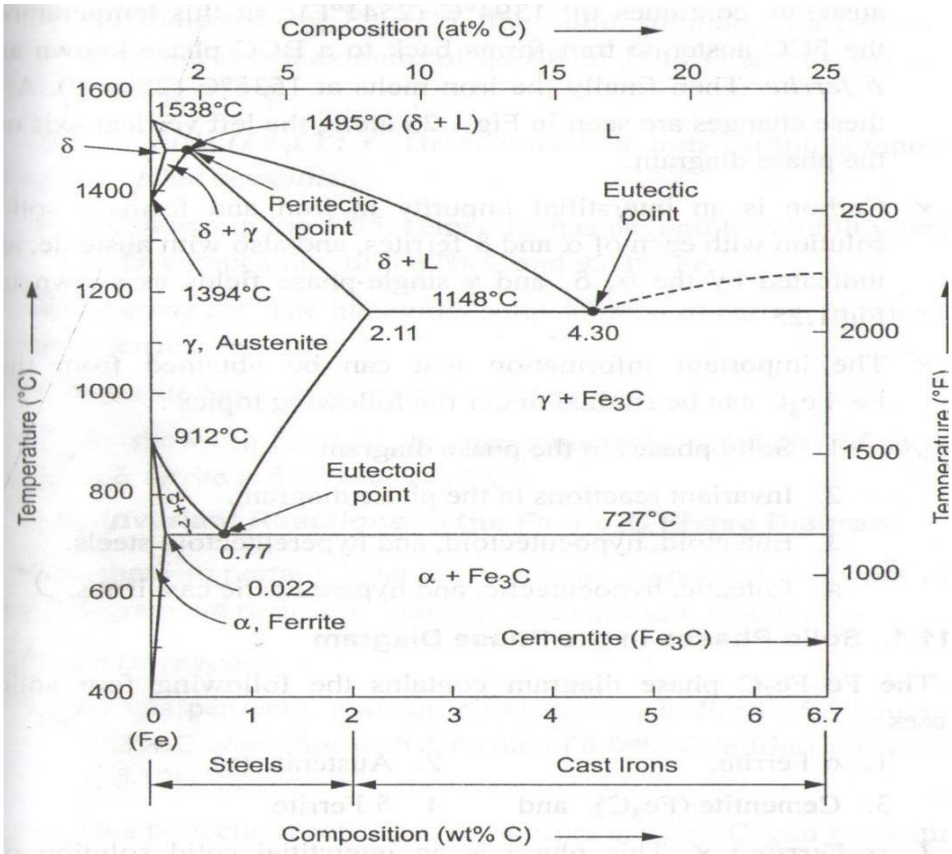
Note that tie lines can be drawn only in the two phase coexistence regions (fields). Though they may be extended to mark the temperature.

To find the fractions of solid and liquid we use the lever rule.

The portion of the horizontal line in the two phase region is akin to 'lever' with the fulcrum at the nominal composition (C0)

The opposite arms of the lever are proportional to the fraction of the solid and liquid phase present (this is lever rule).

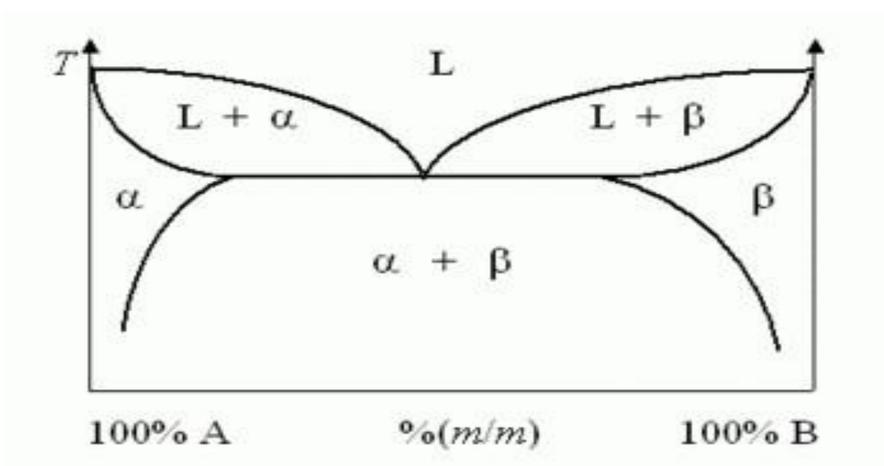
IRON-IRON CARBIDE EQUILIBRIUM DIAGRAM

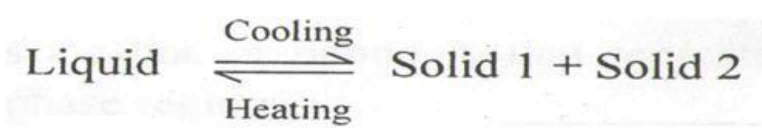


- It is used to show conditions at which thermodynamically distinct phases can occur at equilibrium.
- Common components of a phase diagram are lines of equilibrium or phase boundaries, which refer to lines that mark conditions under which multiple phases can coexist at equilibrium.
- Phase transitions occur along lines of equilibrium.
- Triple points are points on phase diagrams where lines of equilibrium intersect.
- The water phase diagram has a triple point corresponding to the single temperature and pressure at which solid, liquid, and gaseous water can coexist in a stable equilibrium.
- Solidus Line: the temperature below which the substance is stable in the solid state.

- Liquidus Line: the temperature above which the substance is stable in a liquid state.
- There may be a gap between the solidus and Liquidus within the gap, the substance consists of a mixture of crystals and liquid
- The Possible phases are:
  - α- Ferrite(B.C.C)
  - γ- Austenite(F.C.C)
  - δ- Ferrite(B.C.C)
  - Liquid Fe-C
  - Fe_3C (or) Cementite

EUTECTIC REACTION

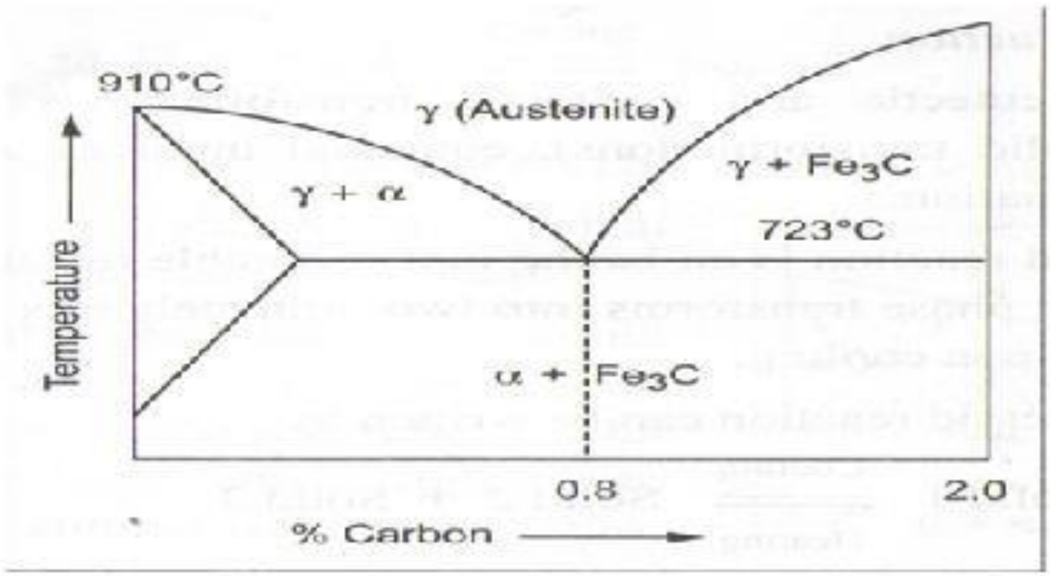


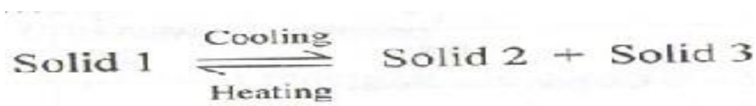


- In a eutectic reaction liquid phase is directly converted in to solid phase having two different structures when cooling.
- The eutectic temperature and composition determine a point on the phase diagram called the eutectic point.

- In a eutectic reaction the solid phase having two different structures converted into single liquid phase while heating.
- The two different structures such as Austenite and Cementite at 4.3% carbo

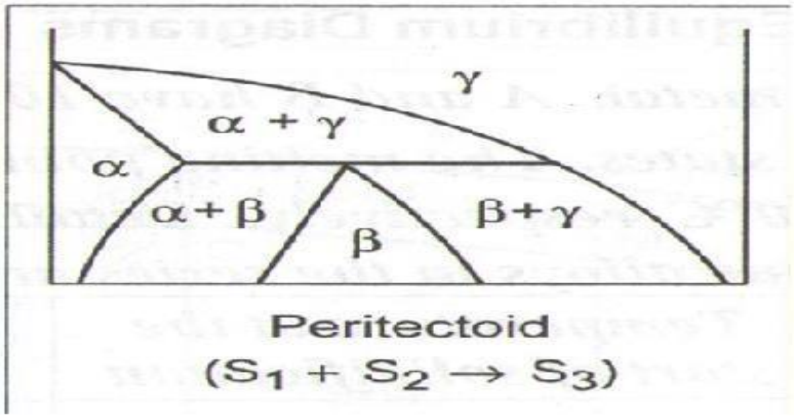
ECTECTOID REACTION





- In a eutectoid reaction the single solid phase having one structures converted into two different structure solid phase while cooling.
- In a eutectoid reaction two different structure solid phases is directly converted in to single structure solid phase while when cooling.
- The eutectoid temperature and composition determine a point on the phase diagram called the eutectoid point.
- That is Austenite is a single structure and two different structures such as Ferrite and Cementite .
- This reaction occurs in a 0.8% of carbon at 723°C.

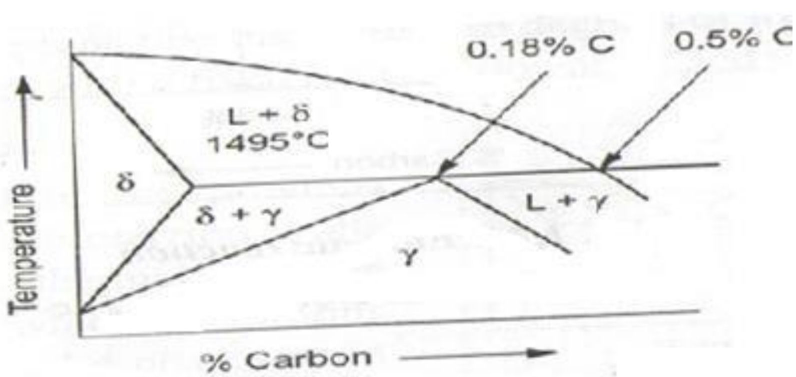
PERITECTOID REACTION



$$\text{Solid 1} + \text{Solid 2} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Solid 3}$$

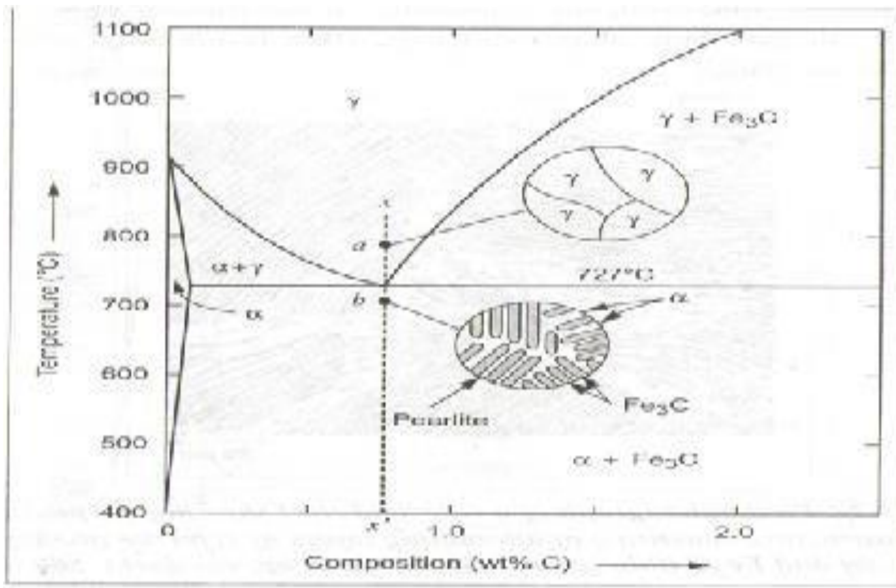
- Theperitectoid reaction is an isothermal reversible reaction in which two solid phases transform into a third solid phase upon cooling.
- Theperitectoid reaction is an isothermal reversible reaction in which single solid phases transform into a two solid phase upon heating.
- It is reversible reaction of eutectic reaction

PERITECTIC REACTION



$$\text{Liquid} + \text{Solid 1} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Solid 2}$$

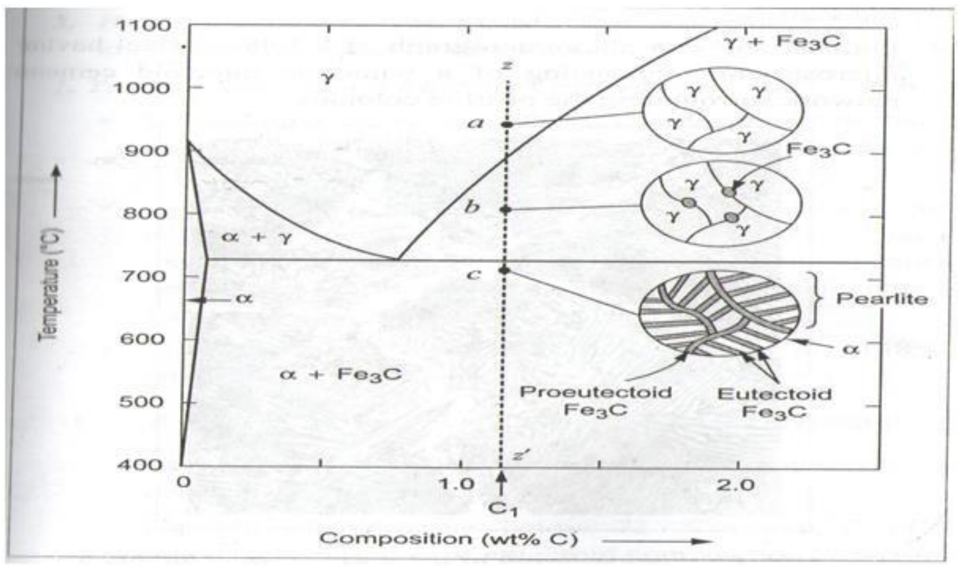
- In the pertectic reaction a solid and liquid phases combine to from another solid phase while cooling and reversible while heating.
- It is reversible reaction of eutectoidreaction.
- This reaction occurs in a 0.18% of carbon at 1495°C.
- **TRANSFORMATION OF EUTECTOID STEEL**



- A sample of a 0.8% eitecotid steel is heated to above 700°C and maintained for sufficient period of time.
- It converted in to homogeneous austenite. This process is called as austenitizing.

- On further cooling the entire structure to transform austenite to a lamellar structure that is ferrite and cementite.
- The a lamellar structure nothing but ferrite and cementite.

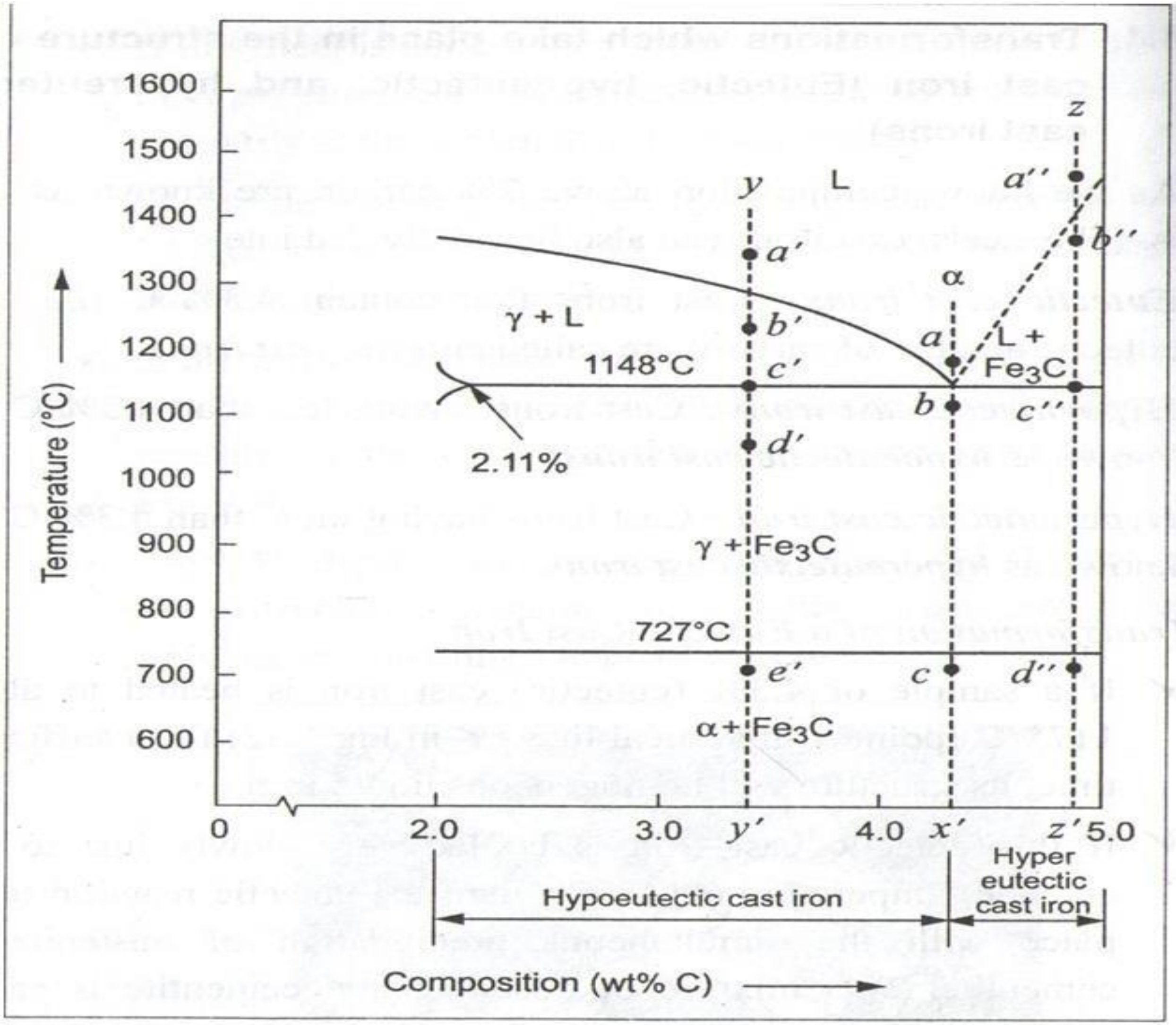
TRANSFORMATION OF HYPEREUTECTOID STEEL



- To the right of the eutectoid reaction between 0.8% to 2.11% of cabon hyper eutectoid takes places.
- Hence three stages a, b,c are marked depend upon curves are draw.
- The sample of 1.2% C is taken.
- At the point C the proeutectoid that is cementite an pearlite structure.
- At the point B cementite and austenite is present.
- At the point A fully austenite is present.

TRANSFORMATION OF EUTECTIC (4.3%C) show in xx’,
HYPOEUTECTIC(2.0% to 4.3%C) show in yy’,

HYPEREUTECTIC(4.3% to 5.0%C) show in yy’,



UNIT-II

HEAT TREATMENT

Heat Treatment Is the Process Of Giving The Controlled Heating And Cooling To Derive Different Properties In The Materials.

PURPOSE OF HEAT TREATMENT:-

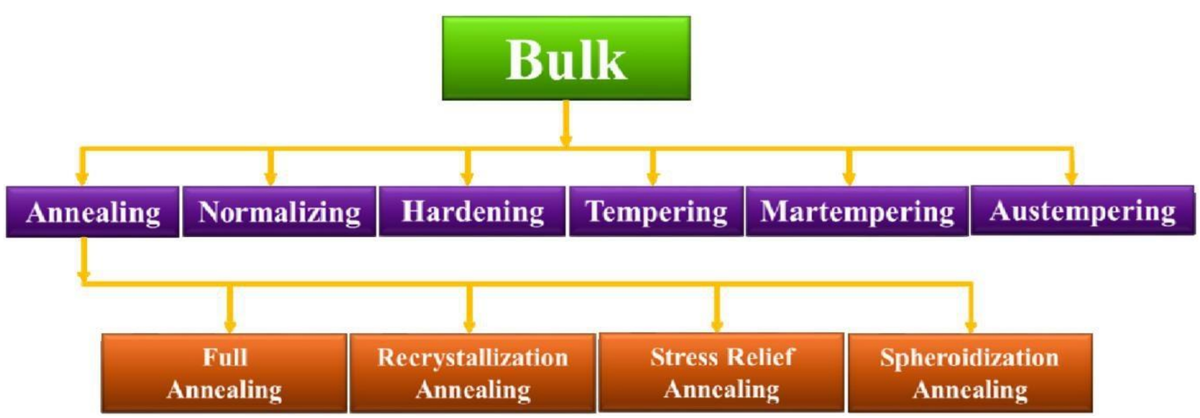
- To Relive Internal Stress
- To Improve Machine Ability.
- To Improve Refine Grain Size.
- To Improve Mechanical Properties.
- To Improve resistance to wear etc.

STAGES OF HEAT TREATMENT PROCESS.-

Stage 1: heating a metal or alloy beyond the critical temperature.

Stage 2: holding at that temperature for a sufficient period of time to allow necessary changes to occur.

Stage 3: cooling the metal or alloy at a required rate to obtain the desired properties.



ANNEALLING

- The Process of Slow Cooling Is Knows As Annealing.
- The Materials Are Heated Above Critical Temperature.
- Then The Temperature maintain For a Period Of Time.
- Then allow it to cool slowly to room temperature In Side the Furnace It Self..
- Cooling is usually done in the furnace itself by decreasing the temperature 10°C to 30°C per hour.

Types Of Annealing:-

- full annealing
- process annealing
- stress relief annealing
- recrystallisation annealing

Full Annealing:-

- The Material Is Heated Above 723°C To 910°C.
- Hence The Structure is converting into homogeneous single phase austenite.
- Then The Temperature maintains For a Period of Time.
- Then allow it to cool slowly to room temperature In Side the Furnace It Self..
- Cooling is usually done in the furnace itself by decreasing the temperature 10°C to 30°C per hour.

Process annealing:-

- The Material Is Heated Above 550°C To 650°C.
- Then The Temperature maintains For a Period of Time.
- Then allow it to cool slowly to room temperature.
- The method is very cheaper more rapidly used.

Stress relief annealing:-

- The Material Is Heated Above 500°C To 650°C.
- Then The Temperature maintains For a Period of Time.
- Then allow it to cool slowly to room temperature In Side the Furnace It Self.
- The method is very cheaper more rapidly used.
- Hence Internal Stress Are Removed**.**
-

Recrystallisation annealing:-

- The Material Is Heated Above 500°C To 650°C.
- Then The Temperature maintains For a Period of Time.
- Then allow it to cool slowly to room temperature In Side the Furnace It Self.
- Hence Distorted Grains Of Cold Worked Metals Are Replaced By New Grains.

NORMALIZING

Normalizing is similar to full annealing but cooling is done by still air rather then furnace.

Purpose of Normalizing:-

- To Improve Refine Grain Size.
- To Improve Mechanical Properties.
- To Increase The Strength Of The Steel.

Operation:-

- Material Is Heated Above 40°C - 50°C Above Its Upper Critical Temperature. .
- Then The Temperature maintains For a Period of Time.
- Then allow it to cool slowly to room temperature.
  - But cooling is done in still air.
  - The Normalizing process consisting of ferrite and pearlite for hypoeucetoid steels. and pearlite and cementite for hyper eutectoid steel.

QUENCHING

- The Process of fast or instant Cooling Is Knows as Annealing.
- Cooling is done by contact the material with quenching medium.
- Quenching medium as solid liquid and gas.

Selection of Quenching medium:-

- Boiling point.
- Viscosity.
- Desired rate of cooling.

Stages of Quenching:-

Stages 1: vapour-jacket stages.

- The work piece of hot metal is first inserted into tank having quench medium.
- The vaporizes and forms a gaseous layer separating the metal and liquid
- In this stage cooling is very slow.

Stages 2: vapour-transport cooling stage.

- The hot metal is cooled to a temperature.
- In this stage bubbles nucleate and remove the gaseous layer.
- In this stage cooling rate more.

Stages 3: liquid cooling stage.

- The metal cools below the boiling point of quenchant.
- In this stage all heat transfer occurs through conduction across the solid and liquid interface.
- In this stage is the slowest cooling rate is obtained.

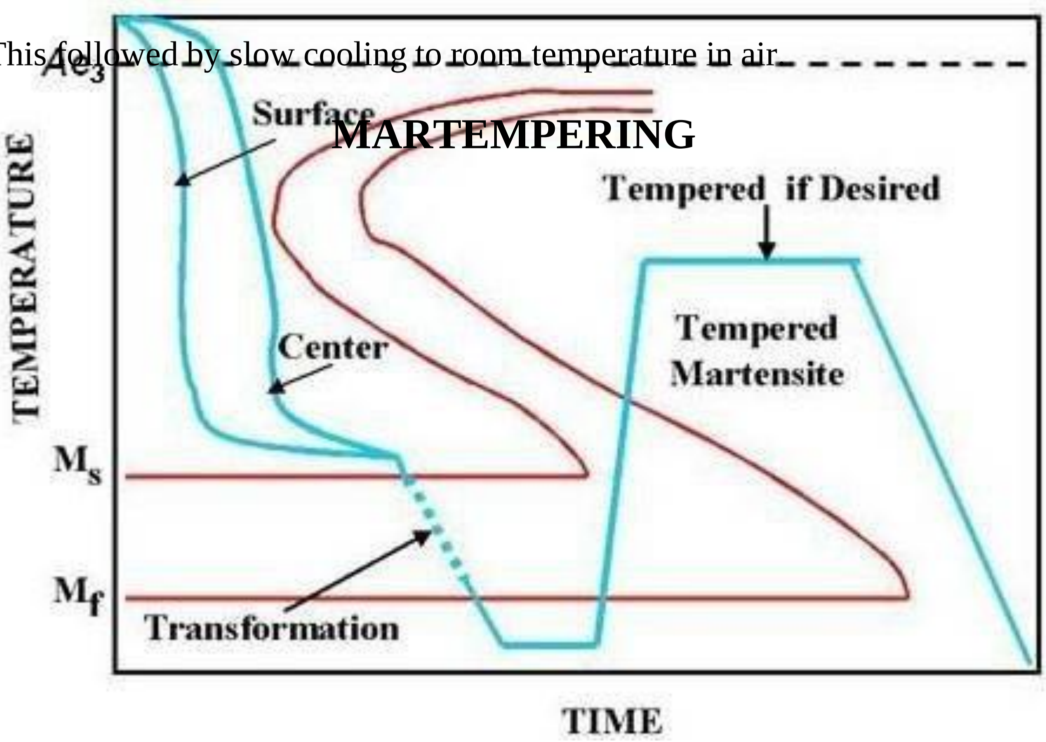
TEMPERING

- The tempering is done to reduce hardness of steel and improve ductility and toughness.
- In this process the heating is done for the metals which are already hardened steel.

Operation:-

- The hardened metal is heated to a temperature between 250°C To 650°C.
- Then The Temperature maintains For a Period of Time (1-2 hours)
- This followed by slow cooling to room temperature in air.

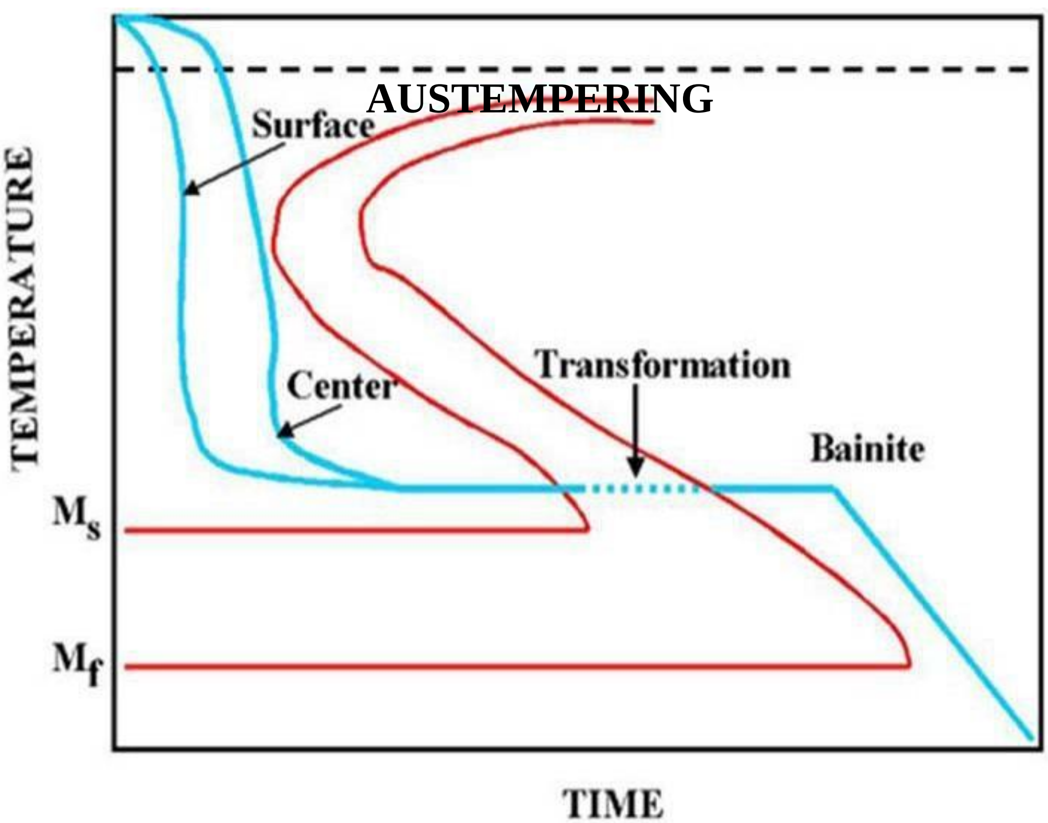
MARTEMPERING



Martempering is a heat treatment for steel involving austenitisation followed by step quenching, at a rate fast enough to avoid the formation of ferrite, pearlite or bainite to a temperature slightly above the martensite start (Ms) point. A schematic of above the process is shown in Fig.4 7.

The advantage of martempering lies in the reduced thermal gradient between surface and center as the part is quenched to the isothermal temperature and then is air cooled to room temperature.

Residual stresses developed during martempering are lower than those developed during conventional quenching because the greatest thermal variations occur while the steel is in the relatively plastic austenitic condition and because final transformation and thermal changes occur throughout the part at approximately the same time. Martempering also reduces or eliminate susceptibility to cracking.



Austempering process is similar to the Martempering process except that the sample after the homogenization of temperature from surface and temperature, instead of quenching,

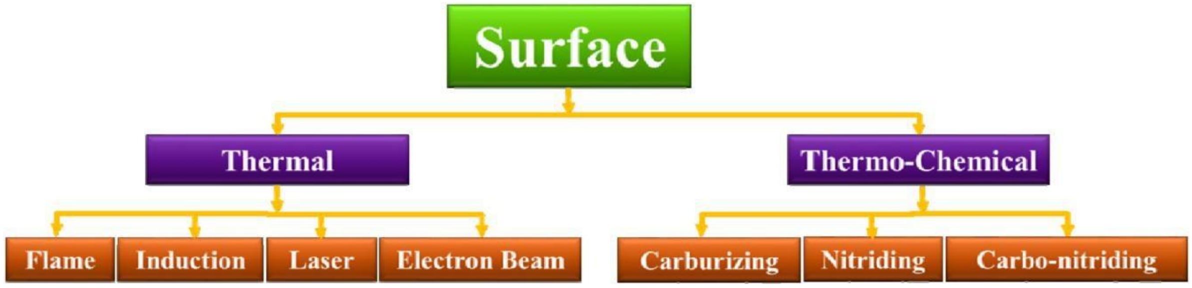
held at that temperature above Ms for sufficient time (in the bainite transformation range) to produce banitic microstructure.A schematic of this process is shown in Fig. An advantage of this process over Martempering is that the tempering can be avoided. Other advantages include, Higher ductility, Resistance to shock, Uniform hardness, Tougher and more wear restiance, Higher impact and Fatigue Strengths.

SURFACE HARDENING

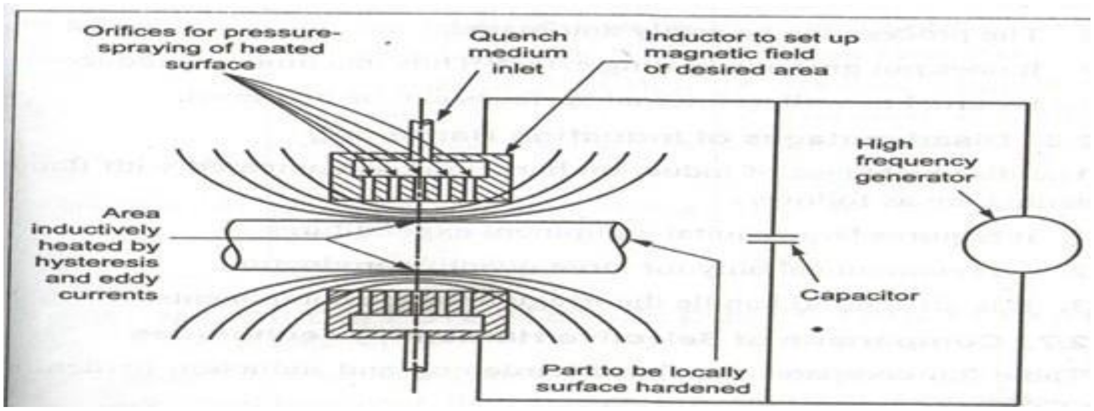
Numerous industrial applications require a hard wear resistant surface called the *case*, and a relatively soft, tough inside called the *core*. Example: Gears

They are two different categories. They are thermo chemical and thermo mechanical treatments. Thermo chemical treatment is related to change in chemical composition and In Thermo mechanical treatment, there is no change of chemical composition of the steel and are essentially shallow- hardening methods.

A detailed flow chart is given below related to surface hardening treatments.

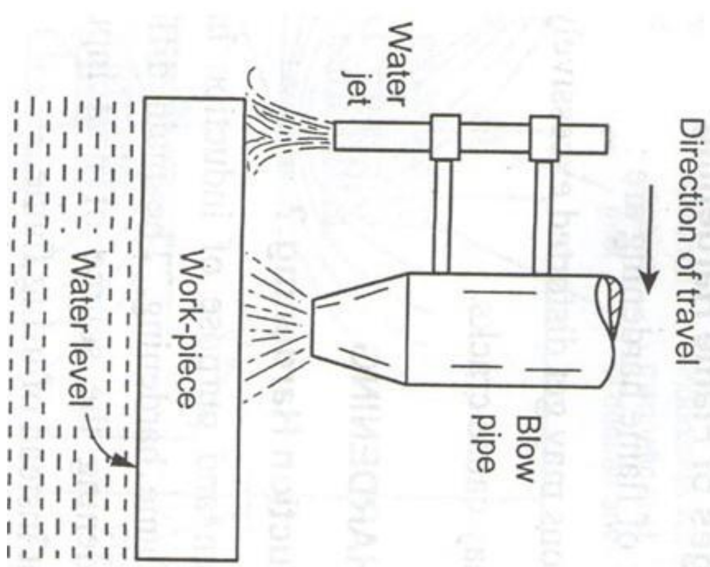


INDUCTION HARDENING



- Material Is Heated Above Its Upper Critical Temperature.
- Heating IS Done By Passing High Frequency Alternating Current Current
- Cyclic Magnetic Field That Is Generated Induces By Alternating Current That Heats The Work Piece.
- Hence Surface Is Quenches By Water Spray That Follows The Flame.

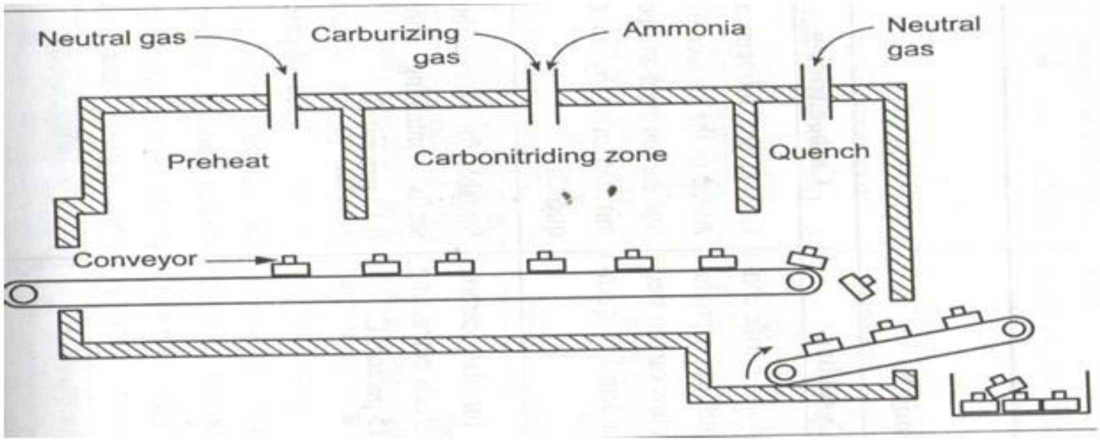
FLAME HARDENING



- Material Is Heated Above Its Upper Critical Temperature.
- Oxy-Acetylene Tourch Is Used As A Heating Source.

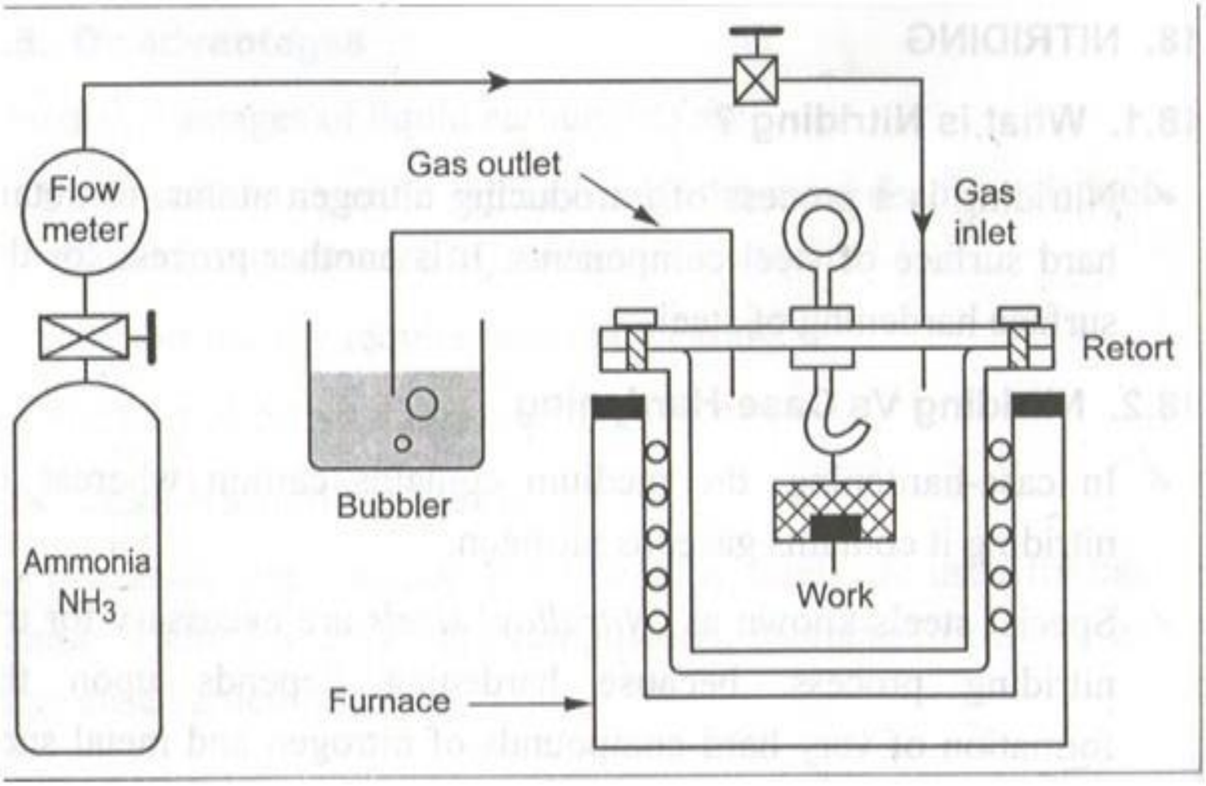
- Hence Surface Is Quenches By Water Spray That Follows The Flame.
- Steels Having 0.3% To 0.6% Carbon Are Hardened By Flame Hardened.

CARBONITRIDING



- It is carried out in a gas- atmosphere furnace.
- Hence propane (or) methane mixed with the ammonia.
- The organic gas serves as the source of carbon and the ammonia gas serves of nitrogen.
- The work piece is heated to 850°C at 2 to 10 hours. This is followed by quenching and then tempering is employed at 180°C.
- Case Depth Is About 0.5mm.
- Hardness after heat treatment Rc65
- Typical uses:-
  - Gears.
  - Nuts.
  - Bolts.

NITRIDING SYSTEM



- First The Work piece are heat treated to produce the required properties in the core.
- The steel part are heated and maintained at about 500°C for between 40 to and 100 hrs.
- The treatment takes place in gas-tight chamber.
- Hence ammonia gas is allowed to circulate
- The ammonia decomposes and releasing single atoms of nitrogen.
- The atomic from of nitrogen(N) is absorbed on the surface of steel components.
- Hence harden surface can obtained.

CYANIDING

- Nitrogen And Carbon Is Used To Obtain Hard Surface
- The Work piece Is Immersing in the Cyanide Bath.

- Hence hard surface is obtained .

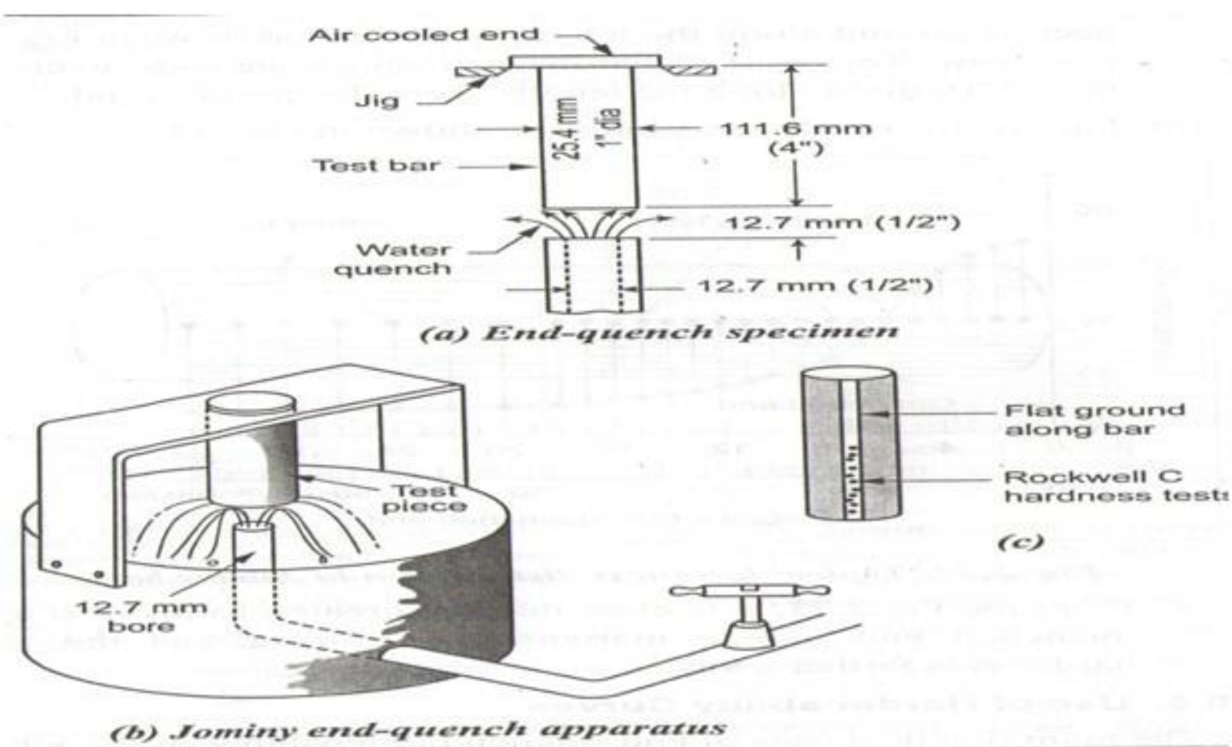
Operation:-

- The Steel Components Are Heated Above A Temperature Of 950°C
- Here Sodium Cyanide And Sodium Carbonate Is Used As Molten Bath**.**
- During this treatment both carbon and nitrogen diffuse into the surface of the steel.
- Hence hard iron nitrides contribute to the surface to be hardened.

Applications:-

Gear. Shafts, pins, braked, cams, sleeves , springs,

END-QUENCH SPECIMEN& JOMINY END QUENCH APPARATUS



TESTING PROCEDURE:-

- The standard test piece is heated to above upper critical temperature until it become completely austenitic.
- Here it is quenched at one end only by standard jet of water at 25°C.

- Thus Different Rates Of Cooling Are Obtained Along The Length Of The Test-Piece.
- Hence Now Rockwell C Hardness Reading Are Taken Every 1.5mm (1/16 Inch) Along The Length From The Quenched End.
- Hence hardened is more on quenched end and lower on the other end.

UNIT-III

FERROUS AND NON FERROUS METALS

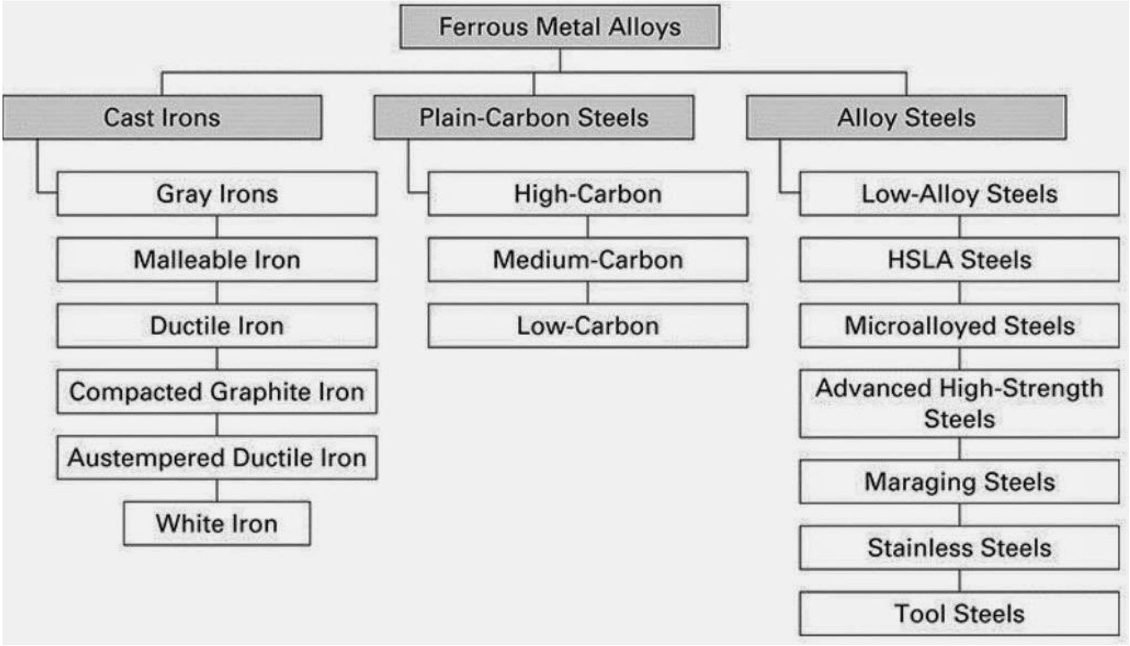
Materials are made up of elements. Materials are anything that have weight and occupy some space. An every increasing variety of materials is available, each having its own characteristics application, advantages and limitations.

Classification of metals:-

Ferrous and non-ferrous

Ferrous:-

- The metals, which contain iron as their main constituent, are called ferrous metals
- Ferrous materials are the most important metals/alloys in the metallurgical and mechanical industries because of their very extensive use.



CAST IRON

It is primarily an alloy of iron and carbon. The carbon content in cast iron varies from 1.5 to 4 per cent. Small amounts of silicon, manganese, sulphur and phosphorus are also present in it. Carbon in cast iron is present either in free state like graphite or in combined state as cementite. Cast iron contains so much carbon or its equivalent that it is not malleable. One characteristic (except white cast iron) is that much of carbon content is present in free form as graphite. Largely the properties of cast iron are determined by this fact.

Melting point of cast iron is much lower than that of steel. Most of the castings produced in a cast iron foundry are of grey cast iron. These are cheap and widely used.

The characteristics of cast iron which make it a valuable material for engineering applications

- Very good casting characteristics.
- Low cost
- High compressive strength
- Good wear resistance
- Excellent machinability

The main limitation of this metal is brittleness and low tensile strength and thus cannot be used in those components subjected to shocks

The varieties of cast iron in common use are:

- Grey cast iron
- White cast iron
- Malleable cast iron
- Nodular cast iron
- Chilled cast iron
- Alloy cast iron

Grey Cast Iron

It is the iron which is most commonly used in foundry work. If this iron is machined or broken, its fractured section shows the greyish colour, hence the name “grey” cast iron. The grey colour is due to the fact that carbon is present in the form of free graphite. A very good characteristic of grey cast iron is that the free graphite in its structure acts as a lubricant. This is suitable for those components/products where sliding action is desired. The other properties are good machinability, high compressive strength, low tensile strength and no ductility.

In view of its low cost, it is preferred in all fields where ductility and high strength are not required. The grey cast iron castings are widely utilized in machine tool bodies, automobile cylinder blocks and flywheels, etc.

White Cast Iron

It is so called due to the whitish colour shown by its fracture. White cast iron contains carbon exclusively in the form of iron carbide Fe_3C (cementite). From engineering point of view, white cast iron has limited applications. This is because of poor machinability

and possessing, in general, relatively poor mechanical properties. It is used for inferior castings and places where hard coating is required as in outer surface of car wheels. Only crushing rolls are made of white cast iron. But it is used as raw material for production of malleable cast ir

Malleable Cast Iron

Malleable cast iron is produced from white cast iron. The white cast iron is brittle and hard. It is, therefore, unsuitable for articles which are thin, light and subjected to shock and vibrations or for small castings used in various machine components. The malleable cast iron is produced from white cast iron by suitable heat treatment, i.e., annealing. This process separates the combined carbon of the white cast iron into noddles of free graphite.

The malleable cast iron is ductile and may be bent without rupture or breaking the

section. Its tensile strength is usually higher than that of grey cast iron and has excellent machining qualities. Malleable cast iron components are mainly utilized in place of forged steel or parts where intricate shape of these parts creates forging problem. This material is principally employed in rail, road automotive and pipe fittings etc.

Nodular Cast Iron

It is also known as “spheroidal graphite iron” or Ductile iron or High strength “Cast iron”. This nodular cast iron is obtained by adding magnesium to the molten cast iron. The magnesium converts the graphite of cast iron from f l ake to spheroidal or nodular form. In this manner, th e mechanical properties are considerably improved. The strength increases, yield point improves and brittleness is reduced. Such castings can even replace steel components.

Outstanding characteristics of nodular cast iron are high fluidity which allows the castings of intricate shape. This cast iron is widely used in castings where density as well as pressure tightness is a highly desirable quality. The applications include hydraulic cylinders, valves, pipes and pipe fittings,

Chilled Cast Iron

Quick cooling is generally known as chilling and the iron so produced is “chilled iron”. The outer surface of all castings always gets chilled to a limited depth about (1 to 2 mm) during pouring and solidification of molten metal after coming in contact with cool sand of mould. Sometimes the casting is chilled intentionally and some becomes chilled accidentally to a small depth.

Chills are employed on any faces of castings which are required to be hard to withstand wear and friction. Chilled castings are used in producing stamping dies and crushing rolls railway, wheels cam followers, and so on.

STAINLESS STEELS

The only material known to engineers which possesses a combination of various properties such as: wide range of strength and hardness, high ductility and formability, high corrosion resistance, good creep resistance, good thermal conductivity, good machinability, high hot & cold workability and excellent surface finish is stainless steel. Alloy steels have been developed for a specific purpose. We shall study them as follows: They are known as stainless since they do not corrode or rust easily in most of environment and media. Stainless steels can be further divided into the following three

Ferritic stainless steel:

It is that steel when properly heat treated and finished, resists oxidation and corrosive attacks from corrosive media. Ferritic stainless steels contain 12–18% chromium, 0.15 to 0.2% carbon besides iron and usual amounts of manganese and silicon. The steels are stainless and relatively cheap. They are magnetic in nature. Structure of these steels consist of ferrite phase which cannot be hardened by he treatment. These steels are actually iron-chromium alloys and cannot be hardened by heat treatment. Such type of steel is utilized in manufacture of dairy equipment food processing plants, etc.

Martensitic stainless steel:

These steels contain 12–18% chromium and 0.1 to 1.8% carbon. These steels can be hardened by heat treatment but their corrosion resistance is decreased. Steels with 12 to 14% chromium and 0.3% carbon are widely used for table cutlery, tools and equipment. Steel with little less carbon percentage and higher percentage of chromium are used as springs, ball bearings and instruments under high temperature and corrective conditions.

Austentic stainless steels:

These are the most costliest among all stainless steels. In these steels besides chromium, nickel is also added. Nickel is a very strong austenitic stabilizer and therefore the microstructure of these steels is austentic at room temperature. These steels contain 12 to 21% chromium and 8 to

15% nickel and carbon less than 0.2%. The most familiar alloy of this group is known as 18:8 stainless steel i.e. 18% chromium and 8% nickel plus other. Other elements like carbon, manganese and silicon in very small quantities.

TOOL STEELS

Tool steels are specially alloyed steels designed for high strength, impact toughness and wear resistance at room and elevated temperatures. They are normally used in forming and machining of metals. So the requirements in a tool steel are that it should be capable of becoming very hard and further that it should be able to retain its hardness at high temperatures normally developed during cutting of materials. This property is known as "red hardness". Further, tool steel should not be brittle for smooth working.

High Speed Steel (H.S.S.)

It is the name given to the most common tool steel. As the name implies, it can cut steel at high cutting speeds. These steels are high in alloy content, have excellent hardenability, maintain their hardness at elevated temperatures around 650°C, are quite resistant to wear and contain relatively large amounts of tungsten or molybdenum, together with chromium, cobalt or vanadium. They are used to produce cutting tools to be operated for various machining operations such as turning, drilling, milling, etc. A typical composition of H.S.S. is tungsten 18%, chromium 4% and vanadium 1%, carbon 0.75 to

0.9% and rest iron.

Molybdenum High Speed Steel

This steel contains 6% tungsten, 6% molybdenum, 4% chromium and 2% vanadium and have excellent toughness and cutting ability. The molybdenum high speed steel are better and cheaper than other types of steel. It is particularly utilized in drilling and tapping operations.

NON-FERROUS METALS AND ALLOYS

Non-ferrous metals are those which do not contain significant quantity of iron or iron as base metal. These metals possess low strength at high temperatures, generally suffer from hot shortness and have more shrinkage than ferrous metals. They are utilized in industry due to following advantages:

- High corrosion resistance
- Easy to fabricate, i.e., machining, casting, welding, forging and rolling
- Possess very good thermal and electrical conductivity
- Attractive colour and low density

The various non-metals used in industry are: copper, aluminium, tin, lead, zinc, and nickel, etc., and their alloys.

Copper

The crude form of copper extracted from its ores through series of processes contains 68% purity known as Blister copper. By electrolytic refining process, highly pure (99.9%) copper which is remelted and casted into suitable shapes. Copper is a corrosion resistant metal of an attractive reddish brown colour.

Properties and Uses

(1) High Thermal Conductivity: Used in heat exchangers, heating vessels and appliances, etc.

(2) High Electrical Conductivity: Used as electrical conductor in various shapes and forms for various applications.

(3) Good Corrosion Resistance: Used for providing coating on steel prior to nickel and chromium plating

(4) High Ductility: Can be easily cold worked, folded and spun. Requires annealing

After cold working as it loses its ductility.

Aluminium

Aluminium is white metal which is produced by electrical processes from clayey mineral known as bauxite. However, this aluminium ore bauxite is available in India in plenty and we have a thriving aluminium industry.

Properties and Uses

- Like copper it is also corrosion resistant.
- It is very good conductor of heat and electricity although not as good as copper.
- Possesses high ductility and light weight so widely utilized in aircraft industry.
- Needs frequent annealing if cold worked since it becomes hard after cold working.
- In view of its ductility and malleability it has replaced copper in electrical transmission and appliances to some extent.
- It is used in manufacturing of household utensils including pressure cookers.

Zinc

The chief ores of zinc are **blende** (ZnS) and **calamine** ($ZnCO_3$). Zinc is a fairly heavy, bluish-white metal principally utilized in view of its low cost, corrosion resistance and alloying characteristics. Melting point of zinc is 420°C and it boils at 940°C.

Properties and Uses

(1) High corrosion resistance: Widely used as protective coating on iron and steel.

(2) **High fluidity and low melting point:** Most suitable metal for pressure die casting generally in the form of alloy.

(3) **When rolled into sheets**, zinc is utilized for roof covering and for providing a damp proof non-corrosive lining to containers.

(4) **The galvanized wires, nails, etc.** are produced by galvanizing technique and zinc is also used in manufacture of brasses.

Nickel

About at least 85% of all nickel production is obtained from sulphide ores.

Properties and Uses

- Pure nickel is tough, silver coloured metal, harder than copper having some but less ductility but of about same strength.
- It is plated on steel to provide a corrosion resistance surface or layer.
- Widely used as an alloying element with steel. Higher proportions are advantageously added in the production of steel such as monel or in conel.
- It possesses good resistance to both acids and alkalis regarding corrosion so widely utilized in food processing equipment.

Magnesium

Principal ores of magnesium are **magnesite**, **carnallite** and **dolomite**. Magnesium is extracted by electrolytic process.

Properties and Uses:

- It is the lightest of all metals weighing around two-thirds of aluminium.
- The tensile strength of cast metal is the same as that of ordinary cast aluminium, i.e., 90 MPa.
- The tensile strength of rolled annealed magnesium is same as that of good quality cast iron.
- Magnesium can be easily formed, drawn forged and machined with high accuracy.
- In powdered form it is likely to burn, in that situation adequate fire protection measures should be strictly observed.
- Its castings are pressure tight and achieve good surface finish. Magnesium castings include motor car gearbox, differential housing and portable tools.

Vanadium

It occurs in conjunction with iron pyrite, free sulphur and carbonaceous matter.

Properties and Uses:

- It is silvery white in colour.
- Its specific gravity is 5.67.
- Its melting point is 1710°C.
- When heated to a suitable temperature it can be hammered into any shape or drawn into wires.
- It is used in manufacture of alloy steels.
- Vanadium forms non-ferrous alloys of copper and aluminium from which excellentcastings can be produced.

Antimony

Chief ore of antimony is **stibnite**. To a small extent, antimony is obtained as a by-product in refining of other metals such as lead, copper silver and zinc.

Properties and Uses

- It is silvery white, hard, highly crystalline and so brittle that it may be readily powdered.
- Its specific gravity is 6.63 and melting point is 630°C.
- It is generally used as an alloying element with most of heavy metals.
- Lead, tin and copper are the metals which are most commonly alloyed with antimony.

ALLOYS OF COPPER

Copper alloys are among the best conductors of heat and electricity and they have good corrosion resistance. The common types of copper alloys are brasses and bronzes. The various alloys of copper are discussed as follows:

Brass

All brasses are basically alloys of copper and zinc. Commercially there are two main varieties of brasses:

(1) Alfa brass: Contains upto 36% Zn and rest copper for cold working.

(2) Alfa-Betabrass: Contains 36 to 45% Zn and remainder is copper for hot working.

The tensile strengtn increases in content of Zn upto 30% zinc. With further increase in zinc content beyond 30%, the tensile strength continues to increase upto 45% of Zn, but ductility of brasses drops significantly. β- phase is less ductile than α-phase but it is harder and stronger.

Thus, there are various types of brasses depending upon proportion of copper and zinc. Fundamentally brass is a binary alloy of copper with as much as 50% zinc. Various classes of brasses such as cartridge brass, Muntzmetal leaded brass, Admirality brass, naval brass and nickel brass depending upon the proportion of copper and zinc plus third alloying metal are available for various uses. Suitable type of brasses can undergo the processes of casting, hot forging, cold forging, cold rolling into sheets, drawing into wires and extrusion for obtaining requisite special cross-section bars. The melting point of brass varies according to its composition but most of the brasses in the common range liquefy between temperatures of 840°C to 960°C. By adding small quantities of other elements, the properties of brass may be greatly affected. For example, addition of 1 to 2% zinc improves the machinability of brass. Brass has a greater strength than that of copper but has a lower thermal and electrical conductivity. Brasses possess very good corrosion resistance and can be easily soldered. Brasses are used in hydraulic fittings, pump linings, utensils, bearings and bushes, etc.

Bronze

The alloy of copper and tin are usually termed **bronzes**. The useful range of composition is 75 to 95% copper and remainder tin. In general, it possesses superior mechanical properties and corrosion resistance to brass. The alloy can be easily cold rolled into wire, rods and sheets. With increase in tin content, the strength of this alloy and its corrosion resistance increases. It is then known as hot working bronze. Bronze is generally utilized in hydraulic fittings, bearings, bushes, utensils, sheets, rods and many other stamped and drawn products.

Phosphor_bronze:

When bronze contains phosphorus, it is known as phosphor bronze. Phosphorus present in such alloy increases the strength, ductility and soundness of castings. Various compositions of this alloy are available for different applications. The composition of the alloy varies according to whether it is to be forged, wrought or cast. A common type of phosphor bronze has the following composition as per Indian standards. Copper = 93.6%, tin = 9%, and phosphorus = 0.1 to 0.3%. The alloy possesses good wearing qualities and high elasticity. The alloy is resistant to salt water corrosion. Cast phosphor bronze is utilized for production of bearings and **gears**. Bearings of bronze contain 10% tin and small addition of lead. This is also used in making gears, nuts, for machine lead screws, springs, pump parts, linings and many other such applications.

Gun metal:

Gun metal contains 2% zinc, 10% tin and 88% copper. It is a very famous composition. Sometimes very small amount of lead is also added to improve cast ability and machinability. The presence of zinc improve its fluidity. This bronze is used for bearing bushes, glands, pump valves and boiler fittings, etc.

Silicon bronze:

Silicon bronze has an average composition of 3 per cent silicon, 1 percent manganese and rest copper. It possesses good general corrosion resistance of copper with higher strength and toughness. It can be cast rolled, stamped, forged and pressed either hot or cold and can be welded by all the usual methods. Silicon bronze is widely utilized for parts of boilers, tanks, stoves or where high strength as well as corrosion resistance isrequired.

Bell metal:

This alloy contains 20 to 21% tin and rest copper. It is hard and resistant to surface wear. It can be readily cast, is generally utilized for casting bells, gongs and utensils, etc.

Manganese Bronze:

It is an alloy of copper, zinc and manganese. It contains 55 to 60% copper, 40% zinc, with 3.5% manganese. This alloy is highly resistant to corrosion. It is stronger and harder than phosphor bronze. It has poor response to cold working but can be easily hot worked. It is generally utilized for producing bushes, plungers, feed pumps and rods, etc. Worm gears are frequently made of manganese bronze.

Muntz Metal:

The composition of this alloy is 60 percent copper and 40 per cent zinc. Sometimes a small quantity of lead is also added. This alloy is stronger, harder and more ductile than normal brass. While hot working between 700°C to 750, it responds excellently for process but does not respond to cold working. This alloy is utilized for a wide variety of small components of machines, bolts, rods, tubes, electrical equipment as well as ordinance works. It is widely employed in producing such articles which are required to resist wear.

Alloys of Aluminium

Aluminium may be alloyed with one or more alloying elements such as copper, manganese, magnesium, silicon and nickel. The addition of small quantities of alloying elements converts the soft and weak aluminium into hard and strong metal, while it retains its light weight. The main alloys of aluminium are: Duralumin, Y-alloy, Magnalium and Hindalium which are discussed as follows:

Duralumin:

A famous alloy of aluminium containing 4% copper, 0.5% manganese, 0.5% magnesium and a trace of iron with remainder as aluminium is known as duralumnin. It possesses high strength comparable with mild steel and low specific gravity. However, its corrosion resistance is much lower as compared with pure aluminium. The strength of this alloy increases significantly when heat treated and allowed to age for 3 to 4 weeks it will be hardened. The phenomenon is termed age hardening. To improve upon the corrosion resistance of it, a thin film of aluminium is rolled on the duralumin sheets. These sheets are known as Alclad by trade name and are widely used in aircraft industry.

It is widely utilized in wrought conditions for forging, stamping, bars, tubes and rivets. It can be worked in hot condition at 500°C. However, after forging and annealing it could also be cold worked. Due to light weight and high strength this alloy may be used in automobile industry.

Y-Alloy:

It is also known as copper-aluminium alloy. The addition of copper to pure aluminium improves its strength and machinability. Y-alloy contains 93% aluminium, 2% copper, 1% nickel and magnesium. This alloy is heat treated as well as age hardened just like duralumin. A heat treatment of Y-alloy castings, consisting of quenching in boiling water from 510°C and then aging for 5 days develops very good mechanical characteristics in them. Since Y-alloy has better strength at elevated temperature than duralumin therefore it is much used in aircraft cylinder heads and piston. It is also used in strip and sheet form.

(3) **Magnalium:** It is produced by melting the aluminium 2 to 10% magnesium in a vacuum and then cooling it in vacuum or under a pressure of 100 to 200 atmospheres. About 1.75% copper is also added to it. Due to its light weight and good mechanical characteristics, it is mainly used for aircraft and automobile components.

(4) **Hindalium:** It is an alloy of aluminium and magnesium with small quantity of chromium. It is manufactured as rolled product in 16 gauge mainly used in manufacture of anodized utensils.

Alloys of Nickel German

silver:

The composition of this alloy is 60% Cu, 30% Ni and 10% zinc. It displays silvery appearance and is very ductile and malleable. It is utilized for electrical contacts, casting of high quality valves, taps and costume jewellery. It is also used in producing electrical wires.

Monel metal:

It contains 68% Ni, 30% Cu, 1% iron and remainder small additions of Mn and other elements. It is corrosion resistant and possesses good mechanical properties andmaintains them at elevated temperatures.

Nichrome:

It is an alloy of nickel and chromium which is utilized as heat resistant electrical wire inelectrical applicances such as furnaces, geysers and electric iron, etc.

Inconel and incolony:

These alloys principally contain, Ni, Cr, Fe, Mo, Ti and very small proportions of carbon.These are used as high temperature alloys. Inconel does not respond to heat treatment.

NON METALLIC MATERIALS

Typical Commodity and Engineering Polymers/Plastics-Properties and Applications

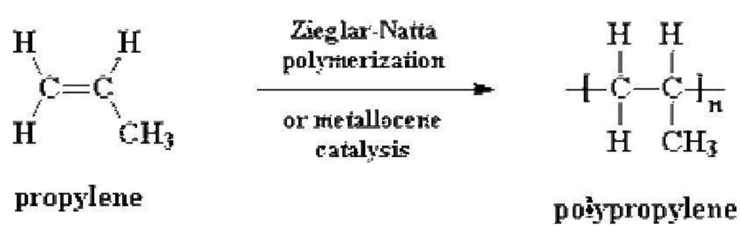
Polymers such as polyethylene, polypropylene, PVC etc which does not have excellent mechanical and thermal properties are used in high volume in non-structural applications are termed commodity polymers. Polymers which possess properties such as

i) Excellent mechanical properties over temperatures from below -40°C (-40°F) to above 148°C (300°F), ii) Self-extinguishing, non-dripping characteristics, iii) Excellent durability, dimensional stability and low water absorption, iv) Resistance to aqueous chemical environments and v) Excellent impact strength, are classified as engineering polymers/plastics . Ex., polyphenylene oxide, PEEK, polyethylene terephthalate, polyphenylene sulphide, Teflon, Kevlar, polyimide, polyeterimide, ABS etc.

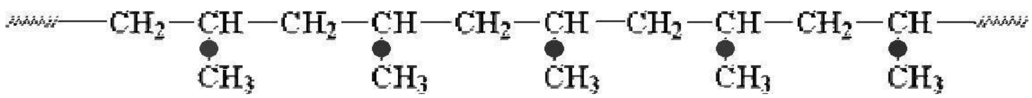
Polypropylene

Polypropylene(thermoplastic, melting temperature: 174 ^{o}C; glass transition temperature: - ^{o}C) is a versatile commodity polymer. It serves double duty, both as a plastic and as a fiber. As a plastic it's used to make things like dishwasher-safe food containers. It can do this because it doesn't melt below 160 ^{o}C, or 320 ^{o}F. Polyethylene, a more common plastic, will anneal at around 100 ^{o}C, which means that polyethylene dishes will warp in the dishwasher. As a fiber, polypropylene is used to make indoor-outdoor carpeting, used around swimming pools and miniature golf courses. It works well for outdoor carpet because it is easy to make colored polypropylene, and because polypropylene doesn't absorb water, like nylon does.

It is slightly more susceptible than polyethylene to strong oxidizing agents. It offers the best stress-crack resistance of the polyolefins. Products made of polypropylene are brittle at 0°C and may crack or break if dropped from benchtop height. Polypropylene can be made from the monomer propylene by Ziegler-Natta polymerization and by metallocene catalysis polymerization.

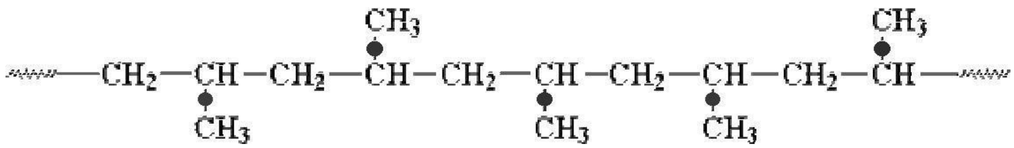


In Isotactic polypropylene all the methyl groups are on the same side of the chain, like this:



isotactic polypropylene

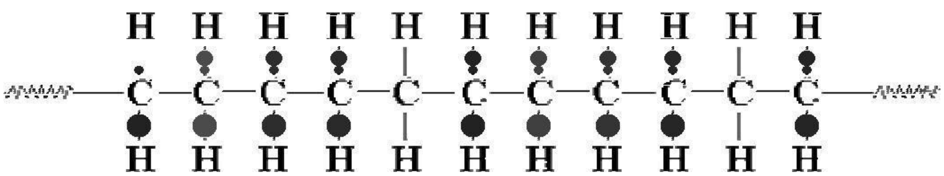
Atactic means that the methyl groups are placed randomly on both sides of the chain like this:



atactic polypropylene

Polyethylene

Polyethylene is probably the polymer seen mostly in daily life. Polyethylene is the most popular plastic in the world. This is the polymer that makes grocery bags, shampoo bottles, children's toys, and even bullet proof vests. For such a versatile material, it has a very simple structure, the simplest of all commercial polymers. A molecule of polyethylene is nothing more than a long chain of carbon atoms, with two hydrogen atoms attached to each carbon atom.



Sometimes it's a little more complicated. That is some of the carbons, instead of having hydrogens attached to them, will have long chains of polyethylene attached to them. This is called branched, or low-density polyethylene, or LDPE. When there is no branching, it is called linear polyethylene, or HDPE. Linear polyethylene is much stronger the branched polyethylene, but branched polyethylene is cheaper and easier to make. Linear polyethylene is normally produced with molecular weights in the range of 200,000 to 500,000, but it can be made even higher. Polyethylene with molecular weights of three to six million is referred to as ultra-high molecular weight polyethylene, or UHMWPE. UHMWPE can be used to make fibers which are so strong they replaced Kevlar for use in bullet proof vests. Large sheets of it can be used instead of ice for skating rinks. Branched polyethylene is often made by free radical vinyl polymerization. Linear polyethylene is made by a more complicated procedure called Ziegler-Natta polymerization. UHMWPE is made using metallocene catalysis polymerization. But Ziegler-Natta polymerization can be used to make LDPE, too. By copolymerizing ethylene monomer with a alkyl-branched comonomer such as one gets a copolymer which has short hydrocarbon branches. Copolymers like this are called linear low-density p olyeth yle ne, or LLDPE. LLDPE is often used to make things like plastic films.

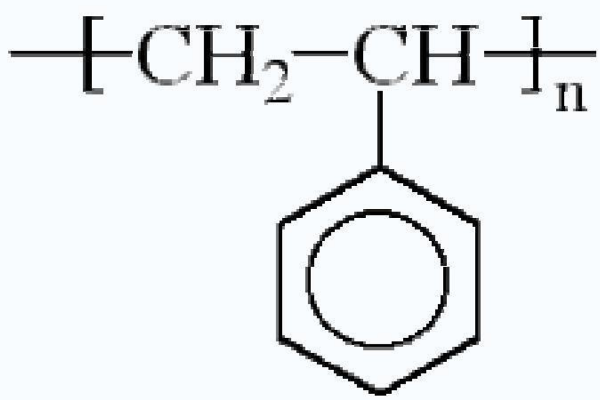
Linear low-density polyethylene (LLDPE) combines the toughness of low-density polyethylene with the rigidity of high-density polyethylene.

Cross-linked high-density polyethylene (XLPE) is a form of high-density polyethylene wherein the individual molecular chains are bonded to each other (using heat, plus chemicals or radiation) to form a three-dimensional polymer of extremely high molecular weight. This structure provides superior stress-crack resistance and somewhat improves the toughness, stiffness and chemical resistance of HDPE. XLPE is a superior material for moulding very large storage tanks.

4.3 Polystyrene

Polystyrene is normally a solid thermoplastic, but can be melted at higher temperature for molding or extrusion, then resolidified. It is an amorphous and its glass transition temperature is $100^{\circ}C$.

Polystyrene is an aromatic polymer. Polystyrene is also used as a building material, with electrical appliances (light switches and plates), and in other household items. Polystyrene can be transparent or can be made to take on various colors. It is economical and is used for producing plastic model assembly kits, plastic cutlery, CD "jewel" cases, and many other objects where a fairly rigid, economical plastic of various colors is desired. For architectural and engineering modelling, polystyrene is extruded into forms of standard modelling scale with the cross-sections of a miniature I-beam as well as rods and tubes. It is also formed into sheets with various patterns for this purpose as well. The blank sheets of polystyrene are referred to as "plasticard". Polystyrene fabricated into a sheet can be stamped (formed) into economic, disposable cups, glasses, bowls, lids, and other items, especially when high strength, durability, and heat resistance are not essential. A thin layer of transparent polystyrene is often used as an infra-red spectroscopy standard.

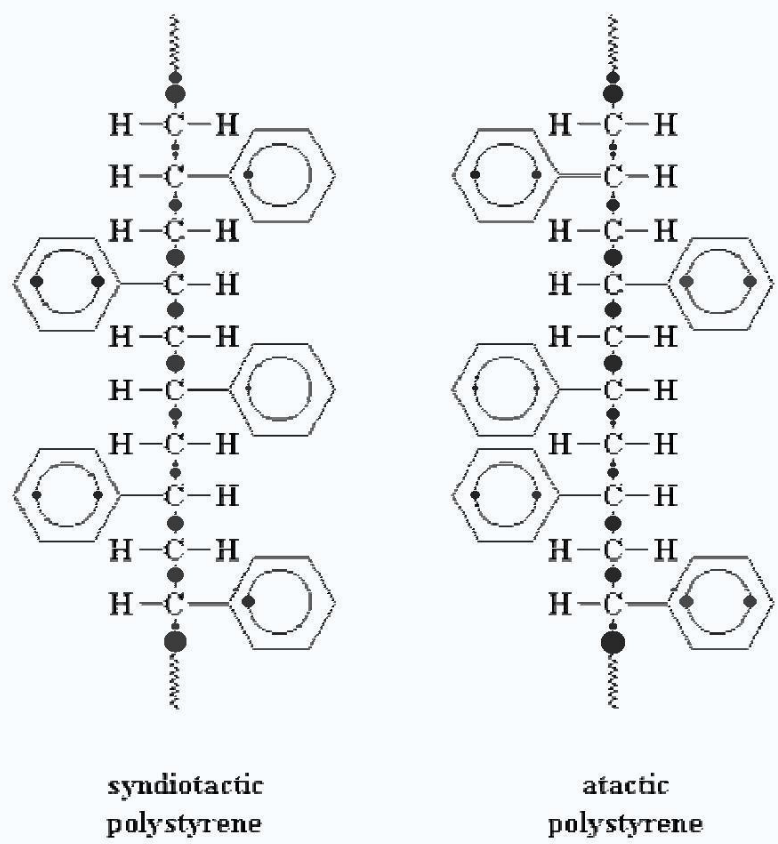


Polystyrene

The outside housing of the computer is probably made of polystyrene. Model cars and airplanes are made from polystyrene, and it also is made in the form of foam packaging and insulation (Styrofoam™ is one brand of polystyrene foam). Clear plastic drinking cups are made of polystyrene. So are a lot of the molded parts on the inside of car, like the radio knobs. Polystyrene is also used in toys, and the housings of things like hairdryers, computers, and kitchen appliances.

Expanded polystyrene is very easily cut with a hot -wire foam cutter, which is easily made by a heated and taut length wire, usually nichrome due to its thermal conductance. Hot wire foam cutters work by heating special wire (usually nichrome or stainless steel) to the point where it can vaporize foam immediately adjacent to it. The foam gets vaporized before actually touching the heated wire, which yields exceptionally smooth cuts. Polystyrene, shaped and cut with hot wire foam cutters, is used in architecture models, actual signage, amusement park and movie sets, airplane construction, and much more.

Polystyrene's most common use, however, is as expanded polystyrene, which is a mixture of about 5% polystyrene and 95% gaseous blowing agent. This is the lightweight material of which coffee cups and takeaway food containers are made. The voids filled with trapped air give expanded polystyrene low thermal conductivity. This makes it ideal as a construction material and is used in structural insulated panel building systems. It is also used as insulation in building structures, as packing material for cushioning inside boxes, as non-weight bearing architectural structures (such as pillars) and also in crafts.



Syndiotactic polystyrene has a regular structure, so it can pack into crystal structures. The irregular atactic polystyrene can't.

and model building, particularly architectural models. Foamed between two sheets of paper, it makes a more-uniform substitute for corrugated cardboard tradenamedF o a m c ore. Expanded polystyrene used to contain CFCs but other, more environmentally-safe blowing agents are now used. Because it is an aromatic hydrocarbon, polystyrene is flammable and burns with an orange-yellow flame giving off soot, as opposed to non-aromatic hydrocarbon polymers such as polyethylene, which burn with a light yellow flame (often with a blue tinge) and no soot. Production methods include sheet stamping (PS) and injection molding (both PS and HIPS).

Toughening of Polystyrene

Pure polystyrene is brittle, but hard enough that a fairly high-performance product can be made by giving it some of the properties of a stretchier material, such as polybutadiene rubber. The two materials cannot normally be mixed due to the amplified effect of intermolecular forces on polymer solubility (see plastic recycling), but if polybutadiene is added during polymerization it can become chemically bonded to the polystyrene, forming a graft copolymer which helps to incorporate normal polybutadiene into the final mix, resulting in high-impact polystyrene or HIPS, often called "high-impact plastic" in advertisements. Common applications include use in toys and product casings. HIPS is usually injection molded in production.

Polyvinyl chloride(PVC)(VINYL Plastic)

$$- -[-CH_2-CH(Cl)-]_n--$$

Polyvinyl chloride is obtained by suspension or bulk free radical polymerization of vinyl chloride.
. It is an amorphous polymer(thermo plastic) having glass transition temperature around 87 ^{o}C. It is an unstable polymer compared to the other commodity polymers like PE, PP etc. Its commercial success is attributed to the discovery of suitable stabilizers and other additives. Lead compounds (lead stearate etc), organo tin compounds(dibutyl tin dilaurate etc) etc are used as stabilizers. About 50% of produced PVC is used as rigid resins (ex in PVC pipes etc). Flexible PVC is made via plasticization using plasticizers such as dioctyl phthalate, dioctyl adipate, tricresyl phosphate etc. Building construction market account for 30 % of its production. which include pipe and fittings, siding, carpet backing, windows gutters, wall coverings etc . Bottles and packaging sheet are also major rigid markets. Flexible vinyl is used in wire and cable insulation, film and sheet, floor coverings, synthetic leather products, coatings, blood bags, medical tubing and many other applications. Plasticized PVC is melt processed. PVC has a good resistance to hydrocarbons. Its application is widened using fillers, pigments, impact modifiers etc.

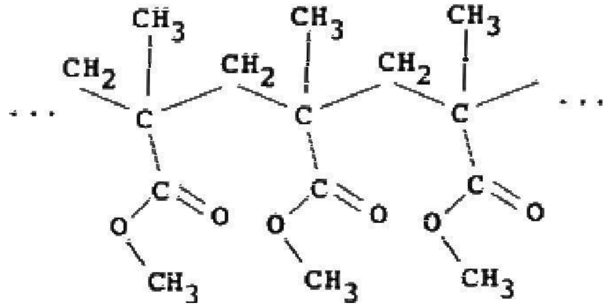
Poly(methyl methacrylate) (Diakon, Lucite, Oroglas, Perspex, Plexiglas)

PMMA is an amorphous(glass transition 105^{o}C), transparent and colourless thermoplastic that is hard and stiff but brittle and notch-sensitive. It has good abrasion and UV resistance and excellent optical clarity but poor low temperature, fatigue and solvent and scratch resistances. Though flammable, it has low smoke emission. General purpose grades can be extruded and injection moulded. Monomer casting is also used to achieve much higher molecular weights - which are not melt processable because of their extremely high melt viscosity - with somewhat improved properties. The monomer cast items most commonly encountered are sheets and novelty displays in which e.g. insects or watch parts are embedded. Thin films are normally made from impact modified grades, which incorporate a small proportion of elastomer(s), in order to improve their flexibility. Perspex CQ is a particular example of monomer cast sheet which was, until recently, made for intra-ocular use and they are additive-free and the purest (and probably the highest molecular weight) grade of PMMA available. Applications include sinks, baths, displays, signs, glazing (especially aircraft), lenses and light covers. Cast sheet is also used for guards and the like.

PMMA is used for instance in the rear lights of cars. The spectator protection in ice hockey stadiums is made of PMMA as are the very largest windows and fish tanks in the world. The material is used to produce laserdiscs, and sometimes also for DVDs, but the more expensive polycarbonate (also used for CDs) has better properties when exposed to moisture. PMMA has a good degree of compatibilty with human tissue, and can be used for replacement intraocular lenses in the eye when the original lens has been removed in the treatment of cataracts. Hard contact lenses are frequently made of this material; soft contact lenses are often made of a related polymer, in which acrylate monomers are used that contain one or more hydroxyl groups to make them hydrophilic.

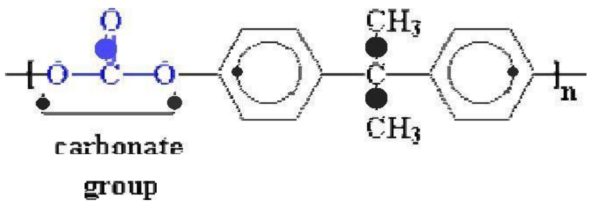
It is a rigid, transparent material and better at transmitting light than inorganic glass. PMMA is more transparent than glass. When glass windows are made too thick, they become difficult to see through. But PMMA windows can be made as much as 13 inches (33 cm) thick, and they're still perfectly transparent. This makes PMMA ideal in the construction of large aquariums where windows must be thick in order to withstand the pressure of millions of gallons of water. The material is often used in place of glass. These optical qualities are the basis for its principal applications: from building to furniture, road signs, the car industry, navy, electrical appliances, laboratory equipment. Unlike glass, PMMA does not filter UV light. Some manufacturers coat their PMMA with UV films to add this property. Up to wavelengths of 2800 nm, PMMA allows most IR (infrared) light to pass. Longer wavelengths of IR energy, up to 25,000 nm are essentially blocked. Special formulations of colored PMMA exist to allow specific IR wavelengths to pass while blocking visible light (for remote control or heat sensor applications, for example). PMMA can be joined using cyanoacrylate cement (so-called "Superglue"), or by using liquid dichloromethane to dissolve the plastic at the join which then fuses and sets, forming an almost invisible weld. PMMA can also be easily polished, by which method cut edges (which turn opaque) can be returned to transparency.

In orthopedics, PMMA bone cement is used to affix implants and to remodel lost bone. It is supplied as a powder and a liquid; when mixed, they yield a dough which then hardens. Dentures are often made of PMMA. In cosmetic surgery, tiny PMMA microspheres suspended in some biological fluid are injected under the skin to permanently reduce wrinkles or scars.



POLYCARBONATE

It is an amorphous engineering thermoplastic with excellent combination of properties. It is one of the engineering plastic to compete with die cast metals. Polycarbonate, or specifically polycarbonate of bisphenol A, is a clear plastic used to make shatterproof windows, lightweight eyeglass lenses



Polycarbonate of bisphenol A is a thermoplastic. This means it can be molded when it is hot. But the polycarbonate used in eye glasses is a thermoset. Polycarbonate plastic is used to make bullet-proof windows, greenhouse walls, automobile headlamps, and rugged housings for cell phones, pagers, and laptop computers. It is a tough, shatter- and heat-resistant material, and is now being used in as many aesthetic applications as in traditional functional applications. From fashionable designer houseware to eye-catching transparent housing for electronics, polycarbonate plastic has bridged the gap between functionality and style. It has become sought after not only for its outstanding physical properties, but also for its ability to be molded into amazingly intricate shapes and dyed with an imagination's worth of brilliant and transparent colors. Like new clothing material that takes the catwalks of Milan by storm, polycarbonate plastic has become fashionable-it has become chic.

Polycarbonate is a tough, dimensionally stable, transparent thermoplastic that has many applications which demand high performance properties. This versatile thermoplastic maintains its properties over a wide range of temperatures, from -40"F to 280"F. It is available in three types: machine grade; window and glass-filled. It has the highest impact of any Thermoplastic, transparent up to 2" in special grades, outstanding dimensional and thermal stability, and exceptional machinability, stain resistant and non-toxic with low water absorption. Machine Grade is relatively stress free to permit the most demanding machining. It is also available in glass- filled. This polycarbonate is perfect for high performance uses in tough applications over a broad temperature range. Window Grade is optically clear, providing total luminous transmittance and very low haze factor. The high impact strength makes it resistant to repeated blows, shattering and spalling. Polycarbonate is excellent for electrical applications, because of its high dielectric strength and high volume resistivity which decreases only slightly as temperature or humidity is increased

Polycarbonate is unusually tough, due to the nature of its chemical bonding. It is also transparent and almost self extinguishing, with a relatively high continuous use temperature of around 115°C. Chemical resistance is not outstanding and it needs the addition of light stabilizers for any UV resistance. Glass fibers enhance the stiffness but reduce toughness, as might be anticipated. Polycarbonate is a versatile blending material, with blends of PC/PET and PC/ABS available commercially. Applications of polycarbonate include: glazing panels, light fittings, safety helmets and medical components.

Glass Filled

Glass-reinforced polycarbonate is finding principal applications in designs where metals, particularly die-cast aluminum and zinc, are commonly used. The coefficient of thermal expansion is reduced by nearly 75%, thus equaling that of some metals. While glass-reinforced has less impact strength than standard grades, it is still tougher and more impact resistant than most other plastics and die cast aluminum.

Machinability

Parts can be easily machined from standard metal working tools. No special tools are needed, and finished parts can be polished to a high gloss. Water or water-soluble cutting oils should be used when machining polycarbonate, since some standard cutting oils will attack the material. Polycarbonate can be machined on standard metalworking or woodworking equipment. Its unique properties permit it to be machined without chipping, splitting or breaking.

Annealing

Polycarbonate slab (Zelux) has been stressed relieved using Liquo-Temp annealing process. In some instances where extensive machining is required, a secondary annealing of semi-finished parts is highly recommended. Secondary annealing can be accomplished by heating parts at 250"F in a desiccated air circulating oven for one hour per one inch of thickness. After heating, the oven should be turned off and allowed to cool to room temperature spontaneously.

Bonding

Polycarbonate can be mechanically bonded by standard methods. It can also be cemented by using a solvent such as methylene chloride or adhesives such as epoxy, urethane and silicone. Polycarbonate and also be ultrasonically or vibrationally welded.

Applications

Appliances

Coffee filters, shaver housings, chocolate moulds, blenders, table wares, kitchen mixer bowls, grinder bowls, housings for ball point and fountain pens, rim heater grills, motor bracket and housing, camera, binocular casings and parts, flash lights, lenses and casings, water tank for steam iron, fruit juicer parts, high impact vacuum sweeper housings, mixers and power tools, bobbins for textile industries, baby feeding bottles and cutlery.

Automotive

Wind screen wiper brackets, car interior moulded trims, instrument glazing, indicator lamps, wind shield for two wheelers, door handles, head lamp support, traffic light housings and signal lenses, battery cases, helmets, wheel cover, body panels, components for sewing machine, chassis, levers, valves, control cams, directional signs, ventilation and radiator grills, housing for automobile and aerial motors etc.

Electrical & electronics

Wiring devices, insulator panels, plug and socket terminal blocks, coil formers, starter enclosures, electric meter covers, breaker boxes, gears, fuses, telephone housing for mining operations, dials, sight windows, housing for computers, calculators, magnetic disk packs etc.

Food contact articles

Mineral water bottles, microwave oven wares, food storage containers etc

Medical equipment components

Blood bottles, dispensers for inhalers, sterilisable lab wares and containers, tissue culture dishes, surgical disposables, diagnostic cardio-vascular and intravenous devices, housing for blood cleaning filters

Other uses

High temperature and pressure windows , face shields , industrial equipment and housing components, instrument components, electrical insulators and connectors, aircraft & missile components, portholes in pressure chambers, jet pump impellers and diffusers, card guides, assembly line cogs, sporting goods, slide rule components vacuum metallised reflectors, housing for street lamps, lenses and safety glasses, sun glasses, face protective waresaudio compact discs, film and slide cassettes,(the astronauts stepped onto the moon in polycarbonate helmets), windows, bank screens, police shields etc.

Typical physical properties

Specific gravity : 1.2, Tensile strength MPa : 62, Tensile modulus MPa : 2379, Flexural modulus MPa : 2344, Elongation at break (%): 110, Impact strength izod, notched, J/m : 123, Hardness : M70, Deflection temperature under load (1.82 MPa): 132 ^{o}C, Coefficient of linear expansion(mm/mm/ ^{0}C) 122 X 10^{-6}, Water absorption 24 hrs (%) : 0.15, Refractive index: 1.58-1.59, Dielectric strength (KV/mm): 15 Dielectric constant (10^{6} Hz): 2.97, Power factor: 0.0021, Volume resistivity(Ohm.m): 2.1 x 10^{16}, Melting point,(^{o}C): 220-230, Glass transition temperature (^{o}C): 140, Mould shrinkage(%): 0.4 - 0.7.

ABS Plastics

It is a tercopolymer containing styrene, acrylonitrile and butadiene comonomers. The earliest materials were the physical blends of styrene -acrylonitrile copolymer(SAN) and acrylonitrile-butadiene copolymers. Today the ABS referes to a product consisting of discrete cross-linked polybutadiene rubber particles that are grafted with SAN and embedded in a SAN matrix. ABS materials are important for their wide spread use as quality housings for equipments. It possesses the following important properties such as, high impact resistance, good stiffness, excellent surface quality, high dimensional stability at elevated temperatures & good heat distortion temperature, good chemical and stress cracking resistance, good low temperature properties etc. It can be electroplated

Eventhough polypropylene is cheaper than ABS, ABS is preferred where the extreme toughness and superior heat distortion resistance is required. But its main disadvantages are i) lack transparency, ii) poor weathering resistance and iii) poor flame resistance

ABS is used in vehicle construction industry(automotive instrument panels, and other interior components such as fascia panels, door covers, door handles, radiator grilles, ventilation system components, heater housings, seat belt fastenings, console panels, loud speaker housings, interior trim and other uses). ABS is also alloyed with polycarbonate to have enhanced heat resistance and surface hardness in conjuction with good impact strength. Molded ABS products are used in both protective and decorative applications. Examples include safety helmets, camper tops, pipe fittings, home-security devices and housings for small appliances, communications equipment, and business machines.

Chrome-plated ABS has replaced die-cast metals in plumbing hardware and automobile grilles, wheel covers, and mirror housings. Typical products vacuum-formed from extruded ABS sheet are refrigerator liners, luggage shells, tote trays, mower shrouds, boat hulls, and large components for recreational vehicles. Extruded shapes include weather seals, glass beading, refrigerator breaker strips, conduit, and pipe for drain-waste-vent (DWV) systems. Pipe and fittings comprise one of the largest single application areas for ABS.

Based on the property requirement ABS can be categorized into different grades such as General purpose , Fire retardant, Improved heat resistant, Enhanced chemical resistant, static dissipation grade, Extrusion grade, Fire retardant- extrusion etc. ABS is more hygroscopic than polystyrene and absorbs moisture upto 0.3%. It can be processed at 250-260 ^{o}C. It has low heat resistance compared to polystyrene. I t is an amorphous material and show low moulding shrinkage. ABS resins are hard, rigid, and tough, even at low temperatures. They consist of particles of a rubberlike toughener suspended in a continuous phase of styrene-acrylonitrile (SAN) copolymer. Various grades of these amorphous, medium-priced thermoplastics are available offering different levels of impact strength, heat resistance, flame retardance, and platability. Most natural ABS resins are translucent to opaque, and they can be pigmented to almost any color. Grades are available for injection molding, extrusion, blow molding, foam molding, and thermoforming. Molding and extrusion grades provide surface finishes ranging from satin to high gloss. Some ABS grades are designed specifically for electroplating. Their molecular structure is such that the plating process is rapid, easily controlled, and economical.

Compounding of some ABS grades with other resins produces special properties. For example, ABS is alloyed with polycarbonate to provide a better balance of heat resistance and impact properties at an intermediate cost. Deflection temperature is improved by the polycarbonate; molding ease, by the ABS. Other ABS resins are used to modify rigid PVC for use in pipe, sheeting, and molded parts. Reinforced grades containing glass fibers, to 40%, are also available.

Properties: ABS plastics offer a good balance of tensile strength, impact and abrasion resistance, dimensional stability, surface hardness, rigidity, heat resistance, low-temperature properties, chemical resistance, and electrical characteristics. These materials yield plastically at high stresses, so ultimate elongation is seldom significant in design; a part usually can be bent beyond its elastic limit without breaking, although it does stress-whiten. While not generally considered flexible, ABS parts have enough spring to accommodate snap-fit assembly requirements.

Impact properties of ABS are exceptionally good at room temperature and, with special grades, at temperatures as low as -40°F. Because of its plastic yield at high strain rates, impact failure of ABS is ductile rather than brittle. A long-term tensile design stress of 1,000 to 1,500 psi (at 73°F) is recommended for most grades.

General-purpose ABS grades may be adequate for some outdoor applications, but prolonged exposure to sunlight causes color change and reduces surface gloss, impact strength, and ductility. Less affected are tensile strength, flexural strength, hardness, and elastic modulus. Pigmenting the resins black, laminating with opaque acrylic sheet, and applying certain coating systems provide weathering resistance. For maximum color and gloss retention, a compatible coating of opaque, weather-resistant polyurethane can be used on molded parts. For weather able sheet applications, ABS resins can be coextruded with a compatible weather-resistant polymer on the outside surface.

ABS resins are stable in warm environments and can be decorated with durable coatings that require baking at temperatures to 160°F for 30 to 60 min. Heat-resistant grades can be used for short periods at temperatures to 230°F in light load applications. Low moisture absorption contributes to the dimensional stability of molded ABS parts.. Molded ABS parts are almost completely unaffected by water, salts, most inorganic acids, food acids, and alkalies, but much depends on time, temperature, and especially stress level. FDA acceptance depends to some extent on the pigmentation system used. The resins are soluble in esters and ketones, and they soften or swell in some chlorinated hydrocarbons, aromatics, and aldehydes.

POLY ETHER ETHER KETONE(PEEK)

Figure 1. Chemical formula of polyether ether ketone (PEEK) from I.C.I.

PEEK (Tg : 145°C) is an abbreviation for PolyEtherEther-Ketone, a high performance engineering thermoplastic and is an excellent material for a wide spectrum of applications where thermal, chemical, and combustion properties are critical to performance. The tensile properties of PEEK™ exceed those of most engineering plastics and can be reinforced with carbon fiber resulting in a tensile strength of over 29,000 psi (200 MPa) with excellent properties being retained up to 570°F(300°C). The exceptional stiffness of PEEK™ is reflected in its flexural modulus which is among the best of any thermoplastic. Glass or carbon fiber reinforcement gives further improvement up to very high temperatures.

These semi-crystalline polymers have excellent mechanical properties, good thermal stability and good chemical resistance. Despite a Tg of 145°C, the continuous service rating of PEEK is 250°C. PEEK is inherently fire retardant. It is easier to burn a hole through an aluminium sheet than through one made from PEEK. These materials are, however, very expensive and difficult to process. They find application in high temperature wire covering and printed circuit boards. Fibre reinforced grades are used in demanding applications that include valves, pumps and missile nose cones

PEEK combines excellent tribological properties with moldability and outstanding performance at high temperatures. Carbon fiber-reinforced PEEK is probably the only injection moldable bearing material that has a measurable wear factor at over 500°F (260°C), all the thermoplastics fail at or below this temperature. PEEK can retain its flexural and tensile properties at very high temperatures -- in excess of 250"C (482"F). The addition of glass fiber and carbon fiber reinforcements enhances the mechanical and thermal properties of the basic PEEK material. Superior chemical resistance has allowed them to work effectively as a metal replacement in harsh environments. They are inert to all common solvents and resist a wide range of organic and inorganic liquids. When extensive machining is required, a secondary annealing process should be considered.

MECHANICAL

At room temperature, PEEK™ functions as a typical engineering thermoplastic. It is tough, strong, rigid, has good load bearing properties over long periods, and it is resistance to both abrasion and dynamic fatigue.

THERMAL

PEEK™'S continuous service UV rating is 482°F (250°C) for unfilled and 500°F (260°C) for glass filled grades. PEEK™ also offers high temperature mechanical properties making it suitable for some application up to 600°F (315 ^{o}C).

FLAMMABILITY

PEEK™ has a high "Oxygen Index" and meets UL 94 -VO requirements, and demonstrates extremely low smoke emission. It contains no flame-retardant additives or halogens.

CHEMICAL RESISTANCE

PEEK™ has good resistance to aqueous reagents and long-term performance in superheated water at 500°F. (260°C.). Its resistance to attack is good over a wide pH range from 60% sulfuric acid to 40% sodium hydroxide at elevated temperatures. Attack can occur with some concentrated acids.

HYDROLYSIS RESISTANCE

PEEK can be used for thousands of hours at temperatures in excess of 480°F(250°C). in steam or high-pressure water environments without significant degradation in properties.

RADIATION RESISTANCE

Preliminary tests suggest that radiation resistance is extremely good.

Two of the highest performance areas of PEEK are in oil exploration and chemical processing industry.

Oil Exploration

Geologists use data logging equipment to analyze the nature and structure of rock in an attempt to locate oil bearing strata. This may involve sample removal or the use of a probe (acoustic, radiation, electrical) to evaluate the rock. Although all downhole materials must have good wear resistance and mechanical strength, the specific geometries involved and the function of the probe exclude most conventional materials. Therefore, an easily processable material which can withstand temperatures in excess of 392°F (200°C), pressures up to 580 psi, and has excellent resistance to chemically aggressive environments, electricity and radiation is required. Natural PEEK™ polymer and GL30 PEEK™ polymer composite grades are chosen by the oil industry as they are the only materials able to perform well in such a demanding environment.

The GL30 PEEK™ polymer is specially formulated to contain an optimum amount of short glass fiber reinforcement. The mechanical properties of the material are greatly enhanced with little reduction in processability.

Chemical Processing Industry

PEEK polymer composites are becoming increasingly used in the chemical processing industry due to a combination of excellent mechanical properties, processability, a high continuous operating temperature (500°F (260°C), UL746B) and outstanding chemical resistance. Conventional processing methods (injection molding, compression molding) are often used to form compressor plates, seals and pump components used in gas and fluid transport systems. The excellent fatigue resistance and general mechanical properties have been shown to outperform fluoropolymers, while the chemical resistance is vastly superior to metal components for such applications. The ability to form thin films on complex 3-dimensional objects means that PEEK™ polymer is often used to coat metallic parts which operate in chemically aggressive environments. The formation of these thin films is achieved by either electrostatic or plasma spray coating techniques. The chemical processing industry strives for purity of product. The contamination introduced by materials used to handle the substances during production is a selection criteria. PEEK™ polymer is inherently pure, and therefore can be used in long service applications with extremely low levels of contamination introduced to the chemical streams being processed.

PEEK Polymer for the Automotive Industry

The automotive industry constantly strives to increase the performance level and minimize the weight of in-engine components. Therefore, thermoplastic materials are often used to replace parts of the engine which are traditionally constructed from metals. PEEK™ polymer, a polyaryletherketone resin, is the leading high performance thermoplastic for automotive applications. It combines outstanding tribological performance with excellent mechanical properties over a wide temperature range.

Most automotive applications are required to operate at temperatures in excess of 248°F (120°C). Therefore, a high continuous operating temperature (500°F (260°C), UL746B) and excellent mechanical properties at this temperature make PEEK™ polymer the natural choice

Tribological performance may be defined as the friction and wear of interacting surfaces in relative motion. Therefore, the tribology of dry and lubricated contacts is critical for the operation of internal combustion engines. PEEK™ polymer has excellent friction and wear properties which are optimized in the specially formulated tribological grades, namely, 450FC30 and 150FC30 PEEK™ polymer.

ADVANTAGES OF PEEK

Excellent flexural, impact and tensile characteristics.

Very high continuous working temperature.

PEEK's excellent mechanical properties are retained even at temperatures in excess of 250"C (482"F).

Very high heat distortion temperature.

For unreinforced PEEK, the HDT is 160"C (320"F). The addition of 30% glass or carbon fiber reinforcement results in a dramatic increase to 315"C (599"F).

Exceptional chemical resistance.

PEEK is insoluble in all common solvents and, being crystalline, is extremely resistant to attack by a very wide range of organic and inorganic chemicals. A superior dielectric with low loss even at high temperatures and frequencies.

Excellent hydrolysis resistance.

PEEK has an excellent resistance to hydrolysis in boiling water and superheated steam (sterilizationin/ autoclavabiiity) at temperatures in excess of 250"C (482"F).

Good radiation resistance.

Absorbing more than 1000 M rads of irradation with no significant reduction in mechanical properties, PEEK exhibits exceptional resistance to high doses of gamma radiation.

Outstanding wear and abrasion resistance.

PEEK and reinforced PEEK have excellent wear and abrasion resistance characteristics with low coefficient of friction and high limiting PV properties.

Low smoke and toxic gas emissions.

Levels of smoke and toxic gas released during combustion are among the lowest of any thermoplastic material.

APPLICATIONS

PEEK's exceptional property profile enables it to be utilized in many of the most critical areas in general industry, such as, automotive, marine, oilwell, electronics, medical, aero-space etc

FEATURES

• Excellent chemical resistance
• Very low moisture absorption
• Inherently good wear and abrasion resistance
• Unaffected by continuous exposure to hot water or steam

Polytetrafluoroethylene(PTFE)

Polytetrafluoroethylene [Teflon, –(CF2-CF2)n -] is obtained by polymerizing (emulsion polymerization) tetrafloroethylene (ie., When all the hydrogen atoms in polyethylene have been replaced by fluorine, polytetrafluoroethylene (PTFE) is obtained). It is a tough, flexible, non-resilient material of moderate tensile strength but with excellent resistance to heat, chemicals and to the passage of electric current. It is resistant to many chemicals, including acetic acid, ammonia, sulfuric acid, and hydrochloric acid It remains ductile in compression at temperatures as low as 4K(-269oC). The coefficient of friction is low and is reported to be lower than that of any other solid.

PTFE is an outstanding insulator over a wide range of temperature and frequency. Its volume resistively exceeds 10 20 ohm meter. Any current measured is a polarization current rather than a conduction current. It has a low dielectric constant (2.1 at 60 Hz). A melt viscosity of 10 10-1011 poises has been measured at about 350oC. A slow rate of decomposition has been detected at the melting point and this increases with increase in temperature. There are no solvent for PTFE and it is attacked by molten alkali metal at room temperature and in some cases by fluorine. Treatment with solution of sodium metal in liquid ammonia will sufficiently alter the surface of PTFE so that it can be cemented to other materials using epoxy adhesives. Although it has good weathering resistance, it is degraded by high energy radiation. The polymer is not wetted by water and does not absorb measurably. The permeability to gases is very low and water vapour transmission is only half that of low density polyethylene.

It has a high bulk density and exceptional chemical properties.. It can temporarily withstand temperatures of 260oC and still have the same chemical properties. Teflon also retains its chemical properties in cryogenic temperatures of -240oC . It is a chemical inert material, making it relatively safe to use and handle.

Nonpolarity

The carbon backbone of the linear polymer is completely sheathed by the electron cloud of fluorine atoms, much like a wire core is protected by insulation coating. This ensheathment, and the angles at which the carbon-fluorine bonds are disposed, causes the centers of electronegativity and electropositivity to be perfectly balanced across the polymer chain cross section. As a result, no net charge difference prevails. This nonpolarity of the polymer is partly responsible for its lack of chemical reactivity.

Low inter chain forces

The bond forces between two adjacent polymer chains are significantly lower than the forces within one chain. High C-F and C-C bond strengths are among the strongest in single bond organic chemistry. The polymer must absorb considerable energy to disrupt these bonds.

Crystallinity

The high degree of crystallinity in these semicrystalline polymers results in high melting points(327°C), mechanical properties, and an integral barrier to migrating, small, nonpolar molecules. Under certain conditions, these molecules penetrate the plastics. The melting point of Teflon PTFE is one of the highest in organic polymer chemistry.

Insolubility

There is no known solvent for Teflon fluorocarbon resins under ordinary conditions.

Low coefficient of friction

The low coefficient of friction of Teflon results from low interfacial forces between its surface and another material and the comparatively low force to deform.

Low dielectric constant and dissipation factor: Teflon provides low, if not the lowest, values for these parameters. These low values arise from the polymer's nonpolarity as well as the tight electron hold in the ultrapolymer bonds.

Low water absorptivity

For Teflon to absorb water, the surface must remain wet for a long enough time for water to become physico-chemically associated with the polymer chains, and then it must become included in the polymer bulk structure. Water is a very high energy material and Teflon has a very low surface energy. Therefore, these events are energetically incompatible and only occur under special circumstances and to a small extent.

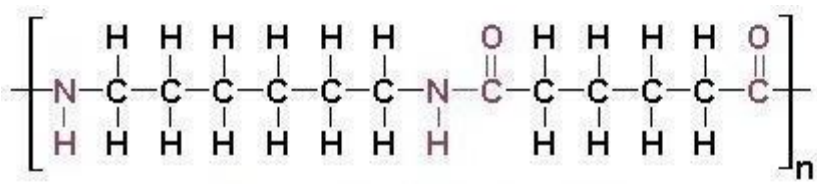
Excellent weatherability

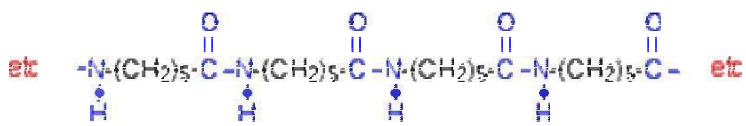
Weather includes light of various wavelengths (IR, visible, UV), water (liquid or gas), other gases, and normal temperatures and pressure. The physical and chemical makeup of Teflon makes it inert to these influences.

Flame resistant

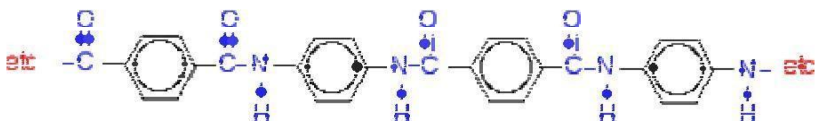
Teflon will burn when exposed to flame, but will not continue to burn when the flame is removed.

Polyamides(NYLONS)





Nylon-6



Kevlar(aromatic nylon)

Polyamides are a group of thermoplastic polymers containing amide groups in the main chain. They are popularly known as nylons. They may be aliphatic (nylon 66, nylon6 etc) or aromatic(Kevlar, nomex etc). Commercially important polyamides are PA-66, PA-6, PA-11, PA-12, PA-610, PA-612 etc. Polyamides have good strength and toughness with excellent fatigue resistance. However, they are prone to absorb moisture, ranging from 8 - 10% for PA6 and PA66 to 2 - 3% for PA11 and PA12 at saturation. Mechanical properties are affected by moisture, with toughness improving with the absorption of moisture whereas modulus is reduced. Polyamides are resistant to hydrocarbons, esters and glycols, but swell and dissolve in alcohols. They are also attacked by acids but generally stable to alkalis. PA6 and PA66 are mainly used in textiles, but they also find application where toughness is a requirement, for example, zip fastener teeth, gears, wheels and fan blades. PA11 is more flexible than PA66 and is typically used for petrol and hydraulic hose as well as powder coatings for metals. Strength and rigidity of these materials can be dramatically enhanced by the addition of glass or carbon fibre reinforcement; the level of saturation water absorption is also reduced. However, the designer needs to be aware of the anisotropic properties that can result in mouldings due to the flow and alignment of the reinforcing phase that occurs during moulding.

Polyamides are fairly readily attacked by strong acids, but are much more resistant to alkaline hydrolysis. Hydrolysis is faster at higher temperatures. Hydrolysis by water alone is so slow as to be completely unimportant. Kevlar is rather more resistant to hydrolysis than nylon .

Properties

Hard and tough thermoplastic, good abrasion resistance, low coefficient of friction, high tensile strength, good dimensional stability, low tendency to warp, smooth appearance of surface, average to high surface gloss, resistant to lubricants, engine fuels, grease etc, good resistance to coolants, refrigerants, paint solvent cleaners, resistant to aqueous solution of many inorganic chemicals.
Uses

Nylon fibres are used in textiles, fishing line and carpets. Nylon films is used for food packaging, offering toughness and low gas permeability, and coupled with its temperature resistance, for boil-in-the-bag food packaging. Moulding and extrusion compounds find many applications as replacements for metal parts, for instance in car engine components. Intake manifolds in nylon are tough, corrosion resistant, lighter and cheaper than aluminium (once tooling costs are covered) and offer better air flow due to a smooth internal bore instead of a rough cast one. Its self-lubricating properties make it useful for gears and bearings.

Electrical insulation, corrosion resistance and toughness make nylon a good choice for high load parts in electrical applications as insulators, switch housings and the ubiquitous cable ties. Another major application is for power tool housings. Glass reinforced polyamides are the material of choice for applications such as power tool housings. Transparent amorphous polyamides are available and find application in sterilisable medical components and sight glasses.

Polyethylene terephthalate(PET)

Polyethylene terephthalate (PET) and polybutylene terephthalate (PBT) are the most common thermoplastic polyesters. They are similar to PA6 and PA66 in many respects but with much lower water absorption. However, they are prone to hydrolysis, and prolonged contact with water at temperatures as low as 50°C has a detrimental effect on properties. Polyethylene terephthalate (PET) is often called just ―polyester‖. Polybutylene terephthalate (PBT) also a (thermoplastic) polyester, the most common resin system used in glass reinforced plastic (GRP) is also a polyester system.

General Properties

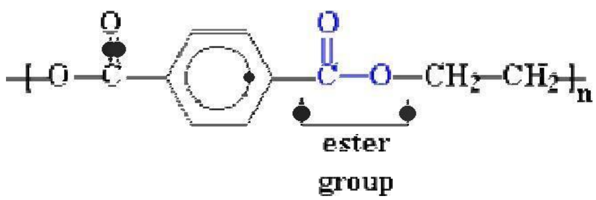
Polyethylene terephthalate (PET) is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis (which hydrolyse it). Its crystallinity varies from amorphous to fairly high crystalline. It can be highly transparent and colourless but thicker sections are usually opaque and off-white.

Polyethylene Terephthalate Films

It is widely known in the form of biaxially oriented and thermally stabilised films usually referred to by their main brand names Mylar, Melinex or Hostaphan. Strictly speaking, these names should be used only for this type of film whose properties are different from, and in several respects superior to, those of –ordinary‖ polyethylene terephthalate (PET) film(Mylar)

Applications

The –Mylar®-type‖ films are used for capacitators, graphics, film base and recording tapes etc. PET is also used for fibres for a very wide range of textile and industrial uses (Dacron®, Trevira®, Terylene®). Other applications include bottles and electrical components.



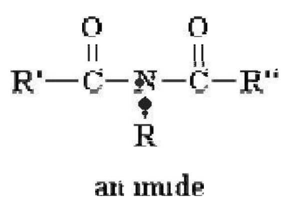
Its melting and glass transition temperature are 265 ^{o}C and 74 ^{o}C respectively. PET is used in the manufacture of biaxially oriented film and bottles, the latter suitable for carbonated drinks. The purpose of the orientation is to enhance rigidity, strength and toughness and also to improve barrier properties, which allows thinner bottles to be made.

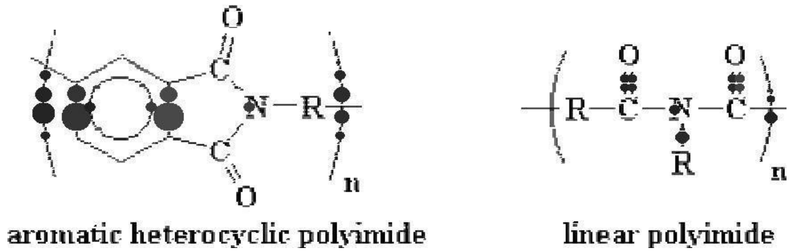
PBT displays a good combination of stiffness and toughness and can withstand continuous service at 120°C. The most important grades are those reinforced with glass. Applications for PBT include electrical connectors, pump components, and gears, as well as under bonnet and exterior parts for cars.

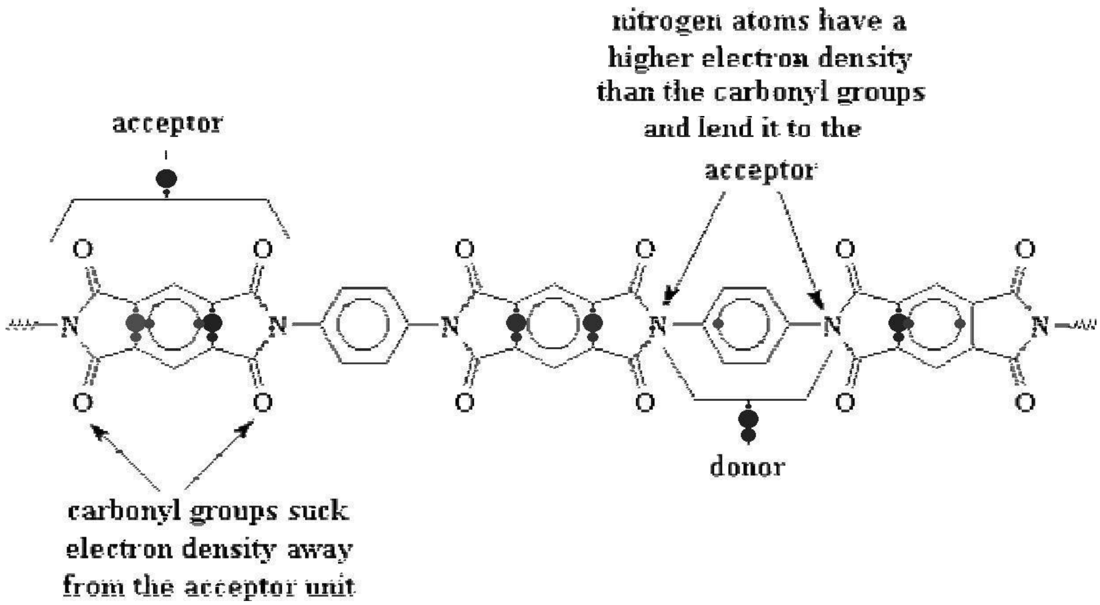
Thermoplastic Polyimides (PI, PAI, PEI)

Polyimide(Vespel-Dupont)(PI)

Polymeric compounds containing multiple imides, ie, NH groups derived from ammonia (NH3) by replacement of two hydrogen atoms by metals or equivalents of acid groups.







Polyimides (PI) are noted for their high temperature performance, retaining their mechanical properties to 250°C. They exhibit low flammability and smoke emission characteristics and offer the lowest minimum service temperature of thermoplastics. They are relatively expensive and can be difficult to process. Thermoplastic polyimide requires high temperatures and pressures and is usually processed by autoclave or compression moulding. They are susceptible to attack by halogenated solvents

Polyimides are a very interesting group of incredibly strong and astoundingly heat and chemical resistant polymers. Their strength and heat and chemical resistance are so great that these materials often replace glass and metals, such as steel, in many demanding industrial applications. Polyimides are even used in many everyday applications. They are used for the struts and chassis in some cars as well as some parts under-the-hood because they can withstand the intense heat and corrosive lubricants, fuels, and coolants cars require. They are also used in the construction of many appliances as well as microwave cookware and food packaging because of their thermal stability, resistance to oils, greases, and fats and their transparency to microwave radiation. They can also be used in circuit boards, insulation, fibers for protective clothing, composites, and adhesives. These polymers have excellent resistant to oxidative degradation, chemicals, strong bases and high energy radiation. It possesses good flame and abrasion resistances. But unfortunately these polymers cannot be moulded by conventional thermoplastic techniques. They are used as wire enamels, insulating varnishes, as coatings for glass cloth etc. Polyimide foams have been used for sound deadening of jet engines. They are used in space craft construction, rockets and weapons technology.

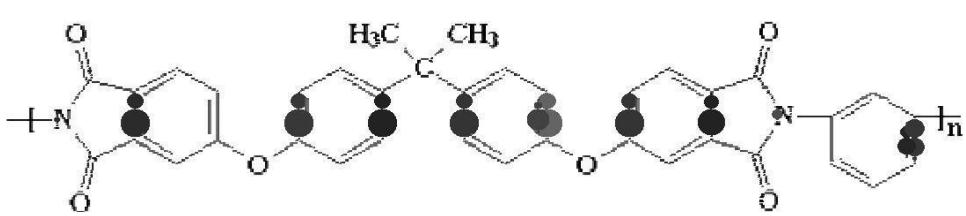
These polymers consists predominantly of ring structures and hence possesses high softening point. They are used in the manufacture of seals, gaskets, piston rings and as a binder in the diamond grinding wheels. Glass and carbon fibre reinforced polyimides are used in aircraft industry. It is also used in soldering and welding equipments. Kapton is a polyimide film made from pyromelltic anhydride and aromatic ether amine. Polyesterimides and polybismaleinimides are modified polyimides.

Polyamide- imide (PAI)

Polyamide-imides are amorphous thermoplastic materials with excellent mechanical properties, especially at elevated temperatures. Trimellitic anhydrides react with aromatic diamines to produce polyamide -imides. Polyamide-imides are applied in demanding engineering applications. Solvay Advanced Polymer's Torlon® is a well-known example of a polyamide-imide engineering resin that is also used to molded, extruded or machined plastic parts or shape stock.

Polyamide-imides are thermoplastic amorphous polymers which enjoy exceptional mechanical, thermal and chemical resistant properties. These properties put polyamide-imides at the top of the price and performance pyramid. Other high performance polymers in this same realm are polyetheretherketones and polyimides. Polyamide-imides hold, as the name suggests, a positive synergy of properties from both polyamides and polyimides, such as high strength, melt processabilty, exceptional high heat capability, and broad chemical resistance. Polyamide-imide polymers can be processed into a wide variety of forms, -from injection or compression molded parts and ingots, -to coatings, films, fibers and adhesives. Properties approaching those of Polyimide, yet melt processable. Highest strength of any unreinforced thermoplastic. Good wear resistance. Low smoke emission. Good radiation resistance.

Polyetherimides(PEI)



Polyetherimides are thermally stable at high temperatures and exhibit good optical properties making them useful in high temperature processing applications, in the fabrication of optoelectronics devices, and in optical applications. Polyetherimides (PEI) are amorphous, high performance thermoplastics with a continuous use temperature of around 170° C. PEI resins can also be melt processed using typical equipment for high volume production.

The strength, creep and impact properties of PEIs make them ideal for under bonnet components. They are also used in high temperature switchgear and electrical connectors. A number of medical equipment components are manufactured using PEIs, taking advantage of their excellent resistance to repeated sterilisation using steam, autoclave, gamma radiation or ethylene oxide. Microwave cookware is another application. They exhibit very high tensile strength without reinforcement, high glass transition(215 ^{o}C), deflection(200 ^{o}C) and softening (219 ^{o}C) temperatures. PEI are competitive with PAI, polysulphones, polycarbonates, polyphenylene sulphides etc. It is used in microwave equipment, printed circuit boards, wire insulation etc.

Polyphenylene Oxide (PP0)

PPO is prepared by oxidative coupling of phenylene oxide monomer.

PPO is a high strength, tough and heat resistant polymer, but in the unmodified state it is extremely difficult to process. It is also relatively expensive. Fortunately, it is miscible with polystyrene, and the resulting amorphous blends are easily processed and cheaper than PPO, with little loss in mechanical properties. Stiffness and strength are approximately 50% higher than high impact ABS, with similar creep behaviour. Modified PPO grades are also self extinguishing when ignited. Resistance to solvents is poor, a characteristic of styrene-based polymers. As well as glass fibre reinforced grades, these materials are available in structural foam grades.

Although unmodified polyphenylene oxide (PPO) is a semi-crystalline thermoplastic, all commercially available grades are blended with (high impact) polystyrene in order to improve melt processability. A wide range of proportions, together with the use of other modifiers, results in a good range of grades with differing properties tailored to the needs of individual end-uses. These blends are amorphous, opaque, pale grey engineering thermoplastics with a balance of properties (and cost) not unlike nylon but without nylon‘s high water absorption and consequent dimensional instability. However they have poor fatigue characteristics and poor solvent, though good hydrolysis, resistance.

Thermal properties

PPO has a higher heat deformation resistance than many general purpose thermoplastics at a lower price than the more expensive 'engineering thermoplastics'. One of the main reasons for using the PPO blends is the outstanding dimensional stability at elevated temperatures and the broad temperature use range.

Fire behavior

Conventional grades are non-self-extinguishing but self-extinguishing grades (generally made by including phosphorous based additives) are available with a slightly lower heat distortion temperature and impact strength. As a general rule the materials are difficult to ignite and burn with a sooty luminous flame and a pungent odour but do not drip. The flammability ranges from UL 94 HB to V-0 depending on the grade tested.

Electrical properties

Moisture absorption is low over a wide range of humidity levels and therefore dielectric properties are excellent over a wide range of moisture and temperature conditions.

Machining

PPO blends can be machined without difficulty using standard machine conditions for polymers. The products are difficult to cut and machining needs to be slow (feeds and speeds).

Surface treatment

PPO blends can be painted with good coatability. A primer and a polyurethane type of paint is recommended.

Welding

PPO can be solvent welded using commercially available solvents and solvent solutions containing 1 to 7% PPO resin.

Bonding

PPO blends can be bonded using a wide range of commercially available adhesives including epoxy adhesives. Mechanical: Machine housings, pump housings and impellers

Applications

Consumer goods: Power tool housings, portable mixers, hairdryers.
Automotive: Instrument panels and seat backs, spoilers, wheel trims, external mirror housings. Electrical: Electrical terminal housings, cable connectors, bulb sockets, coil formers. Miscellaneous: Plastic parts in central heating systems.

Injection moulding

Typical melt temperatures of 250 to 300 ^{o}C are needed with a die head temperature of around 250 ^{o}C for injection moulding of PPO. The injection pressure should be 1000 to 1200 bar and the follow-up pressure should be 50 - 70% of the injection pressure. The back pressure should be set at 30 - 50 bar.
The mould temperature should be in the region of 80 to 105 ^{o}C. Injection speed is generally high but moulds with a long flow path should have adequate mould venting. Mould shrinkage is low (0.005 - 0.01 m/m).

Extrusion

PPO is relatively easy to extrude and can be processed on single or twin screw extruders and on vented or unvented extruders.

UREA-FARMALDEHYDE RESIN

It is a thermosetting resin prepared by heating urea and formaldehyde in the presence of mild alkalies, such as pyridine or ammonia. The urea and formaldehyde undergo a condensation reaction in which they combine to form a water-soluble polymer. This polymer is used to formulate adhesives and coating agents or is mixed.

Physical properties

Relative Vapour Density (air = 1): (at 20°C)1.07(Foe formaldehyde), Specific Gravity (20°C): 1.2–1.4, % Volatile by Volume: 25–50 (water), Solubility in Water (g/L @ 25°C) 50, Water dilutability (mls): 100 minimum, pH (25°C): 7.8–8.2, Viscosity @ 25°C (cps): 150–250, Solids content (%): 64–66. Values may be slightly different depending upon the specific grade of resin. Urea formaldehyde resin is a colourless to milky viscous liquid, with faint formaldehyde odour. It is soluble in water and alcohol. The free formaldehyde content of UF resins is less than 5 per cent and usually less than 0.5 per cent depending on grade.

End uses

It is most commonly used for commercially produced interior and exterior particle boards. UF polymers have proven to be very good adhesives for wooden materials. Due to their high reactivity and cost efficiency, they are the most popular binders for interior products. Adhesives based on formaldehyde are used as binders in the production of reconstituted wood panels such as particleboard, plywood, medium density fibreboard, laminated veneer lumber, finger joints and laminated beams. UF resins and other related formaldehyde resins are also used in resin impregnated decorative paper laminates, glass fibre insulation binders, foundry cores, pulp and papers processing aids, paper sizing, textile treatments, paints and enamels, and miscellaneous joinery applications.

PF RESIN (Phenol-formaldehyde resin, phenolic resin, Bakelite)

It is a thermoset resin prepared by condensation polymerization of phenol and formaldehyde. If acid catalyst is used the resin is called Novolac and if base catalyst is employed the resulting resin is termed as resol. PF is mainly used in mouldings and laminates. Phenolics have poor tracking resistance under conditions of high humidity.

Ceramics materials

Ceramic materials are inorganic, nonmetallic materials. Most ceramics are compounds between metallic and nonmetallic elements for which the interatomic bonds are either totally ionic or predominantly ionic but having some covalent character. The term ceramic comes from the Greek word keramikos, which means burnt stuff, indicating that desirable properties of these materials are normally achieved through a high-temperature heat treatment process called firing.

Ceramic materials are important in today's society. Consider the ceramic engine and what advantages it offers in terms of fuel economy, efficiency, weight savings and performance. Below are three gif's showing a montage of a prototype ceramic engine and some of the internal automotive components made from ceramics.

Aluminum Oxide, Al2O3

Alumina is the most cost effective and widely used material in the family of engineering ceramics. The raw materials from which this high performance technical grade ceramic is made are readily available and reasonably priced, resulting in good value for the cost in fabricated alumina shapes. With an excellent combination of properties and an attractive price, it is no surprise that fine grain technical grade alumina has a very wide range of applications.

Key Properties

Hard, wear-resistant

Excellent dielectric properties from DC to GHz frequencies

Resists strong acid and alkali attack at elevated temperatures Good thermal conductivity

Excellent size and shape capability

High strength and stiffness

Available in purity ranges from 94%, an easily metallizable composition, to 99.5% for the most demanding high temperature applications.

General Information

Aluminum oxide, commonly referred to as alumina, possesses strong ionic interatomic bonding giving rise to it‘s desirable material characteristics. It can exist in several crystalline phases which all revert to the most stable hexagonal alpha phase at elevated temperatures. This is the phase of particular interest for structural applications and the material available from Accuratus.

Alpha phase alumina is the strongest and stiffest of the oxide ceramics. Its high hardness, excellent dielectric properties, refractoriness and good thermal properties make it the material of choice for a wide range of applications.

High purity alumina is usable in both oxidizing and reducing atmospheres to 1925°C. Weight loss in vacuum ranges from 10^{-7} to 10^{-6} g/cm^2.sec over a temperature range of 1700° to 2000°C. It resists attack by all gases except wet fluorine and is resistant to all common reagents except hydrofluoric acid and phosphoric acid. Elevated temperature attack occurs in the presence of alkali metal vapors particularly at lower purity levels.

The composition of the ceramic body can be changed to enhance particular desirable material characteristics. An example would be additions of chrome oxide or manganese oxide to improve hardness and change color. Other additions can be made to improve the ease and consistency of metal films fired to the ceramic for subsequent brazed and soldered assembly.

Key Properties

The major properties of sintered silicon carbide of interest to the engineer or designer, are as follows:

High hardness (second only to diamond)

Low density 40% the density of steel – approximately the same as aluminium Low porosity

Good wear resistance in sliding and abrasive environments

Excellent corrosion resistance in most chemical environments

Low thermal expansion and high thermal conductivity leading to excellent thermal shock resistance.

Applications

Automotive Components and Seal Faces

Due to their greater resistance to both wear and thermal shock, sintered silicon carbide seal faces for automotive water pumps are replacing seal faces made of materials such as aluminium oxide. In many cases the material has proven more suitable in meeting the performance demands of U.S. and European vehicles – i.e. lasting the lifetime of the vehicle without leaking. These components are manufactured by conventional high volume pressing and injection moulding methods to meet the economic constraints of the application.

Armour

Sintered Silicon Carbide has demonstrated an excellent performance record as ceramic material in composite armour protection systems. The properties of sintered silicon carbide, such as its high hardness, compressive strength and elastic modulus, provide superior ballistic capability when confronted with high-velocity projectiles. The low specific density of the material makes it suitable in applications where weight requirements are critical.

Heat Exchanger Tubes

Sintered Silicon Carbide tubes are used in shell and tube heat exchangers in the chemical process industry. The tubes used in these applications are often over 4 m in length.

Mechanical Seals

Pumps must operate in an infinite variety of demanding environments. Sintered Silicon Carbide offers a high performance seal face material that has proven successful in such diverse pumping applications as chemical processing, refining, mining and pulp and paper processing. The material provides superior corrosion and abrasion resistance; shock resistance; and low sliding friction against a wide range of mating materials.

Bearings

For state-of-the-art magnetically driven pumps, sintered silicon carbide is particularly suited for thrust and journal bearing components. Excellent corrosion resistance provides optimum performance in many chemical environments. High thermal conductivity minimizes the likelihood of failure due to thermal shock, and its specific strength makes it safe to use at high rotational speeds. Bearing components are usually produced as tight tolerance precision ground parts.

Blast and Atomisation Nozzles

Sintered Silicon Carbide is probably the most popular ceramic alternative to tungsten carbide for blast nozzle applications. Typically providing long life (50% over WC) due to excellent wear and corrosion resistance. The low wear rate maintains the internal nozzle geometry and provides maximum blasting effectiveness, minimum compressor requirements and reduced downtime due to replacement. Sintered silicon carbide is also about one fifth the weight of Tungsten Carbide, so the blasting operation is also easier for the operator.

Process Industry Valve Applications

The outstanding corrosion resistance of sintered silicon carbide, particularly in acids, makes it an ideal candidate for valve and valve trim applications. Typical demanding applications such as slurry flashing, HF acid handling and rare earth processing use sintered silicon carbide valve components.

Paper Industry Applications
The excellent corrosion and wear resistance of sintered silicon carbide provides hard surfaces that can be machined to smooth, highly polished finishes. These finishes offer low coefficients of friction and compatibility with forming fabrics. Tiles, inserts and palm guides are available in finished and semi-finished forms.

Centrifuge Tiles and Wear Parts
Often used in applications where tungsten carbide and alumina fail to provide optimum lifetime performance.

Semiconductor Production
The benefit of using silicon carbide for semiconductor components includes; the thermal expansion match to silicon, the resistance to wear and chemical corrosion which leads to reduced maintenance and component recycling. The material is well suited as a structural material for low mass silicon wafer handling components and rigid, dimensionally stable platforms due to its lightness in weight and high elastic modulus. Typical applications include vacuum chucks, chemical mechanical polishing blocks, wafer carriers, and thermocouple protection tubes.

Fused Silica, SiO_2

Fused silica is a noncrystalline (glass) form of silicon dioxide (quartz, sand). Typical of glasses, it lacks long range order in its atomic structure. It's highly cross linked three dimensional structure gives rise to it's high use temperature and low thermal expansion coefficient.

.**Key Properties**

- Near zero thermal expansion
- Exceptionally good thermal shock resistance
- Very good chemical inertness
- Can be lapped and polished to fine finishes
- Low dielectric constant
- Low dielectric loss
- Good UV transparency

.
Typical Uses

- High temperature lamp envelopes
- Temperature insensitive optical component supports
- Lenses, mirrors in highly variable temperature regimes
- Microwave and millimeter wave components
- Aeronautical radar windows

Silicon Nitride, Si_3N_4

Silicon nitride is a man made compound synthesized through several different chemical reaction methods. Parts are pressed and sintered by well developed methods to produce a ceramic with a unique set of outstanding properties. The material is dark gray to black in color and can be polished to a very smooth reflective surface, giving parts with a striking appearance. High performance silicon nitride materials were developed for automotive engine wear parts, such as valves and cam followers and proven effective. The cost of the ceramic parts never dropped enough to make the ceramics feasible in engines and turbochargers. The very high quality bodies developed for these demanding high reliability applications are available today and can be used in many severe mechanical, thermal and wear applications.

.**Key Properties**

- High strength over a wide temperature range
- High fracture toughness
- High hardness
- Outstanding wear resistance, both impingement and frictional modes
- Good thermal shock resistance
- Good chemical resistance

.

Typical Uses

- Rotating bearing balls and rollers
- Cutting tools
- Engine moving parts — valves, turbocharger rotors
- Engine wear parts — cam followers, tappet shims
- Turbine blades, vanes, buckets
- Metal tube forming rolls and dies
- Precision shafts and axles in high wear environments
- Weld positioners

General Information

The material is an electrical insulator and is not wet by nonferrous alloys. Silicon nitride is a rather expensive material, but it's performance to cost benefit ratio is excellent in the applications where it can outperform the normally utilized materials with long life and very reliable low maintenance operation.

Zirconium Oxide, ZrO_2

Zirconia is an extremely refractory material. It offers chemical and corrosion inertness to temperatures well above the melting point of alumina. The material has low thermal conductivity. It is electrically conductive above 600°C and is used in oxygen sensor cells and as the susceptor (heater) in high temperature induction furnaces. With the attachment of platinum leads, nernst glowers used in spectrometers can be made as a light emitting filament which operates in air.

Key Properties

- Use temperatures up to 2400°C
- High density
- Low thermal conductivity (20% that of alumina)
- Chemical inertness
- Resistance to molten metals
- Ionic electrical conduction
- Wear resistance
- High fracture toughness
- High hardness

.

Typical Uses

- Precision ball valve balls and seats
- High density ball and pebble mill grinding media
- Rollers and guides for metal tube forming
- Thread and wire guides
- Hot metal extrusion dies
- Deep well down-hole valves and seats
- Powder compacting dies
- Marine pump seals and shaft guides
- Oxygen sensors
- High temperature induction furnace susceptors
- Fuel cell membranes
- Electric furnace heaters over 2000°C in oxidizing atmospheres

Sialon

Sialon, a fine grain nonporous technical grade engineering material, is a silicon nitride ceramic with a small percentage of aluminum oxide added.

Sialon is outstanding in nonferrous metal contact. It is highly thermal shock resistant, strong, and is not wet or corroded by aluminum, brass, bronze, and other common industrial metals.

Key Properties

- Excellent thermal shock resistance
- Not wetted or corroded by nonferrous metals
- High strength
- Good fracture toughness
- Good high temperature strength
- Low thermal expansion

.

Typical Uses

- Thermocouple protection tubes for nonferrous metal melting
- Immersion heater and burner tubes
- Degassing and injector tubes in nonferrous metals
- Metal feed tubes in aluminum die casting
- Welding and brazing fixtures and pins

Key Properties

SiAlONs exploit the following properties:

low density,

high strength

superior thermal shock resistance,

moderate wear resistance

fracture toughness,

mechanical fatigue and creep resistance,

oxidation resistance.

In pressureless sintered materials, the high temperature properties are limited by the glassy phases that form at grain boundaries during sintering. These materials are only suitable to long term use at temperatures of less than 1000°C.

Applications

Cutting Tools

Wear Components

Metal Forming Tools

Composite Materials

Man's evolution has been tied to his progress in materials. Yesterday it was the Stone, Bronze and Iron Ages. Today it is the Age of Composites. However, even in these earlier ages man experimented with and learned to use composite materials.

A composite material is a multiphase material, which is composed of at least two basic elements working together to produce material properties that are different to the properties of those elements on their own. In practice, most composites consist of a bulk material (the ‗matrix‘), and a reinforcement of some kind, added primarily to increase the strength and stiffness of the matrix. This reinforcement is usually in fibre form. Composites maintain an interface between components and act in concert to provide improved specific or synergistic characteristics not obtainable by any of the original components acting alone.

The definition will allow the inclusion of natural materials such as wood which consists of cellulose fibers bonded together with lignin and other carbohydrate constituents, as well as the silk fiber spun by a spider which is as strong as steel on a weight basis consisting of a gel core encased in a solid protein structure as composite materials.

Composites include:

(1) Fibrous (composed of fibers, and usually in a matrix),
(2) Laminar (layers of materials),
(3) Particulate (composed of particles or flakes, usually in a matrix),
(4) Hybrid (combinations of any of the above).

Today, the most common man-made composites can be divided into three main groups:

1. **Polymer Matrix Composites (PMC's)** – These are the most common and will be discussed here. Also known as FRP - Fibre Reinforced Polymers (or Plastics) – these materials use a polymer-based resin as the matrix, and a variety of fibres such as glass, carbon and aramid as the reinforcement.

2. **Metal Matrix Composites (MMC's)** - Increasingly found in the automotive industry, these materials use a metal such as aluminium as the matrix, and reinforce it with fibres such as silicon carbide.

3. **Ceramic Matrix Composites (CMC's)** - Used in very high temperature environments, these materials use a ceramic as the matrix and reinforce it with short fibres, or whiskers such as those made from silicon carbide and boron nitride.

It is when the resin systems are combined with reinforcing fibres such as glass, carbon and aramid that exceptional properties can be obtained. The resin matrix spreads the load applied to the composite between each of the individual fibres and also protects the fibres from damage caused by abrasion and impact. High strengths and stiffnesses, ease of moulding complex shapes, high environmental resistance all coupled with low densities, make the resultant composite superior to metals for many applications. Since PMC‘s combine a resin system and reinforcing fibres, the properties of the resulting composite material will combine something of the properties of the resin on its own with that of the fibres on their own, as surmised in Figure .4.1

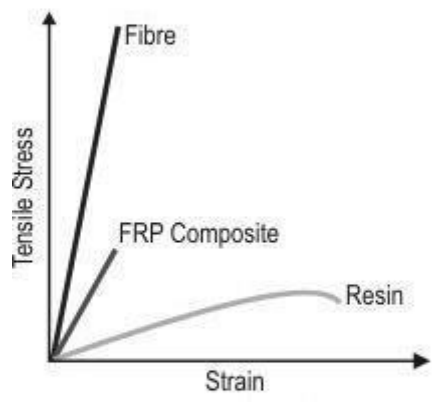


Figure 4.1 Illustrating the combined effect on Modulus of the addition of fibres to a resin matrix

Overall, the properties of the composite are determined by the ,
1) properties of the fibre,
2) properties of the resin,
3) ratio of fibre to resin in the composite (Fibre Volume Fraction (FVF)
4) geometry and orientation of the fibres in the composite

The ratio of the fibre to resin derives largely from the manufacturing process used to combine resin with fibre. However, it is also influenced by the type of resin system used, and the form in which the fibres are incorporated. In general, since the mechanical properties of fibres are much higher than those of resins, the higher the fibre volume fraction (FVF) the higher will be the mechanical properties of the resultant composite. In practice there are limits to this, since the fibres need to be fully coated in resin to be effective, and there will be an optimum packing of the generally circular cross-section fibres. In addition, the manufacturing process used to combine fibre with resin leads to varying amounts of imperfections and air inclusions. Typically, with a common hand lay-up process as widely used in the boat-building industry, a limit for FVF is approximately 30-40%. With the higher quality, more sophisticated and precise processes used in the aerospace industry, FVF‘s approaching 70% can be successfully obtained.

Loads on composites

There are four main direct loads that any material in a structure has to withstand: tension, compression, shear and flexure.

Tension

Figure 4.2 shows a tensile load applied to a composite. The response of a composite to tensile loads is very dependent on the tensile stiffness and strength properties of the reinforcement fibres, since these are far higher than the resin system on its own.

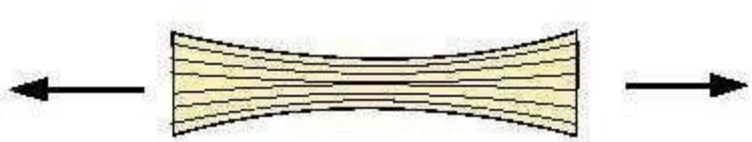


Figure 4.2 Tensile load applied to a composite body

Compression

Figure 4.3 shows a composite under a compressive load. Here, the adhesive and stiffness properties of the resin system are crucial, as it is the role of the resin to maintain the fibres as straight columns and to prevent them from buckling

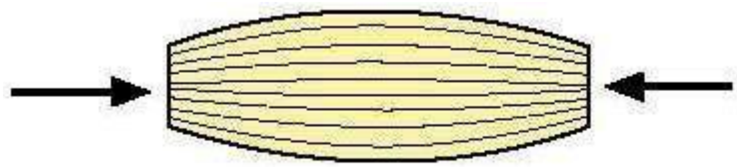


Figure 4.3 - Illustrates the compression load applied to a composite body.

Shear

Figure 4.4 shows a composite experiencing a shear load. This load is trying to slide adjacent layers of fibres over each other. Under shear loads the resin plays the major role, transferring the stresses across the composite. For the composite to perform well under shear loads the resin element must not only exhibit good mechanical properties but must also have high adhesion to the reinforcement fibre. The interlaminar shear strength (ILSS) of a composite is often used to indicate this property in a multiplayer composite (‗laminate').

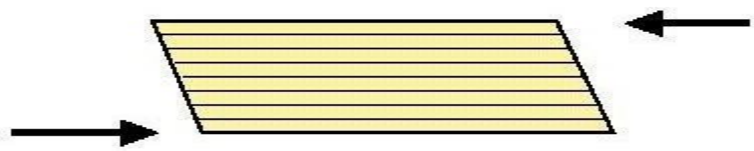


Figure 4.4 - Illustrates the shear load applied to a composite body.

Flexure

Flexural loads are really a combination of tensile, compression and shear loads. When loaded as shown (Figure 4.5), the upper face is put into compression, the lower face into tension and the central portion of the laminate experiences shear

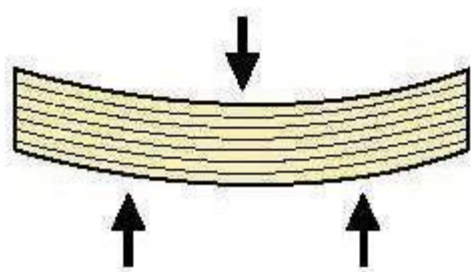


Figure 4.5 - Illustrates the loading due to flexure on a composite body.

A range of strength and stiffness (modulus) figures for different materials are given in Figure 4.6 to compare the spread of properties associated with composites and other structural materials

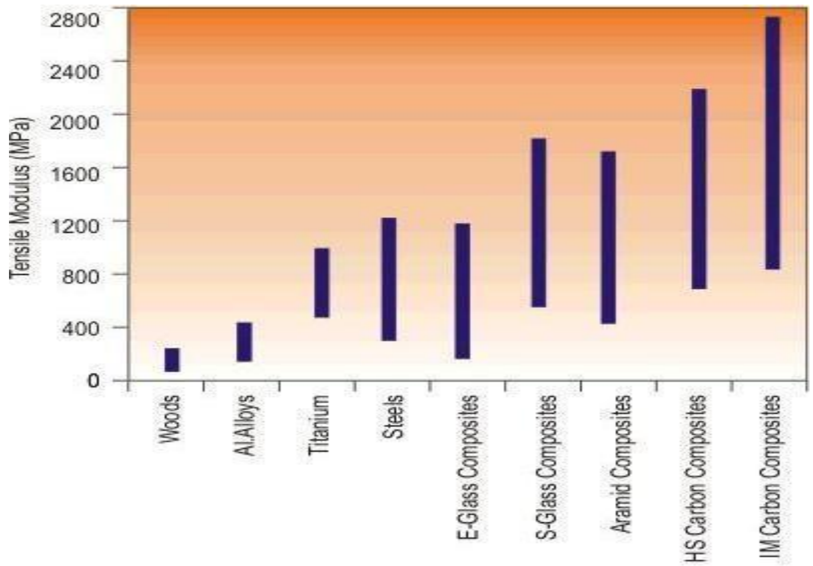


Figure 4.6 – Tensile Strength of Common Structural Materials

Particulate Composites

Particle-reinforced composites are candidate materials for a wide variety of aerospace and nonaerospace applications. The high costs and technical difficulties involved with the use of many fiber-reinforced composites often limit their use in many applications. Consequently, particulate composites have emerged as viable alternatives to conventional fiber-reinforced composites. Particulate composites can be processed to near net shape potentially reducing the manufacturing costs.

They are candidate materials where shock or impact properties are important. For example, particle-reinforced metal matrix composites have shown great potential for many automotive applications. Typically, these materials are aluminum matrix reinforced with SiC or TiC particles.

Reinforced concrete can also be thought of as a particle-reinforced composite. In situ ceramics can be modeled as particulate composites and are candidate materials for many high-temperature applications. The characterization of these materials is fundamental to their reliable use. It has been observed that the overall properties of these composites exhibit scatter because of the uncertainty in the constituent material properties, and fabrication-related parameters.

The observed scatter in the global composite behavior or "response" is usually caused by the existence of uncertainties in the basic or "primitive" variables. Primitive variables are properties or parameters that participate at the lowest or micromechanics level in defining a global or homogenized property. Volume fractions and individual constituent properties such as moduli, thermal expansion coefficients, thermal conductivities, and strengths are examples of primitive

variables.

They are assumed to be independent and have their own statistical distributions. Response variables are those that characterize such composite behavior as the composite moduli, thermal properties, and strengths.

UNIT - V

MECHANICAL PROPERTIES AND TESTING MECHANISMS

5.1 Plastic Deformation

When a material is stressed below its elastic limit, the resulting deformation or strain is temporary. Removal of stress results in a gradual return of the object to its original dimensions. When a material is stressed beyond its elastic limit, plastic or permanent deformation takes place, and it will not return to its original shape by the application of force alone. The ability of a metal to undergo plastic deformation is probably its most outstanding characteristic in comparison with other materials. All shaping operations such as stamping, pressing, spinning, rolling, forging, drawing, and extruding involve plastic deformation of metals. Various machining operations such as milling, turning, sawing, and punching also involve plastic deformation.

Plastic deformation may take place by :

- Slip
- Twinning
- Combination of slip and twinning

Deformation by Slip:

If a single crystal of a metal is stressed in tension beyond its elastic limit, it elongates slightly, a step appears on the surface indicating relative displacement of one part of the crystal with respect to the rest, and the elongation stops. Increasing the load will cause another step. It is as if neighboring thin sections of the crystal had slipped past one another like a sliding cards on a deck. Each successive elongation requires a higher stress and results in the appearance of another step, which is actually the intersection of a slip plane with the surface of the crystal. Progressive increase of the load eventually causes the material to fracture.

Slip occurs in directions in which the atoms are most closely packed, since this requires the least amount of energy.

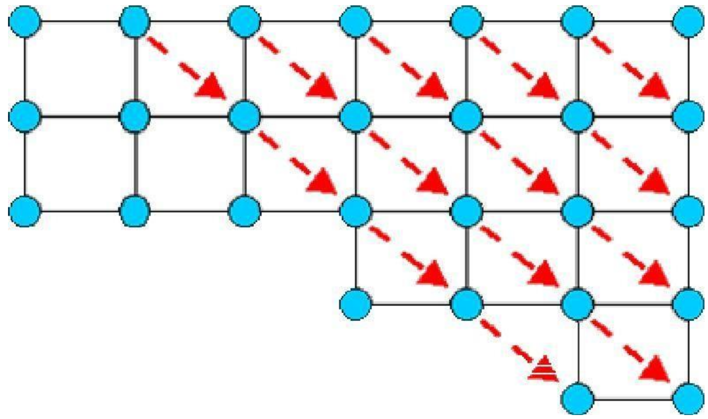


Figure 5.1 The effect of slip on the lattice structure.

Figure 1 shows that when the plastic deformation is due to slip, the atoms move a whole interatomic space (moving from one corner to another corner of the unit cell). This means that overall lattice structure remains the same. Slip is observed as thin lines under the microscopes and these lines can be removed by polishing.

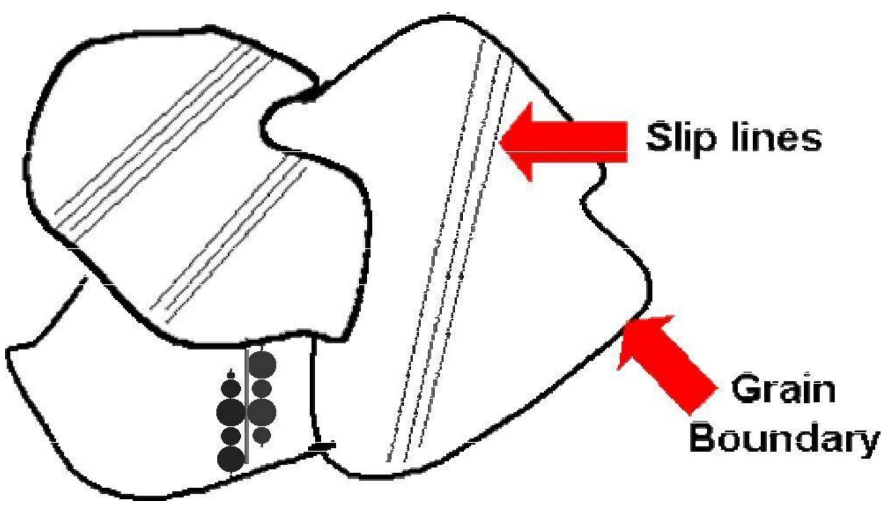


Figure 5.2 Slip appears as thin lines under the microscope.

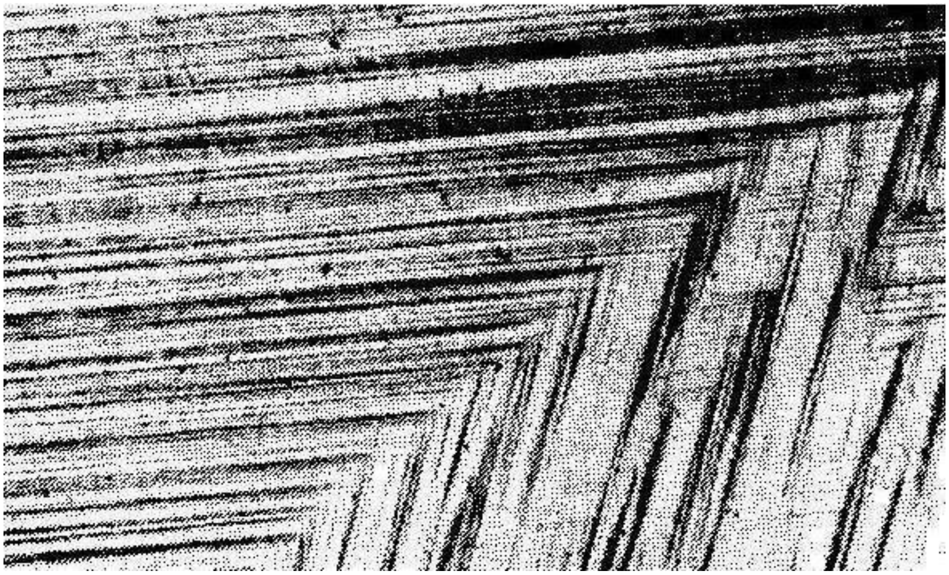


Figure 5.3 Slip lines in copper.

Deformation by Twinning:

When mechanical deformation is created by twinning, the lattice structure changes. The atoms move only a fraction of an interatomic space and this leads to a rearrangement of the lattice structure. Twinning is observed as wide bands under the microscope. These wide bands cannot be removed by polishing.

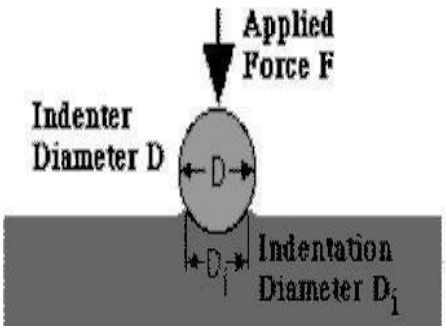
Two kinds of twins are of interest to the metallurgists:

1. Deformation or mechanical twins, most prevalent in close packed hexagonal metals (magnesium, zinc, iron with large amount of ferrite)

2. Annealing twins, most prevalent in F.C.C. (Face centered cubic) metals (aluminum, copper, brass, iron with austenite).These metals have been previously worked and heat treated. The twins are formed because of a change in the normal growth mechanism.

The Brinell Hardness Test

The Brinell hardness test method consists of indenting the test material with a 10 mm diameter hardened steel or carbide ball subjected to a load of 3000 kg. For softer materials the load can be reduced to 1500 kg or 500 kg to avoid excessive indentation. The full load is normally applied for 10 to 15 seconds in the case of iron and steel and for at least 30 seconds in the case of other metals. The diameter of the indentation left in the test material is measured with a low powered microscope. The Brinell harness number is calculated by dividing the load applied by the surface area of the indentation.



$$BHN = \frac{F}{\frac{\pi}{2} D \cdot \left(D - \sqrt{D^2 - D_i^2}\right)}$$

The diameter of the impression is the average of two readings at right angles and the use of a Brinell hardness number table can simplify the determination of the Brinell hardness. A well structured Brinell hardness number reveals the test conditions, and looks like this, "75 HB 10/500/30" which means that a Brinell Hardness of 75 was obtained using a 10mm diameter hardened steel with a 500 kilogram load applied for a period of 30 seconds. On tests of extremely hard metals a tungsten carbide ball is substituted for the steel ball. Compared to the other hardness test methods, the Brinell ball makes the deepest and widest indentation, so the test averages the hardness over a wider amount of material, which will more accurately account for multiple grain structures and any irregularities in the uniformity of the material. This method is the best for achieving the bulk or macro-hardness of a material, particularly those materials with heterogeneous structures.

VICKERS HARDNESS TEST

The Vickers hardness test was developed in 1924 by Smith and Sandland at Vickers Ltd as an alternative to the Brinell method to measure the hardness of materials.[1] The Vickers test is often easier to use than other hardness tests since

the required calculations are independent of the size of the indenter, and the indenter can be used for all materials irrespective of hardness. The basic principle, as with all common measures of hardness, is to observe the questioned material's ability to resist plastic deformation from a standard source. The Vickers test can be used for all metals and has one of the widest scales among hardness tests. The unit of hardness given by the test is known as the Vickers Pyramid Number (HV) or Diamond Pyramid Hardness (DPH). The hardness number can be converted into units of pascals, but should not be confused with a pressure, which also has units of pascals. The hardness number is determined by the load over the surface area of the indentation and not the area normal to the force, and is therefore not a pressure.

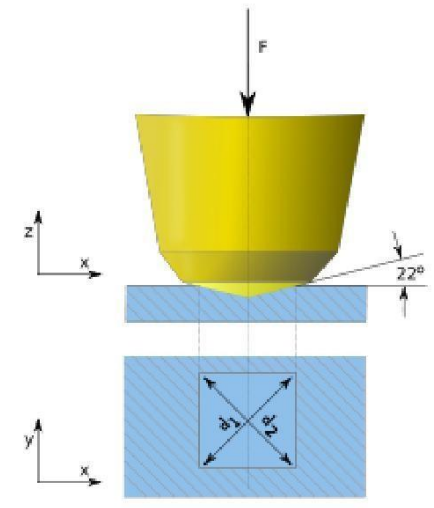
The hardness number is not really a true property of the material and is an empirical value that should be seen in conjunction with the experimental methods and hardness scale used. When doing the hardness tests the distance between indentations must be more than 2.5 indentation diameters apart to avoid interaction between the work-hardened regions.

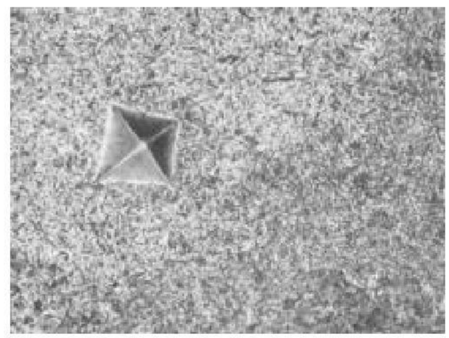
If HV is expressed in SI units the yield strength of the material can be approximated as:

$$\sigma_y = \frac{H_V}{c} \approx \frac{H_V}{3}$$

where c is a constant determined by geometrical factors usually ranging between 2 and 4.

[edit] Implementation





Vickers test scheme

An indentation left in case-hardened steel after a Vickers hardness test.

It was decided that the indenter shape should be capable of producing geometrically similar impressions, irrespective of size; the impression should have well-defined points of measurement; and the indenter should have high resistance to self-deformation. A diamond in the form of a square-based pyramid satisfied these conditions. It had been established that the ideal size of a Brinell impression was 3/8 of the ball diameter. As two tangents to the circle at the ends of a chord 3d/8 long intersect at 136°, it was decided to use this as the included angle of the indenter. The angle was varied experimentally and it was found that the hardness value obtained on a homogeneous piece of material remained constant, irrespective of load. Accordingly, loads of various magnitudes are applied to a flat surface, depending on the hardness of the material to be measured. The HV number is then determined by the ratio F/A where F is the force applied to the diamond in kilograms-force and A is the surface area of the resulting indentation in square millimetres. A can be determined by the formula

$$A = \frac{d^2}{2\sin(136^\circ/2)}$$

which can be approximated by evaluating the sine term to give

$$A \approx \frac{d^2}{1.8544}$$

where d is the average length of the diagonal left by the indenter. Hence,[3]

$$HV = \frac{F}{A} \approx \frac{1.8544F}{d^2}$$

where F is kgf and d is millimetres.

The corresponding units of HV are then kilograms-force per square millimetre (kgf/mm²). To calculate Vickers hardness number using SI units one needs to convert the force applied from kilogram-force to newtons by multiplying by 9.806 65 (standard gravity) and convert mm to m. To do the calculation directly, the following equation can be used:

$$HV = \frac{F}{A} \approx \frac{0.1891F}{d^2}$$

where F is newtons and d is millimetres.

Vickers hardness numbers are reported as xxxHVyy, e.g. 440HV30, or xxxHVyy/zz if duration of force differs from 10 s to 15 s, e.g. 440Hv30/20, where:
440 is the hardness number,

HV gives the hardness scale (Vickers), 30 indicates the load used in kg.

20 indicates the loading time if it differs from 10 s to 15 s

Vickers values are generally independent of the test force: they will come out the same for 500 gf and 50 kgf, as long as the force is at least 200 gf.[5]

Examples of HV values for various materials	
Material	Value
316L stainless steel	140HV30
347L stainless steel	180HV30
Carbon steel	55–120HV5
Iron	30–80HV5

ROCKWELL HARDNESS TEST

The differential depth hardness measurement was conceived in 1908 by a Viennese professor Paul Ludwik in his book Die Kegelprobe (crudely, "the cone trial").[3] The differential-depth method subtracted out the errors associated with the mechanical imperfections of the system, such as backlash and surface imperfections. The Brinell hardness test, invented in Sweden, was developed earlier—in 1900—but it was slow, not useful on fully hardened steel, and left too large an impression to be considered nondestructive.

The Rockwell hardness tester, a differential-depth machine, was co-invented by Connecticut natives Hugh M. Rockwell (1890–1957) and Stanley P. Rockwell (1886–1940). A patent was applied for on July 15, 1914.[4] The requirement for this tester was to quickly determine the effects of heat treatment on steel bearing races. The application was subsequently approved on February 11, 1919, and holds patent number #1,294,171. At the time of invention, both Hugh and Stanley Rockwell (not direct relations) worked for the New Departure Manufacturing Co. of Bristol, CT. New Departure was a major ball bearing manufacturer that, in 1916, became part of United Motors and, shortly thereafter, General Motors Corp. After leaving the Connecticut company, Stanley Rockwell, then in Syracuse, NY, applied for an improvement to the original invention on September 11, 1919, which was approved on November 18, 1924. The new tester holds patent #1,516,207.[5][6] Rockwell moved to West Hartford, CT, and made an additional improvement in 1921. Stanley collaborated with instrument manufacturer Charles H. Wilson of the Wilson-Mauelen Company in 1920 to commercialize his invention and develop standardized testing machines. Stanley started a heat-treating firm circa 1923, the Stanley P. Rockwell Company, which still exists in Hartford, CT. The later-named Wilson Mechanical Instrument Company has changed ownership over the years, and was most recently acquired by Instron Corp. in 1993.

Operation

The determination of the Rockwell hardness of a material involves the application of a minor load followed by a major load, and then noting the depth of penetration, vis a vis, hardness value directly from a dial, in which a harder material gives a higher number. The chief advantage of Rockwell hardness is its ability to display hardness values directly, thus obviating tedious calculations involved in other hardness measurement techniques.

It is typically used in engineering and metallurgy. Its commercial popularity arises from its speed, reliability, robustness, resolution and small area of indentation.

In order to get a reliable reading the thickness of the test-piece should be at least 10 times the depth of the indentation. Also, readings should be taken from a flat perpendicular surface, because round surfaces give lower readings. A correction factor can be used if the hardness must be measured on a round surface.

Stanley P. Rockwell invented the Rockwell hardness test. He was a metallurgist for a large ball bearing company and he wanted a fast non-destructive way to determine if the heat treatment process they were doing on the bearing races was successful. The only hardness tests he had available at time were Vickers, Brinell and Scleroscope. The Vickers test was too time consuming, Brinell indents were too big for his parts and the Scleroscope was difficult to use, especially on his small parts.

To satisfy his needs he invented the Rockwell test method. This simple sequence of test force application proved to be a major advance in the world of hardness testing. It enabled the user to perform an accurate hardness test on a variety of sized parts in just a few seconds.

Rockwell test methods are defined in the following standards: ASTM E18 Metals

Types of the Rockwell Test

There are two types of Rockwell tests:

Rockwell:
The minor load is 10 kgf, the major load is 60, 100, or 150 kgf.

Superficial Rockwell:
The minor load is 3 kgf and major loads are 15, 30, or 45 kgf.

In both tests, the indenter may be either a diamond cone or steel ball, depending upon the characteristics of the material being tested.

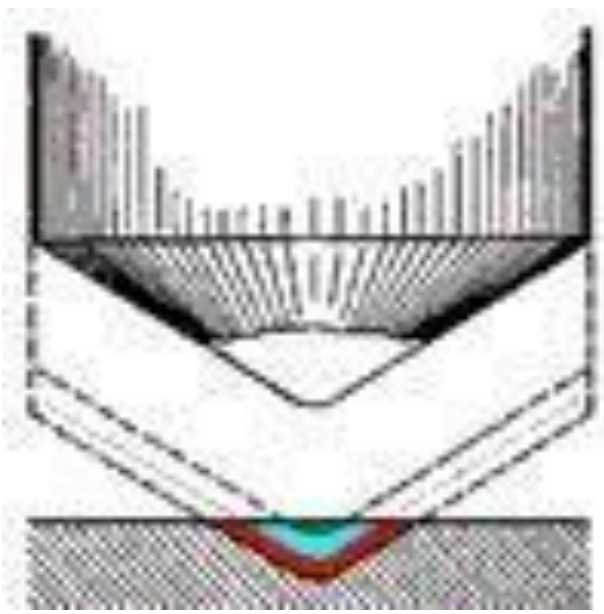
Rockwell Scales

Rockwell hardness values are expressed as a combination of a hardness number and a scale symbol representing the indenter and the minor and major loads. The hardness number is expressed by the symbol HR and the scale designation.

There are 30 different scales. The majority of applications are covered by the Rockwell C and B scales for testing steel, brass, and other metals. However, the increasing use of materials other than steel and brass as well as thin materials necessitates a basic knowledge of the factors that must be considered in choosing the correct scale to ensure an accurate Rockwell test. The choice is not only between the regular hardness test and superficial hardness test, with three different major loads for each, but also between the diamond indenter and the 1/16, 1/8, 1/4 and 1/2 in. diameter steel ball indenters.

If no specification exists or there is doubt about the suitability of the specified scale, an analysis should be made of the following factors that control scale selection:

Scale limitations
Principal of the Rockwell Test



The major load is applied for a specified time period (dwell time) beyond zero The major load is released leaving the minor load applied

The resulting Rockwell number represents the difference in depth from the zero reference position as a result of the application of the major load.

CREEP TEST

Method for determining creep or stress relaxation behavior. To determine creep properties, material is subjected to prolonged constant tension or compression loading at constant temperature. Deformation is recorded at specified time intervals and a creep vs. time diagram is plotted. Slope of curve at any point is creep rate. If failure occurs, it terminates test and time for rupture is recorded. If

specimen does not fracture within test period, creep recovery may be measured. To determine stress relaxation of material, specimen is deformed a given amount and decrease in stress over prolonged period of exposure at constant temperature is recorded

Viscoplasticity is a theory in continuum mechanics that describes the rate-dependent inelastic behavior of solids. Rate-dependence in this context means that the deformation of the material depends on the rate at which loads are applied. The inelastic behavior that is the subject of viscoplasticity is plastic deformation which means that the material undergoes unrecoverable deformations when a load level is reached. Rate-dependent plasticity is important for transient plasticity calculations. The main difference between rate-independent plastic and viscoplastic material models is that the latter exhibit not only permanent deformations after the application of loads but continue to undergo a creep flow as a function of time under the influence of the applied load.

The elastic response of viscoplastic materials can be represented in one-dimension by Hookean spring elements. Rate-dependence can be represented by nonlinear dashpot elements in a manner similar to viscoelasticity. Plasticity can be accounted for by adding sliding frictional elements as shown in Figure 1. In the figure E is the modulus of elasticity, λ is the viscosity parameter and N is a power-law type parameter that represents non-linear dashpot [$\sigma(d\varepsilon/dt) = \sigma = \lambda(d\varepsilon/dt)(1/N)$]. The sliding element can have a yield stress (σy) that is strain rate dependent, or even constant, as shown in Figure 1c.

Viscoplasticity is usually modeled in three-dimensions using overstress models of the Perzyna or Duvaut-Lions types. In these models, the stress is allowed to increase beyond the rate-independent yield surface upon application of a load and then allowed to relax back to the yield surface over time. The yield surface is usually assumed not to be rate-dependent in such models. An alternative approach is to add a strain rate dependence to the yield stress and use the techniques of rate independent plasticity to calculate the response of a material For metals and alloys, viscoplasticity is the macroscopic behavior caused by a mechanism linked to the movement of dislocations in grains, with superposed effects of inter-crystalline gliding. The mechanism usually becomes dominant at temperatures greater than approximately one third of the absolute melting temperature. However, certain alloys exhibit viscoplasticity at room temperature (300K). For polymers, wood, and bitumen, the theory of viscoplasticity is required to describe behavior beyond the limit of elasticity or viscoelasticity.

Creep is the tendency of a solid material to slowly move or deform permanently under constant stresses. Creep tests measure the strain response due to a constant stress as shown in Figure 3. The classical creep curve represents the evolution of strain as a function of time in a material subjected to uniaxial stress at a constant temperature. The creep test, for instance, is performed by applying a constant force/stress and analyzing the strain response of the system. In general, as shown in Figure 3b this curve usually shows three phases or periods of behavior

A primary creep stage, also known as transient creep, is the starting stage during which hardening of the material leads to a decrease in the rate of flow which is

initially very high. $(0 \leq \epsilon \leq \epsilon_1)$.

The secondary creep stage, also known as the steady state, is where the strain rate

is constant. $(\epsilon_1 \leq \epsilon \leq \epsilon_2)$.

A tertiary creep phase in which there is an increase in the strain rate up to the fracture strain. $(\boldsymbol{\varepsilon}_2 \leq \boldsymbol{\varepsilon} \leq \boldsymbol{\varepsilon}_R)$. [edit]

Relaxation test

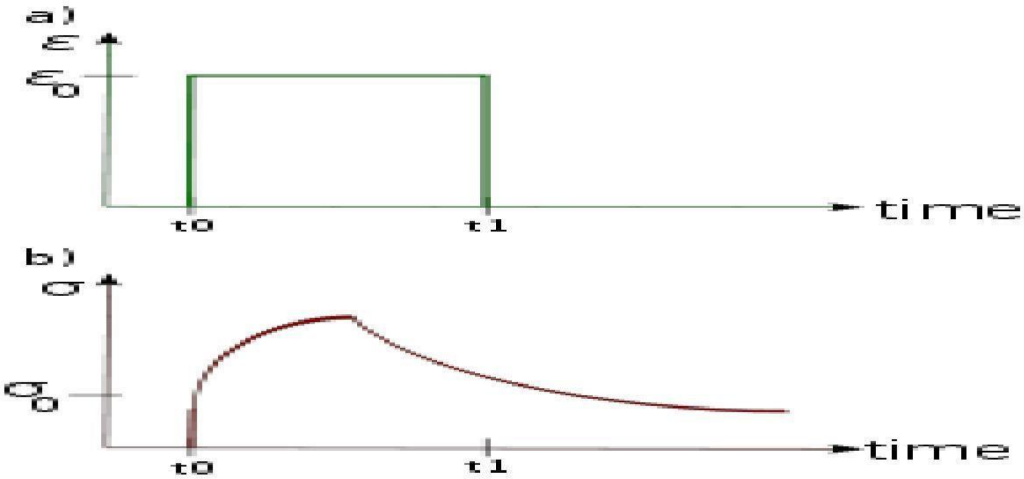


Figure 4. a) Applied strain in a relaxation test and b) induced stress as functions of time over a short period for a viscoplastic material.

As shown in Figure 4, the relaxation test is defined as the stress response due to a constant strain for a period of time. In viscoplastic materials, relaxation tests demonstrate the stress relaxation in uniaxial loading at a constant strain. In fact, these tests characterize the viscosity and can be used to determine the relation which exists between the stress and the rate of viscoplastic strain. The decompositon of strain rate is

$$\frac{\mathrm{d}\boldsymbol{\varepsilon}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{e}}}{\mathrm{d}t} + \frac{\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{vp}}}{\mathrm{d}t} .$$

The elastic part of the strain rate is given by

$$\frac{\mathrm{d}\boldsymbol{\varepsilon}_{\mathrm{e}}}{\mathrm{d}t} = \mathsf{E}^{-1}\,\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}t}$$

For the flat region of the strain-time curve, the total strain rate is zero. Hence we have,

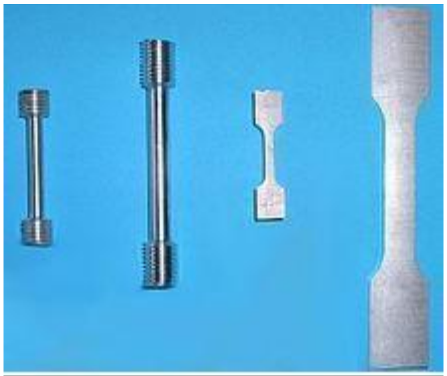
$$\frac{d\varepsilon_{vp}}{dt} = -E^{-1}\ \frac{d\sigma}{dt}$$

Therefore the relaxation curve can be used to determine rate of viscoplastic strain and hence the viscosity of the dashpot in a one-dimensional viscoplastic material model. The residual value that is reached when the stress has plateaued at the end of a relaxation test corresponds to the upper limit of elasticity. For some materials such as rock salt such an upper limit of elasticity occurs at a very small value of stress and relaxation tests can be continued for more than a year without any observable plateau in the stress.

It is important to note that relaxation tests are extremely difficult to perform because maintaining the condition $\frac{d\varepsilon}{dt} = 0$ in a test requires considerable delicacy

Tensile testing, also known as tension testing, is a fundamental materials science test in which a sample is subjected to uniaxial tension until failure. The results from the test are commonly used to select a material for an application, for quality control, and to predict how a material will react under other types of forces. Properties that are directly measured via a tensile test are ultimate tensile strength, maximum elongation and reduction in area. From these measurements the following properties can also be determined: Young's modulus, Poisson's ratio, yield strength, and strain-hardening characteristics.

Tensile specimen



Tensile specimens made from an aluminum alloy. The left two specimens have a round cross-section and threaded shoulders. The right two are flat specimen designed to be used with serrated grips.

A tensile specimen is a standardized sample cross-section. It has two shoulders and a gage section in between. The shoulders are large so they can be readily gripped, where as the gage section has a smaller cross-section so that the deformation and failure can occur in this area.

The shoulders of the test specimen can be manufactured in various ways to mate to various grips in the testing machine (see the image below). Each system has advantages and disadvantages; for example, shoulders designed for serrated grips are easy and cheap to manufacture, but the alignment of the specimen is dependent on the skill of the technician. On the other hand, a pinned grip assures good alignment. Threaded shoulders and grips also assure good alignment, but the technician must know to thread each shoulder into the grip at least one diameter's length, otherwise the threads can strip before the specimen fractures.

In large castings and forgings it is common to add extra material, which is designed to be removed from the casting so that test specimens can be made from it. These specimen not be exact representation of the whole workpiece because the grain structure may be different throughout. In smaller workpieces or when critical parts of the casting must be tested, a workpiece may be sacrificed to make the test specimens. For workpieces that are machined from bar stock, the test specimen can be made from the same piece as the bar stock.

The repeatability of a testing machine can be found by using special test specimens meticulously made to be as similar as possible.

The following tables give the test specimen dimensions and tolerances per standard ASTM E8.

A universal testing machine

The most common testing machine used in tensile testing is the universal testing machine. This type of machine has two crossheads; one is adjusted for the length of the specimen and the other is driven to apply tension to the test specimen. There are two types: hydraulic powered and electromagnetically powered machines.

The machine must have the proper capabilities for the test specimen being tested. There are three main parameters: force capacity, speed, and precision and accuracy. Force capacity refers to the fact that the machine must be able to generate enough force to fracture the specimen. The machine must be able to apply the force quickly or slowly enough to properly mimic the actual application. Finally, the machine must be able to accurately and precisely measure the gage length and forces applied; for instance, a large machine that is designed to measure long elongations may not work with a brittle material that experiences short elongations prior to fracturing.

Alignment of the test specimen in the testing machine is critical, because if the specimen is misaligned, either at an angle or offset to one side, the machine will exert a bending force on the specimen. This is especially bad for brittle materials, because it will dramatically skew the results. This situation can be minimized by using spherical seats or U-joints between the grips and the test machine. A misalignment is indicated when running the test if the initial portion of the stress-strain curve is curved and not linear.

The strain measurements are most commonly measured with an extensometer, but strain gauges are also frequently used on small test specimen or when Poisson's ratio is being measured. Newer test machines have digital time, force, and elongation measurement systems consisting of electronic sensors connected to a data collection device (often a computer) and software to manipulate and output the data. However, analog machines continue to meet and exceed ASTM, NIST, and ASM metal tensile testing accuracy requirements, continuing to be used today.[citation needed]

Process

The test process involves placing the test specimen in the testing machine and applying tension to it until it fractures. During the application of tension, the elongation of the gage section is recorded against the applied force. The data is manipulated so that it is not specific to the geometry of the test sample. The elongation measurement is used to calculate the engineering strain, ε, using the following equation:[4]

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

where ΔL is the change in gage length, L0 is the initial gage length, and L is the final length. The force measurement is used to calculate the engineering stress, σ, using the following equation:

$$\sigma = \frac{F_n}{A}$$

where F is the force and A is the cross-section of the gage section. The machine does these calculations as the force is increase so that the data points can be graphed into a stress-strain curve.

Standards
Metals

ASTM E8 Standard Test Methods for Tension Testing of Metallic Materials ISO 6892 Metallic materials—Tensile testing at ambient temperature
JIS Z2241 Method of tensile test for metallic materials

COMPRESSION TEST

The box compression test (bct) measures the compressive strength of boxes made of corrugated fiberboard as well as wooden boxes and crates. It provides a plot of deformation vs compressive force. Containers other than boxes can also be subjected to compression testing: drum, pail, etc.

Compression tester for shipping containers

A BCT plot is a measure of the strength of a shipping container and is measured in kN .deflection or deformation is measured in mm or inches.

Test Procedures

A common method of conducting the test, as described in several published standard test methods, is to compress a box at a constant rate of 1/2 inch (12.5 mm) per minute between two rigid platens. The platens can be fixed so that they remain parallel or one can be pivoted or "floating". The test can be conducted on empty or filled boxes, with or without a box closure. Conditioning to standard temperature and humidity is important.

The dynamic loads have some relationship with expected field loads.: often factors of 4 or 5 are used to estimate the allowable working load on boxes.

A test can also be conducted with platens that are not mechanically driven but are free to move with a fixed mass (or fixed force) loaded upon them. The results of static load testing can be:

The time to failure

The time to a critical deformation

The ability of a box to protect the contents from compression damage etc

As with any laboratory testing field validation is necessary to determine suitability.

Factors potentially affecting test results

Size and construction of the specific shipping container under test Grade and flute structure of corrugated fiberboard

moisture content of the corrugated board (based on relative humidity) Orientation of the box during the test

Inner supports, if used during testing (wood, corrugated board, cushioning) Contents (when box is tested with contents)

Box closure

Whether the compression machine has "fixed" or "floating" (swiveled) platens. Previous handling or testing of box.

Estimations

Corrugated fiberboard can be evaluated by many material test methods including an Edge Crush Test (ECT). There have been efforts to estimate the peak compression strength of a box (usually empty, regular singelwall slotted containers, top-to-bottom) based on various board properties. Some have involved finite element analysis. One of the commonly referenced empirical estimations was puplished by McKee in 1963. This used the board ECT, the MD and CD flexural stiffness, the box perimeter, and the box depth. Simplifications have used a formula involving the board ECT, the board thickness, and the box perimeter. Most estimations do not relate well to other box orientations, box styles, or to filled boxes. Physical testing of filled and closed boxes remains necessary.

Relevant Standards

ASTM Standard D642 Test Method for Determining Compressive Resistance of Shipping Containers, Components, and Unit Loads.

ASTM Standard D4577 Test Method for Compression Resistance of a Container Under Constant Load

ASTM Standard D7030 Test Method for Short Term Creep Performance of Corrrugated Fiberboard Containers Under Constant Load Using a Compression Test Machine

German Standard DIN 55440-1 Packaging Test; compression test; test with a constant conveyance-speed

ISO 12048 Packaging -- Complete, filled transport packages -- Compression etc.

Department of Mechanical Engineering

Lecture Notes

Subject Code : ME3393

Subject Name: MANUFACTURING PROCESSES

Sem/Year : 03/II

Regulation : 2021

ME3393 **MANUFACTURING PROCESSES**

L	T	P	C
3	0	0	3

COURSE OBJECTIVES:

1. To illustrate the working principles of various metal casting processes.
2. To learn and apply the working principles of various metal joining processes.
3. To analyse the working principles of bulk deformation of metals.
4. To learn the working principles of sheet metal forming process.
5. To study and practice the working principles of plastics molding.

UNIT – I METAL CASTING PROCESSES **9**

Sand Casting – Sand Mould – Type of patterns - Pattern Materials – Pattern allowances – Molding sand Properties and testing – Cores –Types and applications – Molding machines – Types and applications– Melting furnaces – Principle of special casting processes- Shell, investment – Ceramic mould – Pressure die casting – low pressure, gravity- Tilt pouring, high pressure die casting- Centrifugal Casting – CO2 casting — Defects in Sand casting process-remedies

UNIT II METAL JOINING PROCESSES **9**

Fusion welding processes – Oxy fuel welding – Filler and Flux materials—Arc welding, Electrodes, Coating and specifications – Gas Tungsten arc welding –Gas metal arc welding - Submerged arc welding – Electro slag welding– Plasma arc welding — Resistance welding Processes -Electron beam welding –Laser beam Welding Friction welding – Friction stir welding – Diffusion welding – Thermit Welding, Weld defects – inspection &remedies – Brazing - soldering – Adhesive bonding.

UNIT III BULK DEFORMATION PROCESSES **9**

Hot working and cold working of metals – Forging processes – Open, impression and closed die forging – cold forging- Characteristics of the processes – Typical forging operations – rolling of metals – Types of Rolling – Flat strip rolling – shape rolling operations – Defects in rolled parts – Principle of rod and wire drawing – Tube drawing – Principles of Extrusion – Types – Hot and Cold extrusion. Introduction to shaping operations.

UNIT IV SHEET METAL PROCESSES **9**

Sheet metal characteristics – Typical shearing, bending and drawing operations – Stretch forming operations – Formability of sheet metal – Test methods –special forming processes - Working principle and applications – Hydro forming – Rubber pad forming – Metal spinning – Introduction of Explosive forming, magnetic pulse forming, peen forming, Super plastic forming – Micro forming – Incremental forming.

UNIT V MANUFACTURE OF PLASTIC COMPONENTS **9**

Types and characteristics of plastics – Molding of thermoplastics & Thermosetting polymers– working principles and typical applications – injection molding – Plunger and screw machines – Compression molding, Transfer Molding – Typical industrial applications – introduction to blow molding – Rotational molding – Film blowing – Extrusion – Thermoforming – Bonding of Thermoplastics- duff moulding.

TOTAL :45 PERIODS

OUTCOMES:

At the end of the course the students would be able to

1. Explain the principle of different metal casting processes.
2. Describe the various metal joining processes.
3. Illustrate the different bulk deformation processes.
4. Apply the various sheet metal forming process.
5. Apply suitable molding technique for manufacturing of plastics components.

TEXT BOOKS:

1. Kalpakjian. S, "Manufacturing Engineering and Technology", Pearson Education India,4th Edition, 2013
2. P.N.Rao Manufacturing Technology Volume 1 Mc Grawhill Education 5th edition,2018.

Unit-I

METAL CASTING PROCESSES

1. Sand casting, sand mould, Types of patterns, Pattern materials, pattern allowances.
2. Moulding sand properties & testing - cores, Types & Applications
3. Moulding machines - Types & Applications.
4. Melting furnaces - Principles of special casting proc.
   - 1. Shell, investment
   - 2. Ceramic mould
   - 3. Pressure die casting
   - 4. Low pressure, gravity
   - 5. Tilt pouring
   - 6. High pressure die casting.
   - 7. centrifugal casting
   - 8. CO_2 casting
5. Defects in sand casting processes - remedies.

UNIT-I - METAL CASTING PROCESSES.

* In engineering industries, most of the components are made by ferrous and non-ferrous metals such as iron, steel, aluminium etc.
* Complicated shapes may not be produced with conventional machining processes.

SAND CASTING

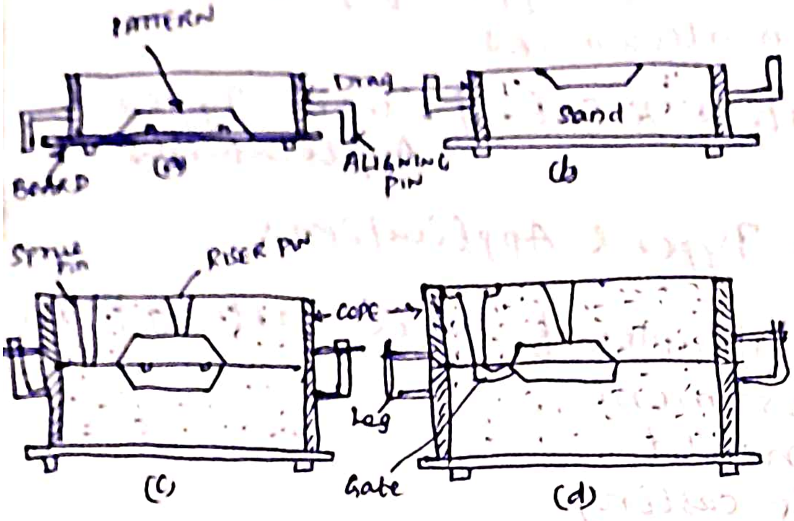
* making components of complicated shapes in large quantities.
* producing metal parts by pouring molten metal into the mould cavity of the required shape & allowing the metal to solidify.
* Solidified metal pieces called casting.
* Plant ⇒ foundary. (having all tools & equipments)

SAND MOULD

The process of making a cavity similar to the product required in sand.

TYPES 1. Green sand mould. 2. Dry sand mould 3. Loam mould

1. Green sand mould.



green sand
↳ Silica sand, clay, water & additives
↳ 10 to 15% of clay
4 to 6% water
remaining sand.
↳ It is porous.
↳ molten metal poured immediately after the mould is prepared.
↳ making small & medium size casting

↳ sand mould (Bench mould)
① Here two piece split pattern is used. One half of the split pattern is placed at the centre of the moulding board in fig(a)
② In fig.(b) The Drag is tilted upside down as shown in fig(b)
③ The cope box is placed correctly in position on the drag using dowel pin as shown in fig(c)
④ Sprue pin, riser pin are removed, pouring basin formed. Pattern are removed using draw spike. gate is cut in the fig(d). Finally the cope & drag assembled. The mould is ready for pouring.

Dry sand mould.

The step-by-step procedure of making dry sand mould is similar to green sand mould.
The only difference is that the mould is dried after it is made by green sand.
The drying may be done oxyacetylene flame for large mould oven for small mould.

Application: Engine cylinders, Engine blocks, machine block & mill rolls.

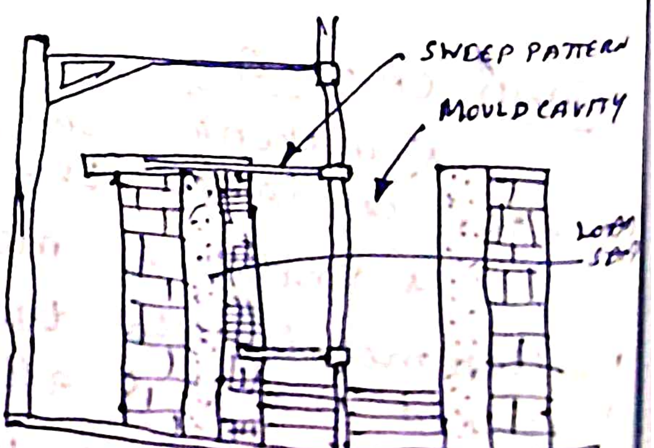
LOAM MOULDING

for very large casting, moulding boxes, patterns ~~and~~ cannot be used.

LOAM SAND MIXTURE

Silica sand, water, graphite powder, more amount of clay.
Loam sand is used.
Pit moulding method

Application
M/c parts, Large cylinders, Large bells, wheels, kettles, gear wheels, Pans etc.



Types of patterns

Pattern is one of the important tools used for making cavities in the mould.
In these cavities, molten metal is poured to produce a casting.
Pattern is slightly larger than the desired casting due to various allowances which are required.

Types of pattern

1. Solid or single piece pattern, 2. Split pattern. 3. Loose piece pattern. 4. Match plate pattern, 5. Sweep pattern. 6. Skeleton pattern. 7. Segmental pattern. 8. Shell pattern.

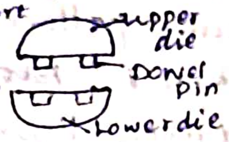
(a) Solid or single piece pattern

* Solid piece without joints, partings, or loose piece.
* No allowances - exact size

Solid or onepiece pattern.

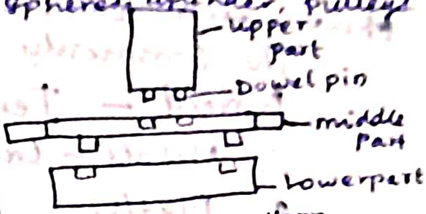
Split pattern

* Complex geometry cannot be removed from mould
* Two part
* Dowel pin
* Three piece pattern.



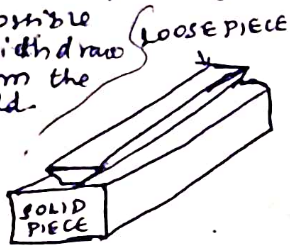
Split pattern.

* making symmetrical shaped castings such as spheres, cylinder, pulleys



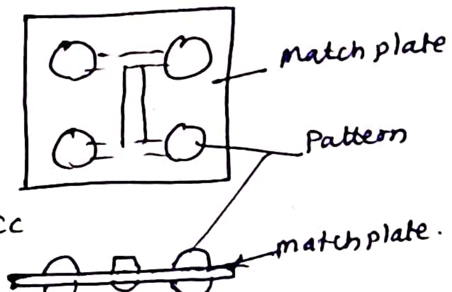
Three piece pattern.

* If a pattern is made from a single piece having projections or back which lie above or below the parting plane, it is impossible to withdraw it from the mould.



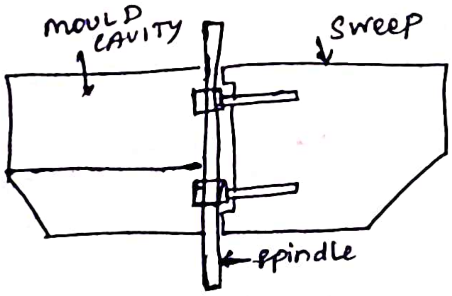
Loose piece pattern.

* A pattern which is made into two halves mounted on both side of plate
* made of Aluminium or wood.
* many patterns can be mounted on the same match plate.
* runner, gates are required.
* Piston rings of IC engines.



match plate pattern

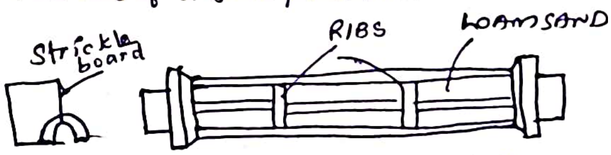
* To generate the surfaces of revolution such as cylinder, cone & spheres in large casting.



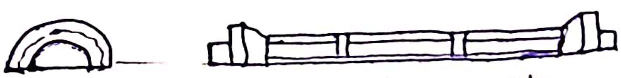
Sweep pattern.

SKELETON PATTERN

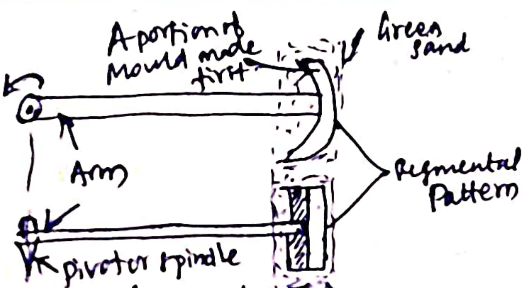
* For large casting of a simple shape.
* used instead of a full pattern.



* ribbed frame of desired casting.
* made of wooden strips. with a lot of openings and ribbed to end supports.



* filled with loam sand. Ramming done inside



Segmental Pattern.

* Segment of whole pattern.
* or called part pattern.
* used for making circular moulds.

App : RING, wheel rim, gear blank etc.

* hollow pattern. outer shape used for making mould

Shell pattern.

* Also called block pattern
* made of metal
* drainage fittings & pipe work

Pattern allowances

Pattern are not made into the exact size of the castings to be produced.

Patterns are made slightly larger than the required castings.

This extra size given on the pattern is called pattern allowance.

The pattern allowances are given for the purpose of compensating the metal shrinkage to provide extra metal which is to be removed in machining, to avoid metal distortion, for easy withdrawal of pattern from mould and for rapping.

If allowances are not given on the pattern, the casting will become smaller than the required size.

The various types of allowances normally provided on the pattern are.

(1) Shrinkage allowances. (2) Machining or finish allowances (3) Draft or taper allowances. (4) Distortion or Camber allowances. (5) Rapping or shake allowances.

1) Shrinkage allowance

Material	Shrinkage allowances
CI	10.4 mm/m
Al	17 mm/m
Brass	15.3 mm/m
Steel	20.8 mm/m
Zinc, lead	25 mm/m.

Shrinkage Allowances.

* The metals shrink on solidification and contract further on cooling to room temp.
* To compensate, the pattern made larger than required casting
* This extra size ⇒ shrinkage allowance

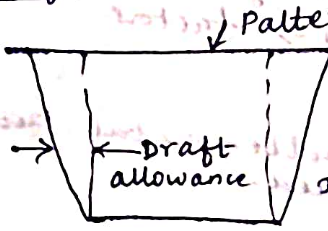
2. Machining Allowance

Materials	Machining Allowances	
	0-300mm	0-600mm
CI	2.5mm	4.0mm
Al	1.6mm	3.2mm
Bronze	1.6mm	3.2mm
Brass	1.6mm	3.2mm
Cast steel	3mm	4.5mm

Typical Machining Allowances.

* All casting to be machined to get the required surface finish
* During machining some of metal removed, for this purpose made larger ⇒ machining Allowance.

Draft or Taper allowances

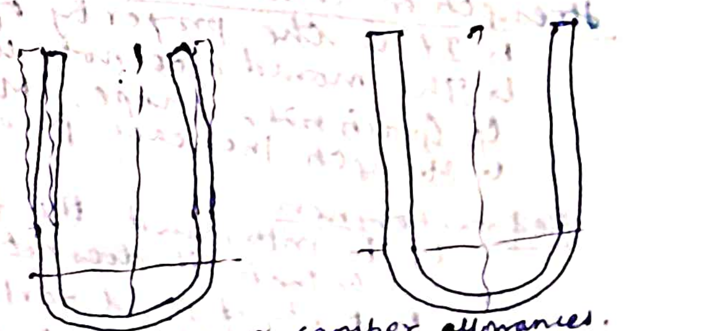


Draft or taper allowances.

If the vertical faces of pattern are perpendicular to the parting line, the edges of the mould may be demaged when the pattern is removed from the sand.

Hence the vertical faces are made into taper for easy removal of pattern.

The slight taper ⇒ vertical side ⇒ Pattern.



Distortion or camber allowances.

The casting may distort or wrap during cooling, if it is an irregular shape, that long casting surfaces U & V shape.

* U shape ⇒ distort ⇒ leg become divergent instead of ~~bending~~ being parallel

Moulding sand properties

A good castings can be produced only with the use of good quality moulding sand.
For this the moulding properties of the sand have to be controlled.

These properties.
- (1) Porosity or Permeability
- (2) Plasticity or flowability
- (3) Adhesiveness
- (4) Strength or cohesiveness
- (5) Refractoriness
- (6) Collapsibility.

Porosity or Permeability
- Permeability ⇒ measure of moulding sand by which the sand allows the steam and gases to pass through it.
- Defects ⇒ blow-holes, surface blows, gas holes.
- To escape the remaining gases.
- Coarse grain exhibit more permeability.

Plasticity or flowability
- to get compacted to a uniform density.
- Flowability ⇒ It assists the moulding sand to flow and pack all around the pattern and take up the required shape.

Thus, it gives the shape of the pattern & retains the shape after removing the pattern.
Adding clay & water to silica sand.

Adhesiveness
- it sticks or adheres to another body.
- it should cling or sticks to the sides of moulding box.
- so it doesnot fall out when the flasks are lifted and turned over.
- Type of binder used in sandmix, clay, moisture.

Strength or cohesiveness
- It is the property of which it sticks together.
- The mould doesnot collapse or partially damaged.
- Grain size, shape, Moisture, Density ⇒ factors.
- strength increase porosity decrease.

Refractoriness
- to withstand the temp. of the molten metal poured so that it does not cracked & fused.
- Purity of sand particles & size.
- Poor ⇒ rough surface finish occur.

Collapsibility
- to decrease in volume to some extend under the compressive forces developed by shrinkage during freezing cooling.

moulding sand properties & testing

Moulding sands have to be in correct size to ensure the proper sand quality as per the required level for accurate & good surface finished castings.

Generally the moulding sand contains silica sand grains, clay content and moisture content

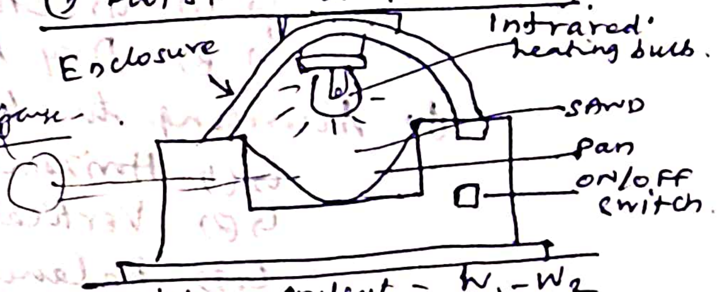
Proper control is necessary on moulding sand properties throughout it's composition.

The sand testing control provides a clear-cut idea about the moulding sand performance. It helps the men to control the properties of moulding sands. The following sand control tests are carried out on moulding sand.

(1) Moisture content test
(2) clay content test
(3) Grain fineness test
(4) Permeability test
(5) Strength test
(6) Deformation & toughness test
(7) Hot strength test
(8) Refractoriness test
(9) Mould hardness test

(1) Moisture content test-

Diagram-



moisture content $= W_1 - W_2$

W_1 = Weight of the sand before drying

W_2 = " " " after "

% of moisture content $= \frac{W_1 - W_2}{W_1} \times 100$.

(2) Moisture teller based on chemical reaction

$$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca(OH)_2$$

(2) Clay content test-

clay content $= W_1 - W_2$

W_1 = Weight of sand before drying

W_2 = Weight " " after drying

% of clay $= \frac{W_1 - W_2}{W_1} \times 100$.

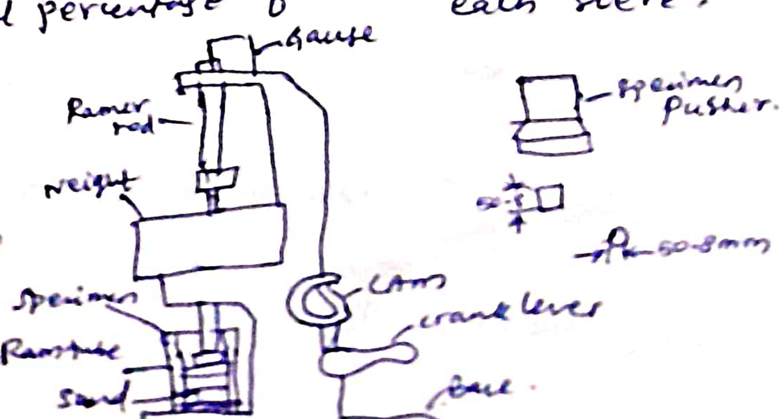
(3) Grain fineness test

AFS (American Foundrymen's Society) fineness number.

$$\text{AFS grain fineness number} = \frac{\text{Total products}}{\text{Total percentage of sand retained on pan \& each sieve}}.$$

(4) Permeability Test

Tendency of sand which allows the escape of gases or air through it, when it is originated in moulding sand due to pouring of hot molten metal.

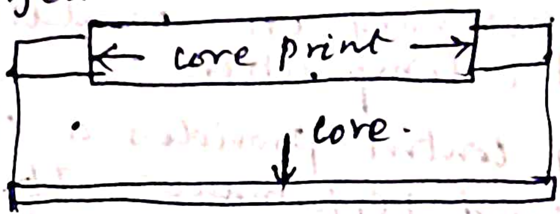


Cores - Types, applications

A core is a body made of sand which is used to make a cavity or a hole in the casting.

The shape of the core is similar to required cavity in the casting to be made.

Core print: The projection on a pattern. Core print forms a seat in mould. The core is supported in the seat formed by the core print.



Core prints are used to make holes in engine blocks, gears & pulleys.

Types of cores:

→ The cores can be classified as follows.

(a) According to the state of core:
- → (1) Green sand core
- → (2) Dry sand core.

(b) According to the position of the core in the mould.
- → (1) Horizandal core
- → (2) Vertical core
- → (3) Balanced core
- → (4) Hanging core
- → (5) Drop core.

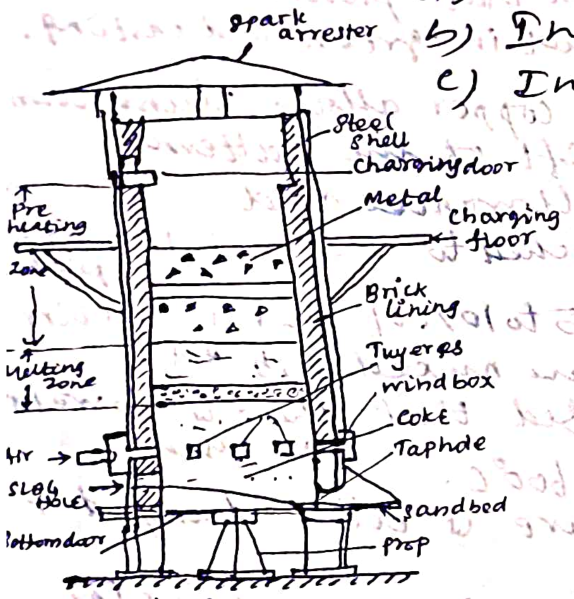
Melting furnaces

Various types of melting furnace are used in the foundary shop.

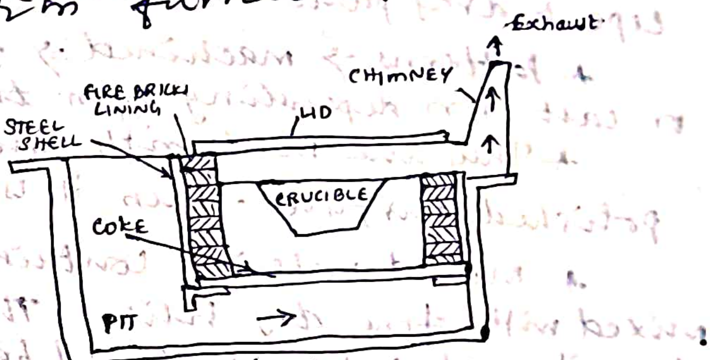
The type of furnace used depends upon the type of metal & quantity of metal to be melted.

The metal melting furnaces used in foundaries are.

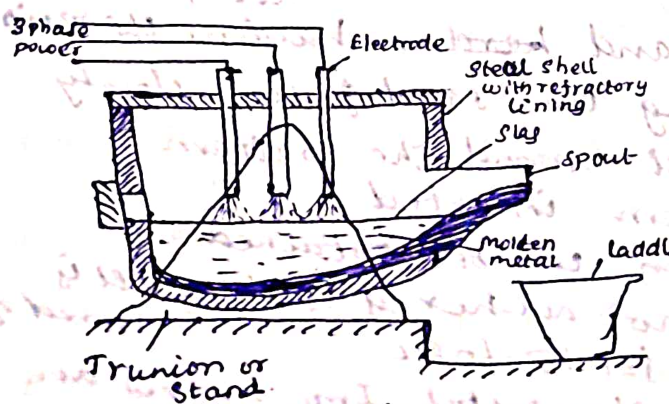
(1) Blast furnace ⇒ For smelting iron to produce pig iron.
(2) Cupola furnace ⇒ For cast iron
(3) Open hearth furnace ⇒ For steel
(4) Crucible furnace ⇒ For non-ferrous metal.
  a) Pit type furnace
  b) Coke fired stationary furnace
  c) Oil fired tilting furnace.
(5) Pot furnace
(6) Electric furnace
  a) Direct arc furnace
  b) Indirect arc furnace
  c) Induction furnace.



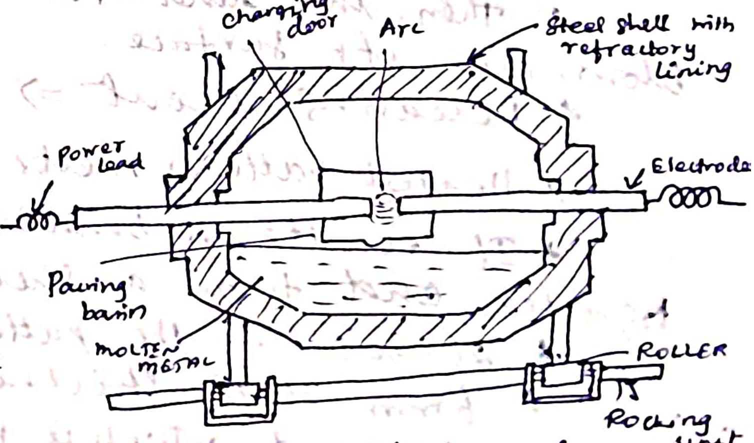
Cupola furnace.



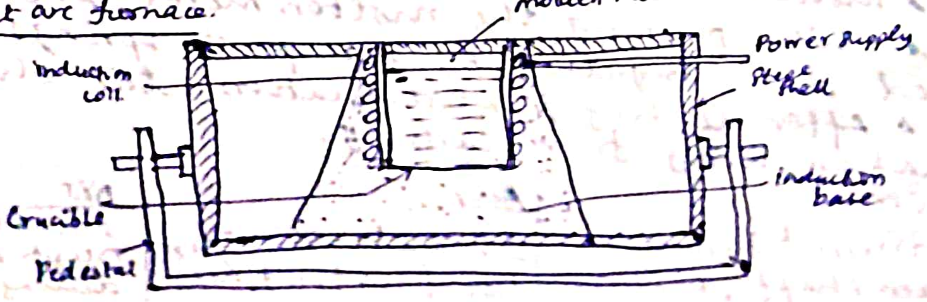
PIT FURNACE



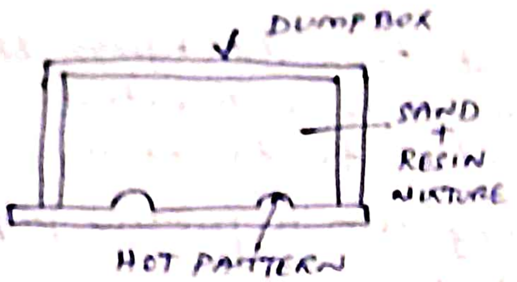
Direct arc furnace.

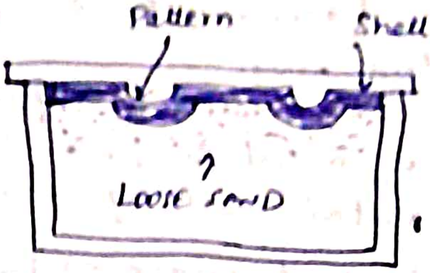


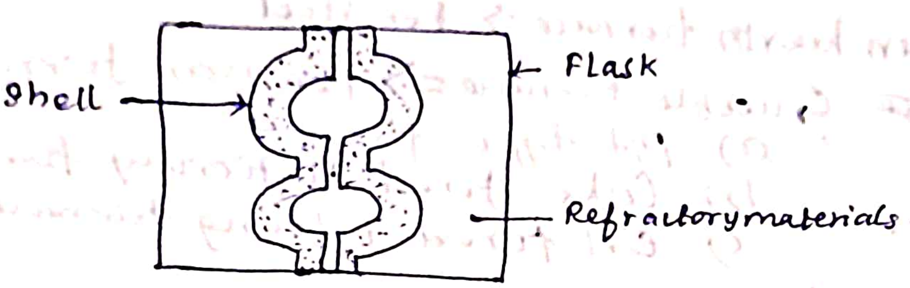
Indirect electric arc furnace.



Shell casting method.



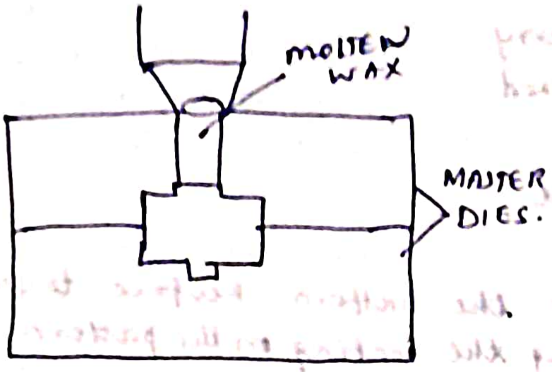




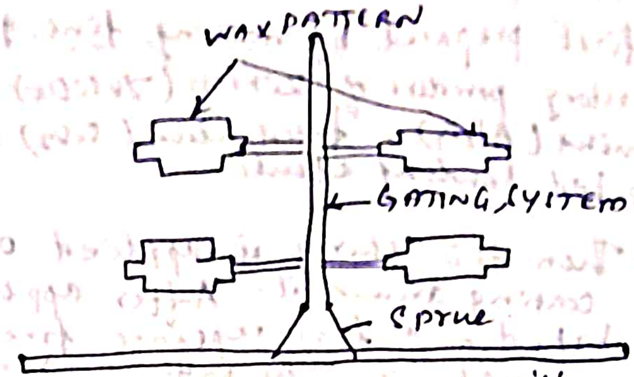
* semiprecise method for producing small castings in large numbers
* It involves the use of match plate pattern similar to cope & drag patterns which are used in green sand casting.
* Patterns ⇒ machined ⇒ from copper alloys, aluminium or cast iron depending upon the lift of the pattern.
* They are made with usual allowances and polished surfaces. Then it is attached to the metal match plate
* Mould material contains ⇒ 5 to 10% of phenolic resin mixed with fine dry silica. These are mixed with either dry oil or alcohol. It should be noted that there is no water is used.
* Pattern is heated to 230 – 600°C.
* Then, the sand-resin mixture is either dumped or blown over it's surface.
* Releasing agent ⇒ silicone is sprayed over the hot pattern.
* Heated pattern, melts and ~~hardens~~ hardens the resin.
* It results in bonding the sand grains closely together and forms a shell around the pattern.
* 20-30 sec – the pattern inverted.
* 6 mm shell thickness can be obtained.
* Extra sand which is not adhered to the shell is removed f...
* Mould heated 300°C for 15-60 sec.
* While pouring the molten metal, two halves are clamped down together by clamps or springs.
* After cooling & solidification, the shells are broken or shaken away from casting.

Application	Adv.	Dis
brake drum & bushing	±0.002 to 0.005 mm/mm	small size
cam, cam shaft, piston.	Good surface finish	Dust & ~~...~~ fume during process

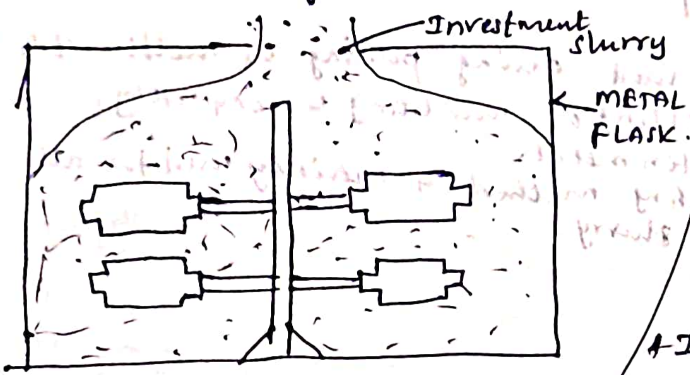
Investment casting method or Lost wax Process



Preparing disposable wax pattern for investment casting.



Investment casting with gating systems.



Investment casting with Investment slurry.

* Casting obtained by this method have very smooth surfaces and possess high dimensional accuracy.
* Precision Investment casting.
* Investment ⇒ Layer of refractory material with which the pattern is covered to make the mould.

* mould cannot be used again and again ⇒ Lost wax process
* Pattern ⇒ disposable ⇒ wax & surrounded with a shell of refractory meterial (ie ceramic) to form the casting mould.
* Once ⇒ refractory material (ie. ceramic) ~~to form the casting mould~~
* ~~Once ⇒~~ hardened ⇒ internal geometry takes the shape of the casting.
* The wax is melted out & molten metal is poured into the cavity where the wax pattern was.
* The metal solidifies within the ceramic mould and then the metal casting is broken out.
* Pattern made of wax is melted out and gets destroyed ⇒ Lost wax method.
* molten wax ⇒ 4 bar die cavity ⇒ Plastic material ⇒ Polystyrene, poly the reneate.
* CLUSTER ⇒ several small wax patterns are prepared & assembeled togather with a gating system along with central sprue. Assembling various small wax pattern are welded by using heated tools.
* SLURRY ⇒ mixing fine ~~silica~~ Silica either with water or ethyl silicate
* Primary coating ⇒ wax pattern dipped in give primary coating of 1mm thick
* DRY ⇒ air for 2 to 3 hrs. ; Oven ⇒ BAKING 2hrs Temp ⇒ 100 to 120°C. ; Heating furnace ⇒ 150°C for dry; 800-900°C to vapourise wax.

Application	Adv	Lim
Nozzle, buckets, vanes blades of gas turbine	Complex shapes can be cast accuracy	Small size casting.
Aircraft engines, frames. fuel system jwellery	No of casting at a time	more expensive.
	Undercut & intricate shapes easily obtained	Holes not possible.

Pressure die casting

- The mould, called die, is used to make a casting which is permanent.
- Molten metal is forced into the mould cavity under high pressure.
- This process is used for casting a low melting temp. material.

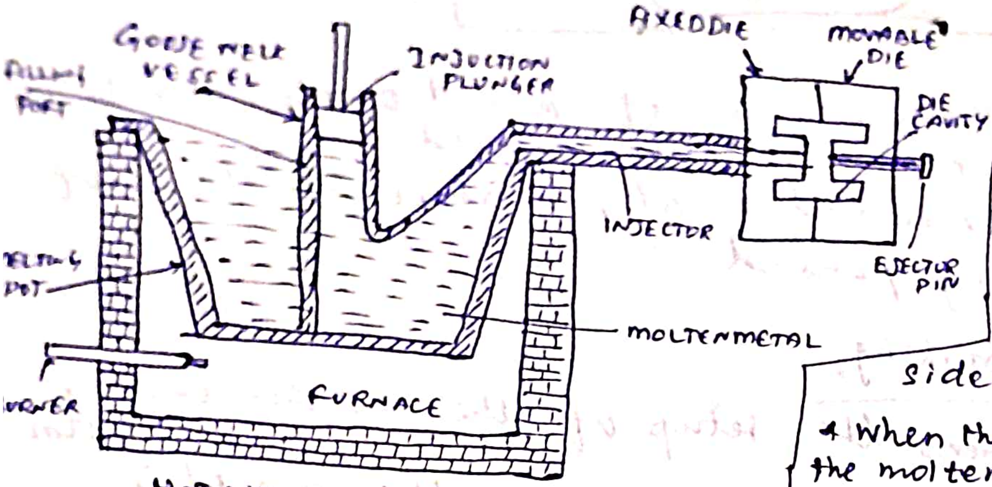
Eg. Aluminium, & Zinc alloys, brass etc.

Types

1) HOT CHAMBER DIE CASTING
2) COLD CHAMBER DIE CASTING.

HOT CHAMBER DIE CASTING

- melting furnace is an integral part of mould.
- There is a gooseneck vessel which is submerged in molten metal.



HOT CHAMBER DIE CASTING.

- Plunger → top of the gooseneck
- When plunger is in upward position, the molten metal will flow into the vessel through a port provided on the side wall.
- When the plunger comes down the molten metal is forced into the dies.

- Since the die is immediately cooled by water and sufficient cooling is provided for solidification.

Then the movable die is moved some distance and finished casting is removed by ejectors.

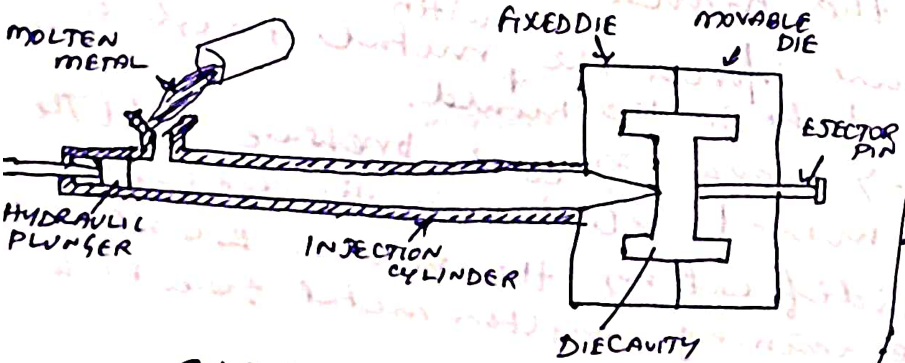
The plunger & movable die operated by Hydraulic system.

The operating pressure of Hydraulic plunger is 15 MN/m²

suitable for Zinc, tin & Lead.

COLD CHAMBER DIE CASTING

- In cold chamber, the metal melting unit is not an integral part of the machine.



COLD CHAMBER DIE CASTING.

- The metal is melted in a separate furnace and brought to the machine for pouring.

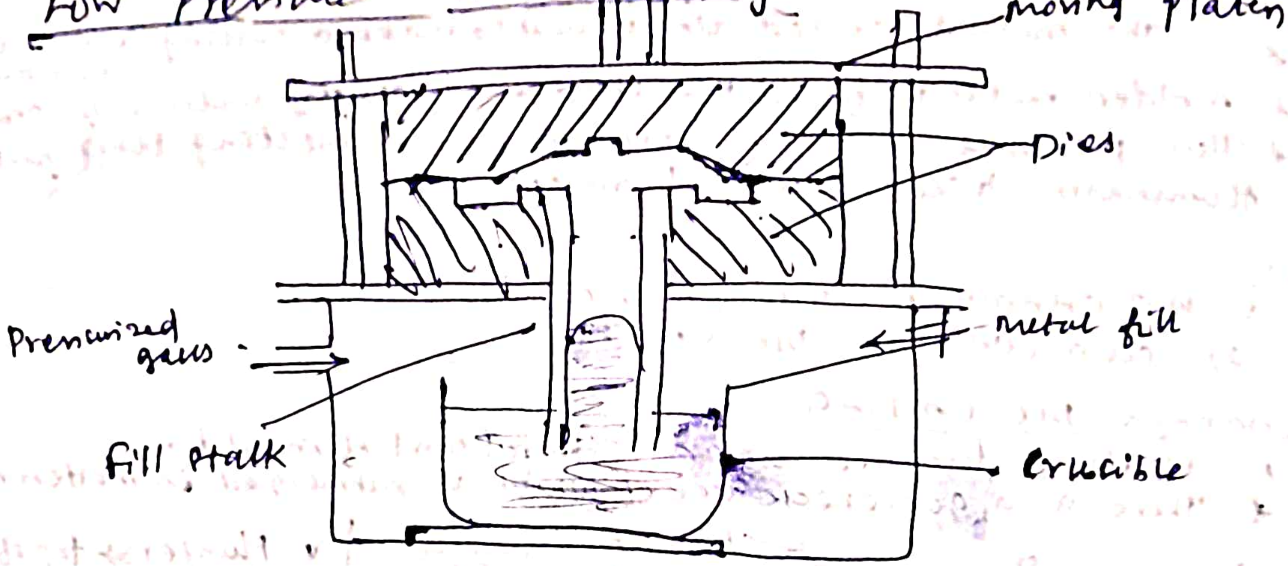
Cold chamber die casting is more suitable for metals with high melting points and corrosive properties such as Aluminium.

So cold chamber die casting is preferred for Aluminium alloys.

Working.

- Cold chamber ⇒ Hydraulic plunger.
- Measured quantity ⇒ molten metal } ⇒ Poured ⇒ Injection cylinder.
- Die - water cooled → immediated solidification of molten metal takes place.

Low pressure die casting



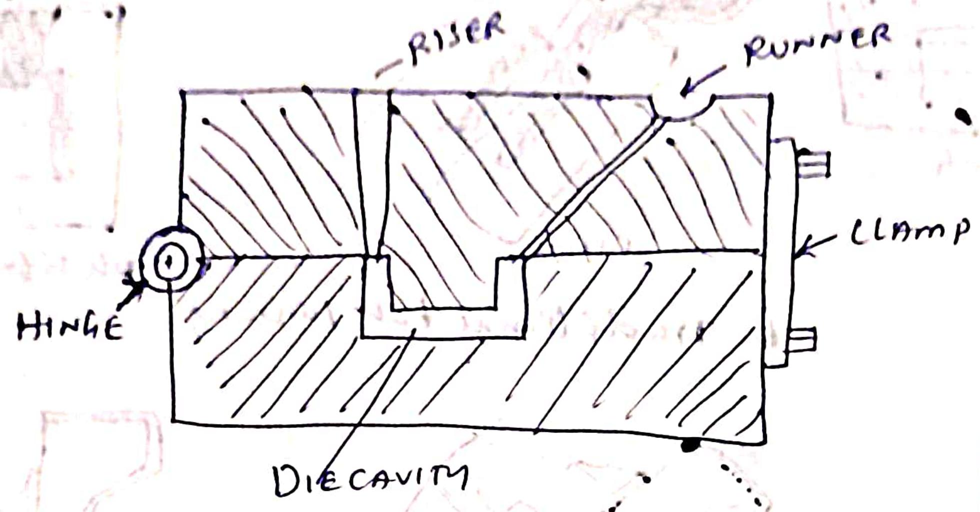
* uses the pressure levels of around 0.7 to 1 bar to feed the molten metal into the mould.
* Usually, the mould is at, or above, the level of the metal being poured.

Principle of working.

* Fig. shows the setup of the LP die casting system
* In Low Pr. castings, the mould is located above the sprue and flows 'up'. the sprue and ~~The sprue~~ into the runner system and the casting cavity
* Metal ⇒ melts ⇒ furnace ⇒ metal alloys.
* Ex - Al ⇒ casting Temp 710°-720°C
* molten metal ⇒ go ⇒ holding furnace ⇒ below the mould.
* Low pressure forces the molten metal through a riser tube in the mould.
* Liquid ⇒ moves ⇒ cont. pressure until the molten metal solidifies in the die cavity.
* On solidification, the pressure is released and the remaining molten metal goes back through the riser tube.

Adv : Automobile hub, cylinder block, cylinder head

Gravity Die casting

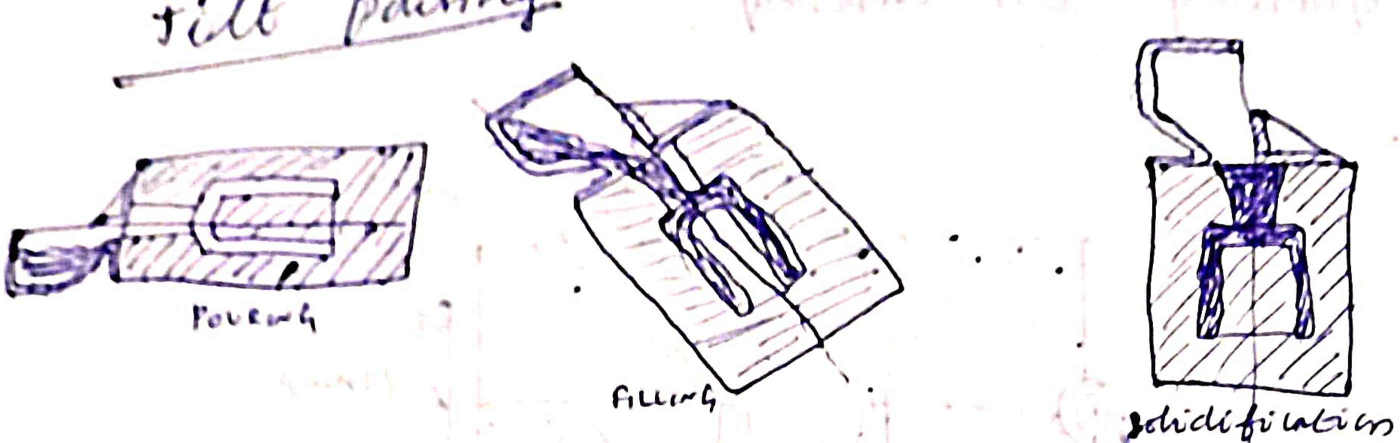


- Also called permanent mould casting.
- mould => Generated => Two halves.
- Hinged at one end
- Producing large no. of castings of similar shape. ~~castings~~
- Mould => Heat resisting CI, Alloy steels, graphite or other material.
- Pouring cup, sprue, gate & riser mad ~~up~~ in the mould itself
- Preheat => mould => Coatings ~~protects~~;
Protects mould surface from erosion & sticking.

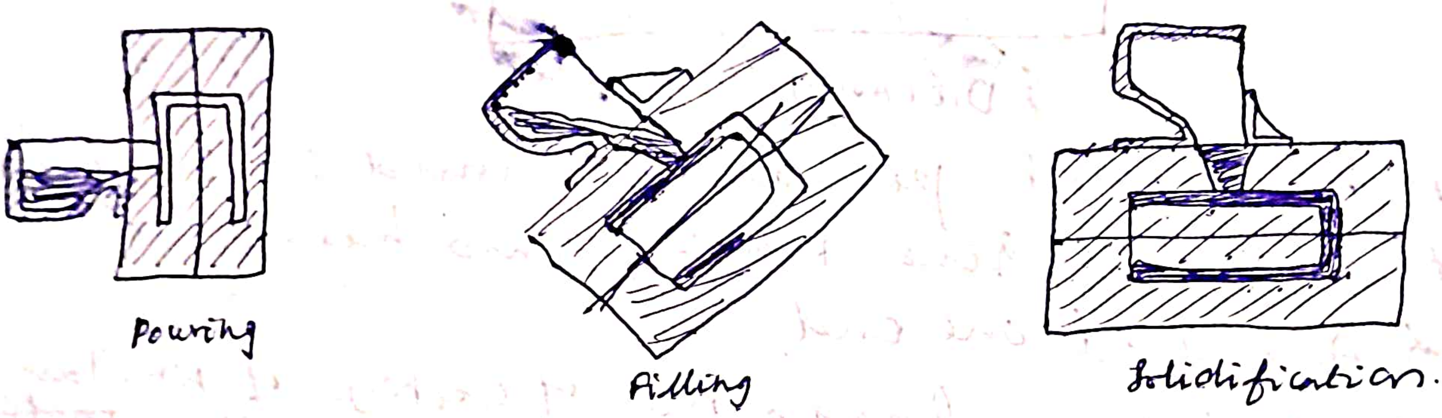
App

- Carburetor bodies,
- Oil pump bodies,
- Pistons
- Connecting rod,
- gear cover, etc.

Tilt pouring



(a) Traditional tilt pouring



(b) Reverse tilt pouring.

* Also called Tilt casting, (gravity die casting)

* Tilt pouring is a process with the unique feature of transferring the liquid metal into a mould by simple mechanical means, under the action of gravity; yet without surface turbulance

GOAL:

To reduce porosity & inclusions by limiting turbulence

High Pressure die casting

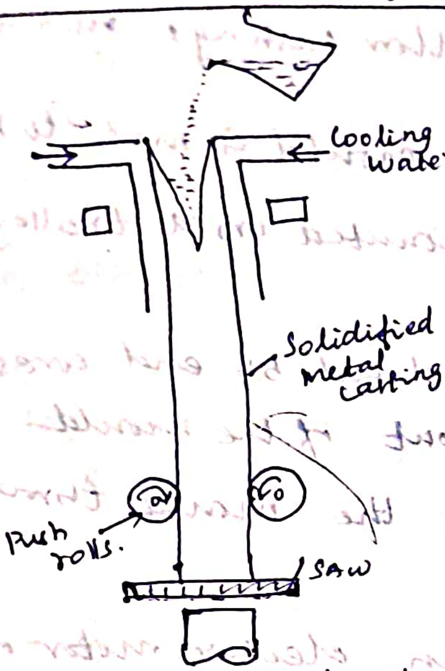
In high pressure casting molten metal is forced with the aid of forcing mechanism.

↳ 1. Centrifugal casting.

↳ 2. Continuous casting process

↳ 3. Carbon dioxide (CO_2) process.

Continuous casting process:

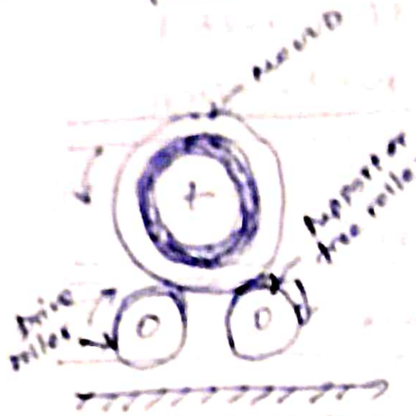


* In this process, molten metal is poured from a laddle continuously into a long vertical mould.
* The mould is made up of copper, brass or graphite.
* The mould is water-cooled.
* Hence the molten metal is immediately solidified.
* This solidified casting comes down continuously.
* Saw or oxy acetylene flame is used to cut the casting of required length.
* X-ray unit controls the pouring rate of molten metal from the laddle.
* Lubricating oil is applied b/w casting & mould wall to reduce friction.
* Guide rolls ⇒ At bottom keep on pulling the casting to match with the cooling rate.

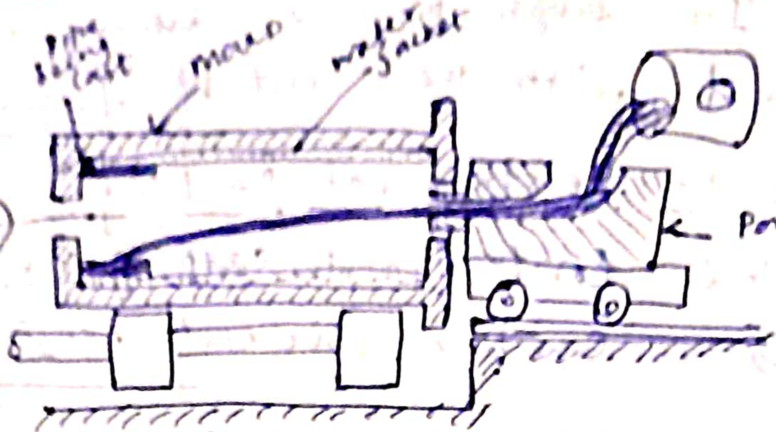
Argon gas is supplied at the top of the mould to prevent atmospheric reaction with the molten metal.

Application	Adv.	Dis Adv.
rods, pipes, slabs, ingots, bars, etc.	1. rate of production high 2. Good surface finish obtained 3. No impurities on castings 4. Grain size & structure can be controlled	1. cost of mould & other equipment high 2. operation & maintenance cost high

Centrifugal casting



side view of rotating parts.



front view of casting process

* Primarily used for making hollow castings such as pipe without using core.
* In this process, a metal mould is made to rotate
* The rotating mould is mounted on a trolley & trolley moves over rails.
* The end of the mould is closed by end cores to prevent the flow of metal out of the mould
* The metal is poured into the mould through a long spout.
* The mould is rotated by an electric motor or mechanical means as well as it moves axially on the rails.

Process

* Due to centrifugal force, the molten metal is thrown to the walls of the mould.
* The outside of the mould is water cooled.
* So the molten metal immediately solidifies.
* It is used for producing cylindrical & symmetrical objects.

Applicat.

Waterpipes, gears, bush bearings, flywheels, piston ring, brake drum, etc.

Adv

1. Core is not required
2. Rate of production is high
3. Pattern, runner and riser are not required
4. This casting can be made.

Dis Adv.

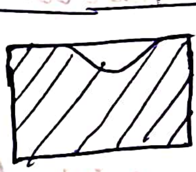
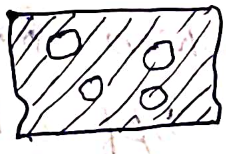
1) suitable for cylindrical & symmetrical shaped casting
2) cost of equipment high

Defects in casting & remedies.

Because of some reasons, castings may have some defects. The defects in a casting may arise due to the defects in one or more of the following.

(1) Design of casting and pattern
(2) Moulding and design of mould and core
(3) Metal composition
(4) Melting & pouring
(5) Gating and riser design.

These defects maybe reduced by a proper control of manufacturing cycle and proper foundary techniques.

Defects	Diagram	Causes	Remedies.
1. Shrinkage It is depression on the casting surface		1. Improper solidification 2. Incorrect pouring temp. 3. Faulty gating, runner & riser systems.	1. Provide proper solidification 2. Pour at correct temp. 3. Modify gating, runner & riser system
2. Blow holes. When the molten metal is poured, gases and steam are formed. If these gases could not come out, blow holes are formed on the interior of the casting.		1. Excess moisture in sand 2. Hard ramming 3. Improper venting 4. Excess binder	1. Control moisture content. 2. Ram properly 3. Provide sufficient vent holes. 4. Control binder content.
3. Scab It is the erosion or breaking down a portion of the mould and the recess filled with metal.		1. Uneven ramming 2. High velocity of pouring	1. Provide uniform ramming 2. Pour at correct velocity

UNIT-II

METAL JOINING PROCESS

Fusion welding processes

welding: The process of joining similar metals by the application of heat is called "welding".

It is a fabrication process that joins materials, usually metals or thermoplastics, by causing coalescence.

It is often done by melting the workpieces and adding a filler material to form a pool of molten material that cools to become a strong joint.

Fusion welding:

In fusion welding, the metal at the joint is heated to a molten state and then it is allowed to solidify.

When heat alone is used during welding, the Pressure is not applied during the welding processes.

It is non pressure welding.

Filler material may be required during welding.

Ex: Gas welding, Arc welding & Electron beam welding

Plastic welding.

In pressure or plastic welding, the metal parts are heated to a plastic state and they are pressed together to make the joint. Here there is no filler materials required.

Ex: Resistance welding, pressure welding & forge welding.

classification of welding process based on method of welding

(1) Gas welding:
- (a) Air-acetylene welding
- (b) Oxy-acetylene welding
- (c) Oxy-hydrogen welding.

(2) Arc welding.
- (a) carbon arc welding
- (b) Plasma arc welding
- (c) Shield metal arc welding
- (d) Tungsten Inert gas (TIG)
- (e) Metal Inert gas (MIG)
- (f) submerged arc welding
- (g) Electro-slag welding.

(3) Resistance welding.
- (a) Spot welding
- (b) Seam welding
- (c) Projection welding
- (d) Resistance butt welding
- (e) Flash butt welding
- (f) Percussion welding.

(4) Solid State welding
- (a) Cold welding
- (b) Diffusion welding
- (c) Forge welding
- (d) Explosive welding
- (e) Friction welding
- (f) Ultrasonic welding
- (g) Hot pressure welding.
- (h) Roll welding

(5) Thermo chemical welding.
- (a) Thermit welding
- (b) Atomic welding

(6) Radiant energy welding.
- (a) Electric beam welding
- (b) Laser beam welding.

Oxy fuel welding.

Gas welding.

↳ one type of welding processes is which the edges of the metals to be welded are melted by using gas flame.

No pressure is applied during welding

The flame is produced at the tip of a welding Torch

· The welding heat is obtained by burning a mixture of oxygen & combustible gas.

The gases are mixed at required proportion

The gases employed are acetylene, hydrogen, propane & butane.

The flame only melts the metals.

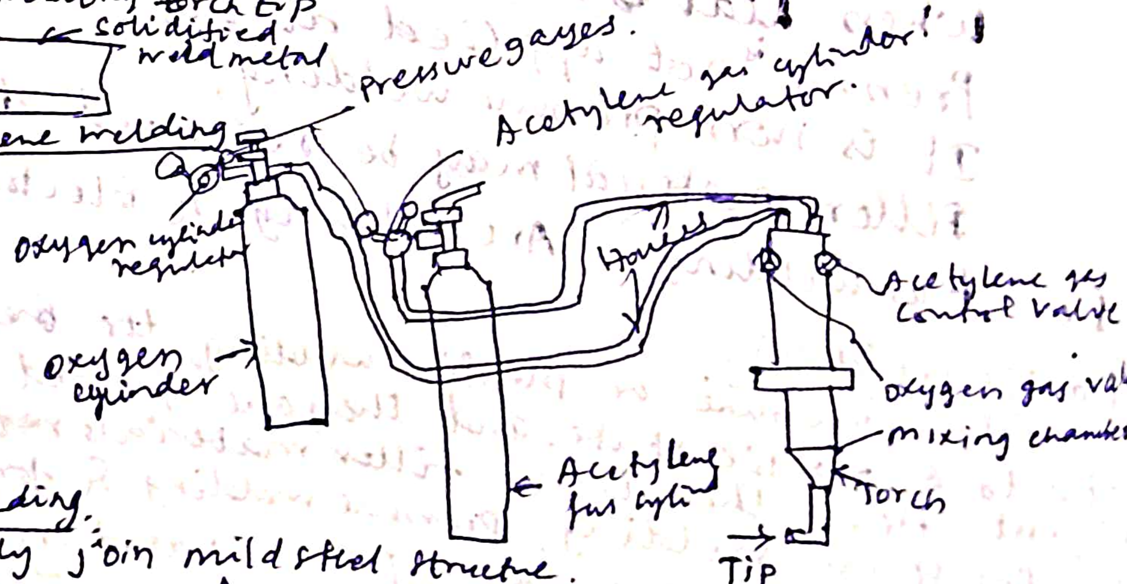
Filler rod: The additional metal required to weld is supplied by filler rod

Flux: During welding to prevent oxidation and remove impurities

metal having 2mm to 50mm thick are welded by gas welding

Types (1) Oxy-acetylene (2) Air-acetylene (3) Oxy-hydrogen

oxy-Acetylene welding



Oxy-Acetylene welding.

↳ permanently join mild steel structure.

↳ Combination of oxy-Acetylene ⇒ flame temp ⇒ 3200°C.

↳ Ideal for welding & cutting.

↳ when the flame comes in contact with steel, it melts the surface and forming a molten pool which allows welding torch

↳ O_2 & C_2H_2 (Acetylene) stored ⇒ high pressure ⇒ separate cylinder

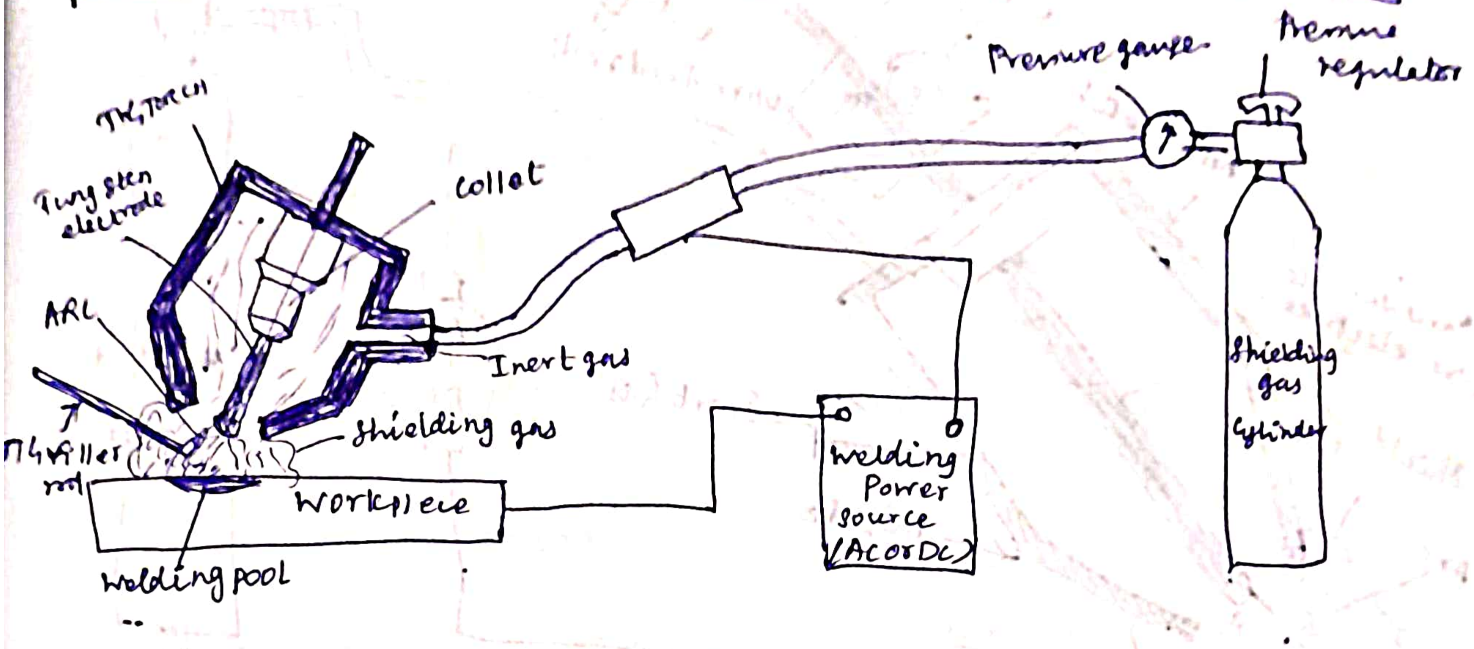
↳ Acetylene is very harmful if it is not handled carefully.

Air-acetylene

↳ Air is used instead of oxygen.

↳ Air taken from the atmosphere is compressed in a compressor & it mixed with acetylene

Gas Tungsten arc welding (GTAW) (TIG)



* Gas Tungsten Arc welding (GTAW) is also called Tungsten Inert Gas (TIG) welding.
* In TIG, the electric arc is produced between a non-consumable tungsten electrode and the workpiece.

Electrode holder: There is an electrode holder in which the non-consumable tungsten electrode is fixed to produce the arc

Inert gas: While supplying the electric power between the electrode & workpiece, the inert gas from the cylinder is passed through the nozzle of the welding head around the electrode.

The inert gas (Argon, Helium, Nitrogen & CO_2) surrounds the arc and it protects the weld from atomospheric effects. and hence defect free joints are made.

Filler metal

Filler metal may or may not be used. when a filler metal is used, it is fed mannually into the weld pool.

Electrode:

* Tungesten electrode is used.
* It has high melting point (3430°C)
* Therefore, it will not melted during welding.

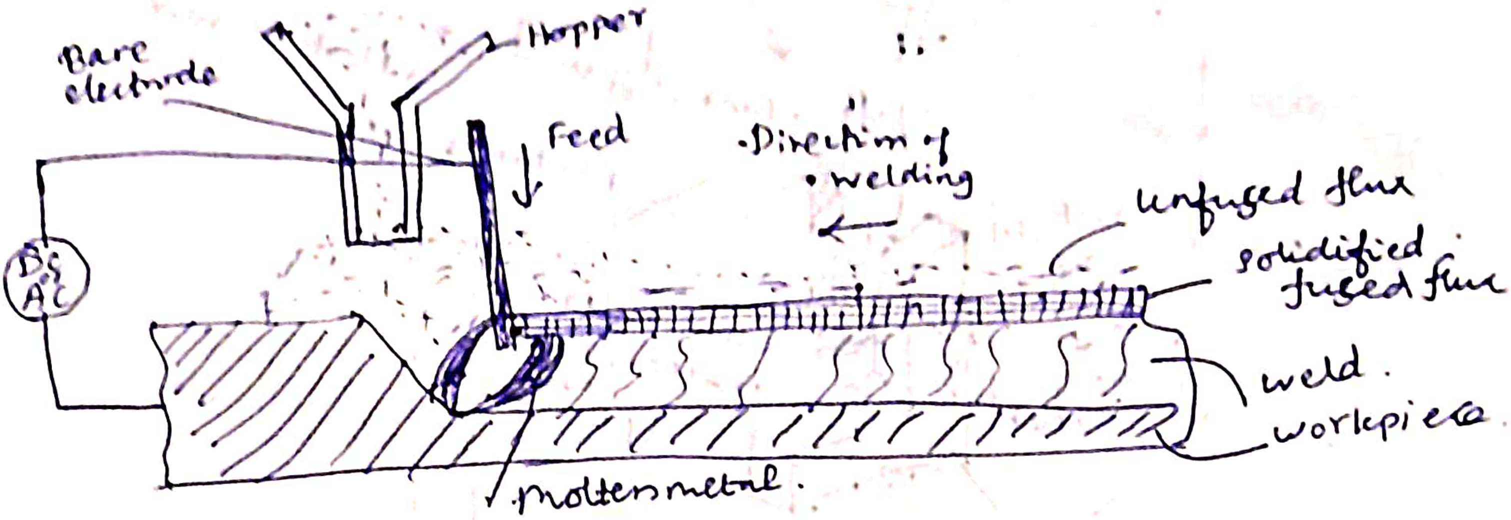
Parameters

* Nozzle (shield) size, gas flow rate, filler rod size, electrode dia, and current are choosen depending on the position of weld & metal thickness.

Advantages

* It is applicable to wide range of materials such as aluminium, stainless steel, manganese & copper alloy.
* more suitable for thin sections
* It does not create as much spatter & spark
* It reduced distortion in the weld joint b/c of concentrated heat source
* No flux is required, welding speed high.

Submerged arc welding



* Flux is mainly used to avoid the oxidation reactions with oxygen present in the atmosphere.
* The complete welding setup is dipped in the flux powder and hence it is named as submerged arc welding or hidden arc welding.
* Electric arc is produced b/w consumable bare electrode & workpiece.
* The metal electrode is continuously fed from the reel by a moving head.
* Flux powder is fed in front of the moving head.
* It supplied from a hopper.
* It completely covered by ~~flux~~ flux.
* Since, the arc is completely covered by flux, there will not be any defect due atmospheric effect.

Flux powder

↳ It is madeup of silica, metal oxides & other compounds fused together and it is crushed to the proper size.
↳ It also act as deoxidiser and scavenger.
↳ also contain powder metal alloying.
↳ covers arc & molten metal.

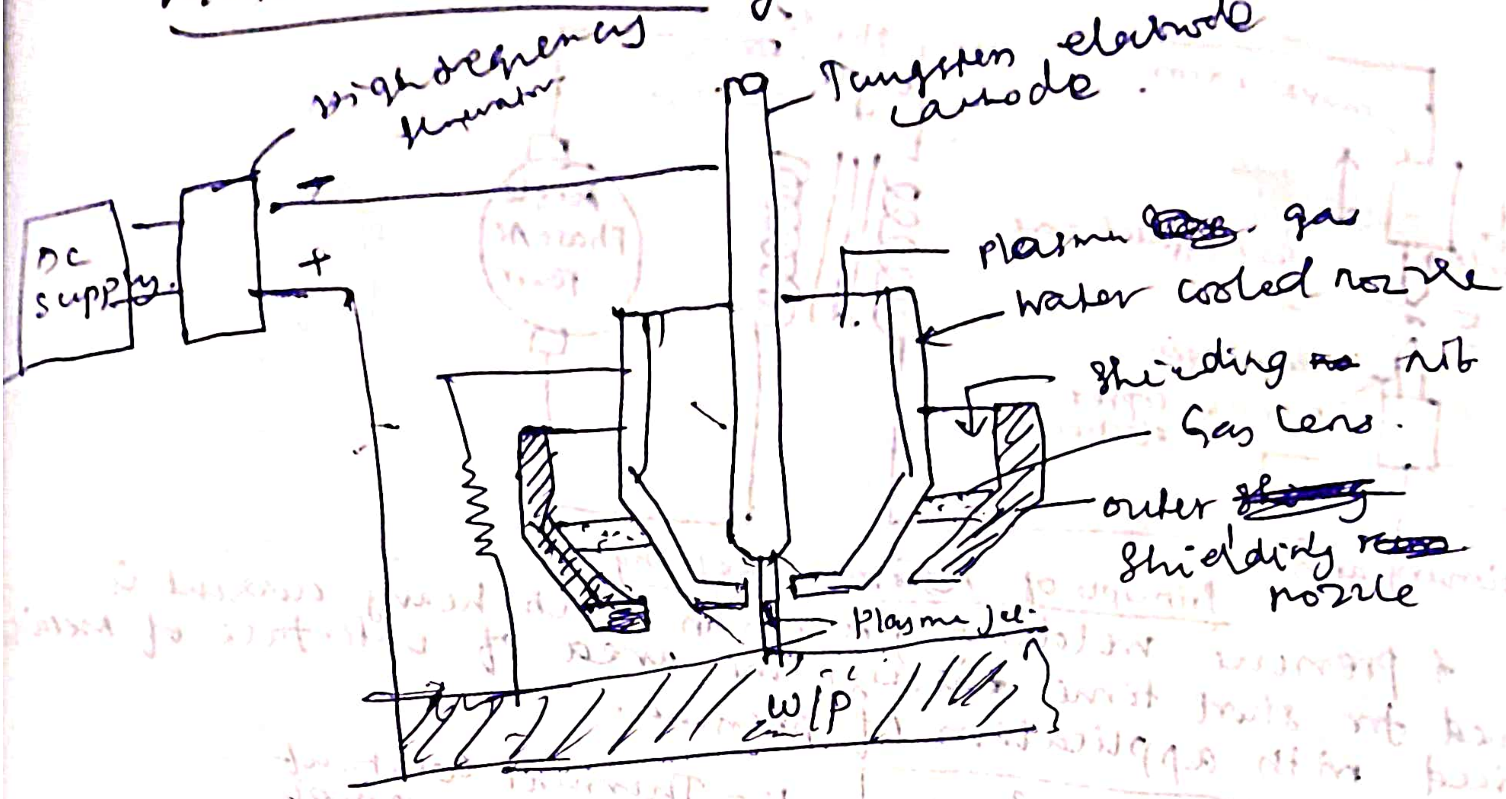
Power supply

Voltage used 25V to 40V.
current DC using 600A to 1000A
AC is 200A.

Application

↳ Heavy steel plate fabrication includes, structural shapes, longitudinal seam of larger dia pipes, machine components of heavy machinery.
↳ pressure vessel, boilers, tanks, etc.

Plasma Arc welding

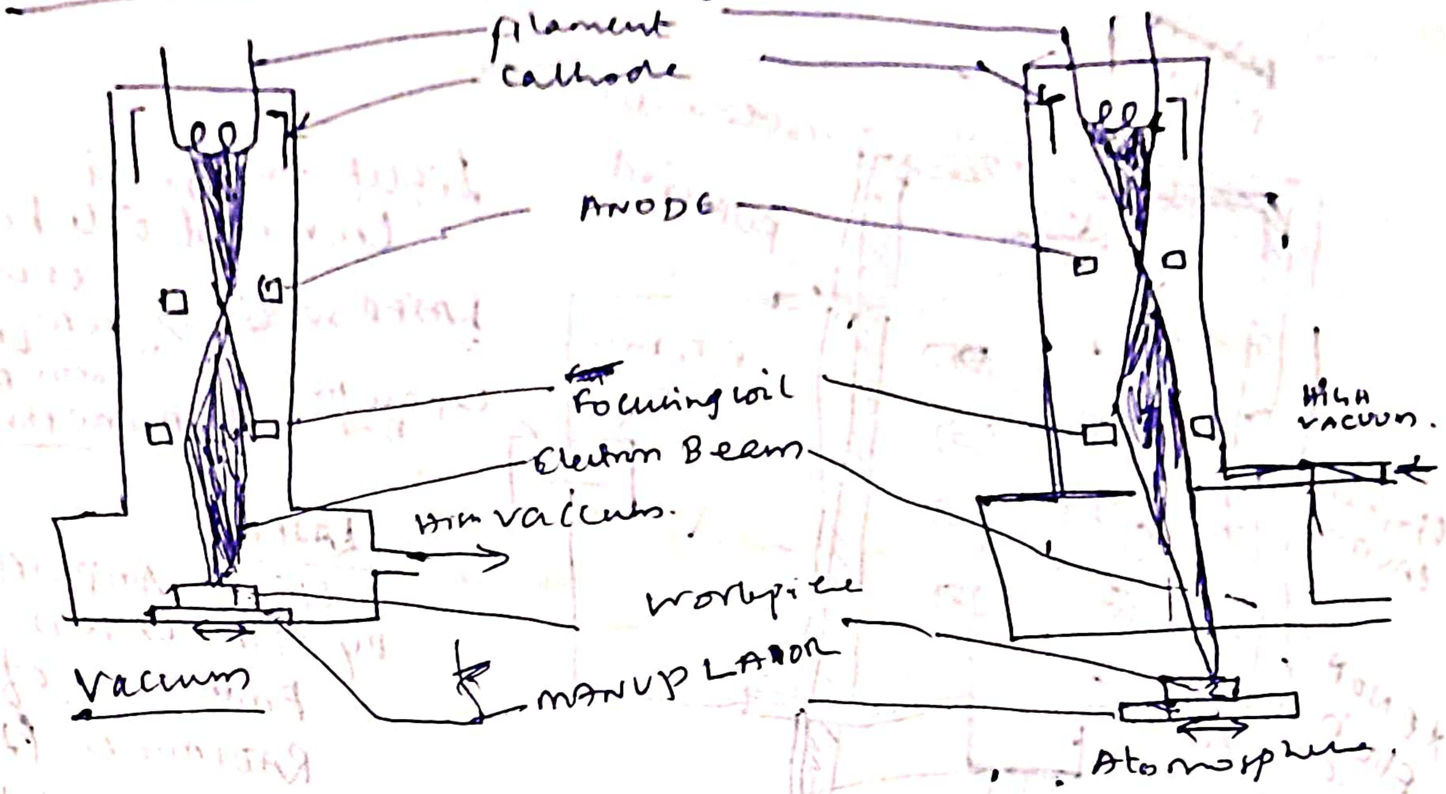


* Non conventional method.
  ↳ suitable for machining cast alloy, waspaloy and carbides.

Principle
↳ High temperature ionized gas.
↳ A plasma is the gas region in which there is practically no resultant charge, (ie) where positive ions and electrons are equal in number.
↳ The region is an electrical conductor and it is affected by electric & magnetic field.
↳ When this high temp. plasma passes through the orifice, the proportion of the ionized gas increases and plasma arc welding is formed.

Working.
↳ When the High heat content PLASMA gas is forced through the torch, an orifice is surrounded by negative tungsten electrode in the form of jet
↳ The Plasma cutting force imposes a swirl on the orific gas flow.
↳ ARC ⇒ initiated ⇒ supply ⇒ electrical energy b/w nozzle and tungsten electrode.
↳ Release High energy & heat.
↳ Heat b/w 10000°C to 30000°C
↳ This heat ⇒ narrow & deep welds can be made with high speed.

Electron beam welding.

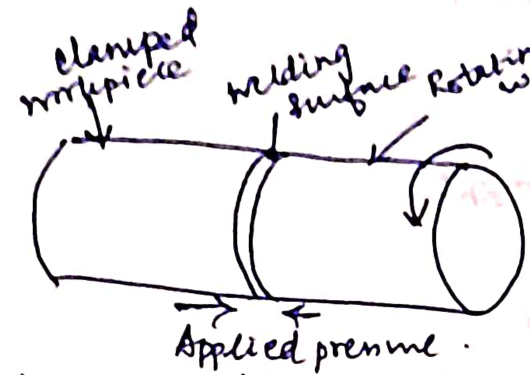


↳ Fusion welding process
↳ A beam of high velocity electrons is used for producing high temp for melting the W/p to be welded.
↳ Electron strikes the W/p, and their kinetic energy is converted into thermal energy by releasing heat.

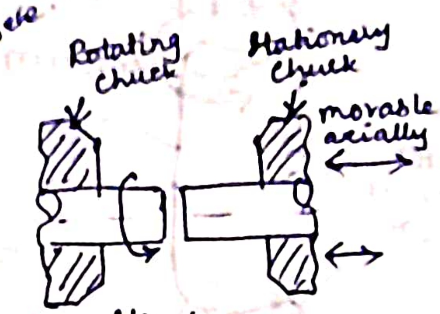
Working: If filament of tungsten or tantalum is heated to high temp. in a vacuum either directly by means of an electric current or indirectly by means of an adjacent heated, a greater number of electrons are given off from the filament.
+ These electrons carry a negative charge which is passed through the anode hole.
+ The greater is the filament current, the higher will be the temp. & greater will be electron emission.

This heat energy is used to weld the metals. kinetic energy is converted into heat energy. KE is $mv^2/2$.
where m = mass of electron (9.1×10^{-28} g)
v = velocity.

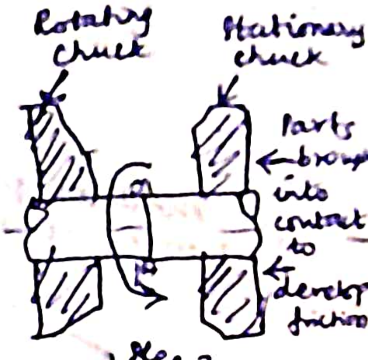
Friction welding



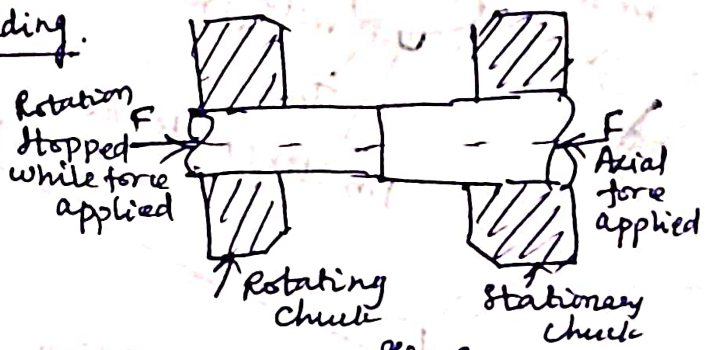
Principle of friction welding.



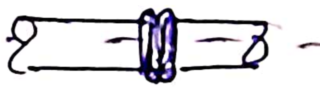
Step-1



Step-2



Step-3



Step-4

Principle.

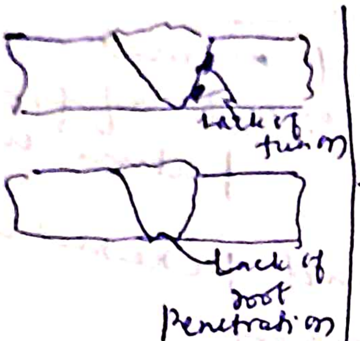
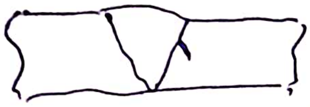
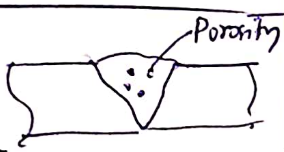
↳ It is a solid state welding process in which coalescence is achieved by frictional heat combined with pressure.

The heat is obtained through mechanical friction b/w rubbing surfaces of w/p in relative motion to one another.

Working.

* Initially components to be welded are held in chucks or clamps.
* One part is rotated at high speed (1500 to 3000 rpm) using a rotating chuck & other part held stationary.
* During welding, the stationary chuck is moved and contacted with rotating component under pressure.
* Heat is produced b/w contact surfaces.
* This heat is used to weld the components under pressure
* Pressure is used to generate sufficient heat to reach a bonding temp. with in few seconds.

Weld defects

		Causes	Remedies
1) Lack of fusion		* Poor adhesion of the weld bead to base metal. * Amount of supply of current	Increasing the weld current & reducing the deposition rate by Lowering the travel speed.
2. Cracks.		Either in weld or base metal adjacent to weld. * Significantly reduces strength.	Excess material restraint should be avoided. & Proper filler material should be utilized.
4. Porosity		Small voids in weldments formed by gases entrapped during solidification * Moisture in the electrode * Incorrect welding technique	Drying the electrode before welding Proper shielding gas supply. Dry & cleaned weld surface.

UNIT-III

Bulk deformation processes

- Hot working and cold working of metals
- Forging processes
- Open, impression and closed die forging
- Cold forging
- Characteristics of the processes.
- Typical forging operations
- Rolling of metals
- Types of Rolling
- Flat strip Rolling
- Shape rolling operations
- Defects in Rolled parts
- Principles of rod & wire drawing
- Tube drawing
- Principles of Extrusion
- Types
- Hot & cold Extrusion
- Introduction to shaping operation

Hot working & Cold working of metals.

HOT WORKING	COLD WORKING
1. Working above recrystallation temperature	Working below recrystallization temp.
2. New crystals are formed after hot working.	There is no recrystalization.
3. It hardens the metal	No hardening happens.
4. Impurities are removed from the metal	Impurities are not removed
5. More elongation of metal takes place.	Less elongation occurs.
6. Large size metals can also be deformed	It is limited to certain size.
7. Surface finish is not good	Good surface finish can be obtained.
8. Blowholes, cracks get welded during hot working.	Ductility is obtained during cold working and it is useful for machining process
9. Internal stress is not formed	Stress formation in the metal occurs.

Forging processes

Forging is the process of mechanical working of metals.

In this process, the desired shape is obtained by the application of a compressive force.

In hot forging, the metal is heated above the recrystallization temperature.

Then it is compressed and squeezed to the required shape by using a hammer or press tool.

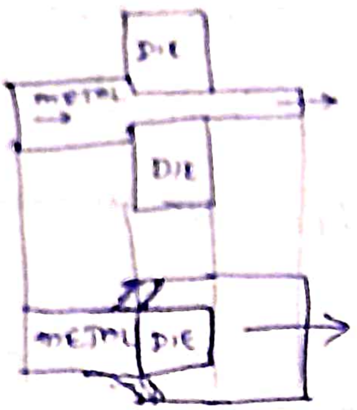
Steps in forging operation.

BILLET → Pre shaped → Rough Forge → Finishing DIE → Trimming DIE → FINAL PRODUCT

Step 1: Forging process starts ⇒ ROUND or SQUARE piece from a forging blank or bar stock.

Open-die forgings } ⇒ often used to perform or preshape the workpiece for closed-die-forging

Open, Impression & closed die forging.



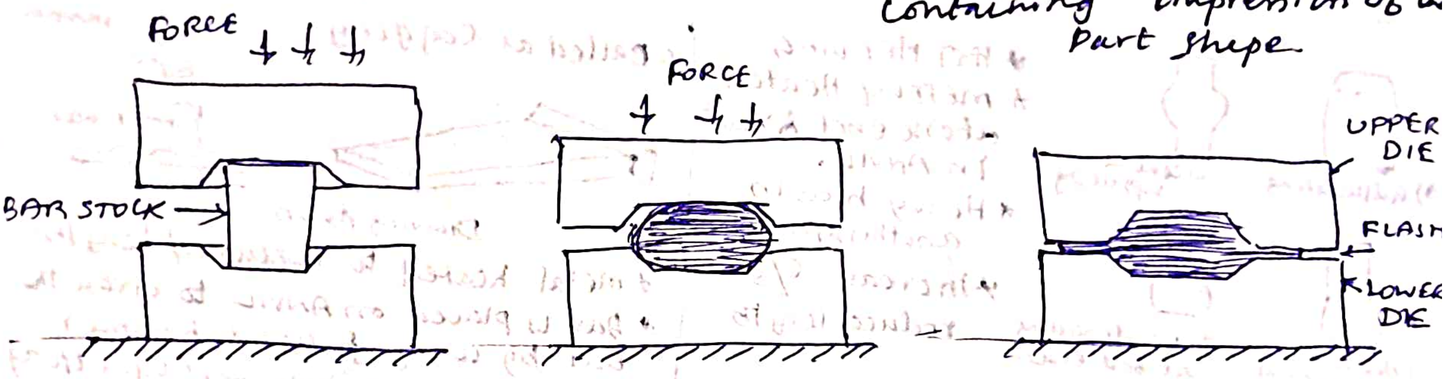
- Carried out between flat dies or dies of very simple shape.
- Heated work is placed on flat surface of Anvil or b/w flat dies and the hammering force is applied.

Types
- (1) Hand forging
- (2) Power forging.

OPEN DIE FORGING

IMPRESSION DIE FORGING

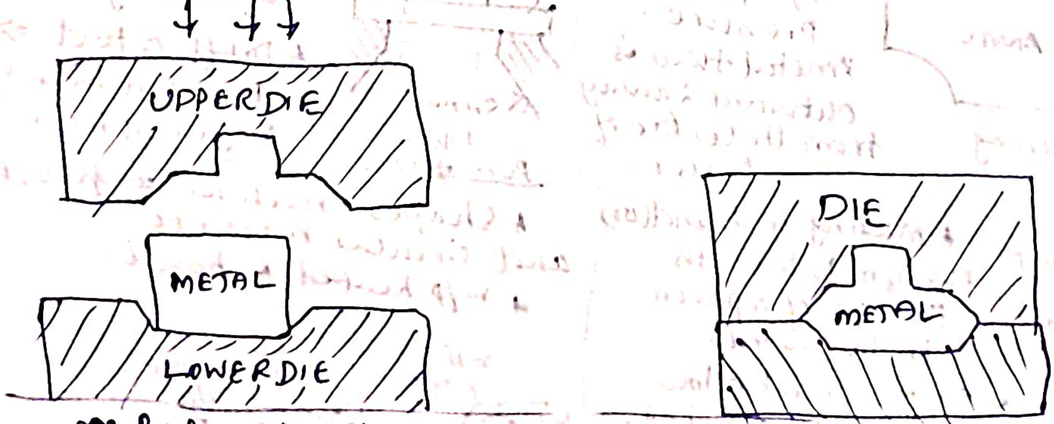
Workpiece obtains the shape of the die-cavities or impression while being forged b/w two or more dies containing impression of the part shape



IMPRESSION DIE FORGING

Advantages
- less metal is lost to flash.
- Flash can account for 20 to 45% of the starting material.

Closed die forging:



(a) Before pressing (b) After pressing.

closed die forging.

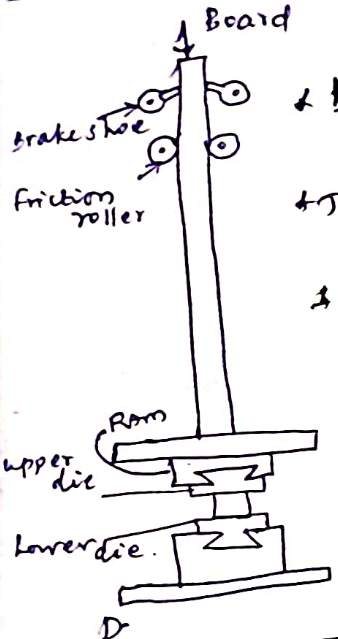
- another form of impression-die-forging also called flashless forging.

Adv:
forging with precision & close dimensional tolerance.

Typical forging operations

Press forging	Drop forging
1. The operation is a time consuming process	The operation is less time consuming process.
2. The operation creates close tolerance on product	The tolerance on product is not as close as in press forging.
3. There is no limit to the size of the part.	It is limited to the smaller size parts.
4. Alignment of dies is easier	Alignment of dies is challengeable
5. Operation is quite	Operation is noisy.
6. Structural quality of the product is superior.	Structural quality of the product is not superior.
7. It needs simple maintenance	Maintenance is difficult.

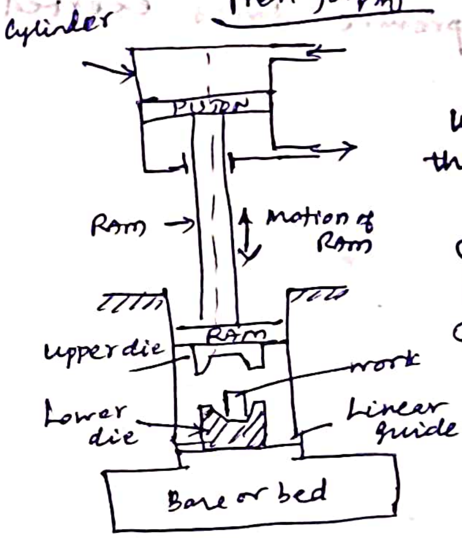
Drop forging



* Both dies are having impressions
* Two rollers are fixed on the board.
* Both rolls are rotate opposite to each other.

Engine valve, coupling, bolts, screws etc.

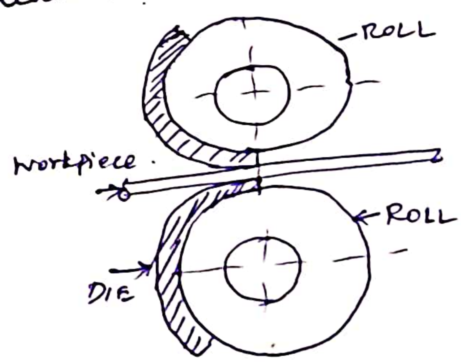
Press forging



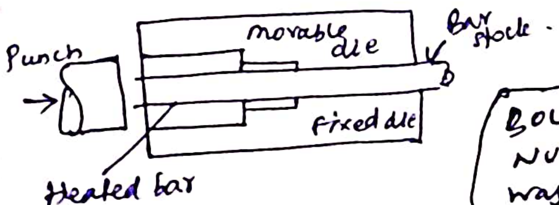
Used for making the components such as
1. Spanner
2. Connecting rod
3. M/c components

Heated metal part is passed b/w two rolls.

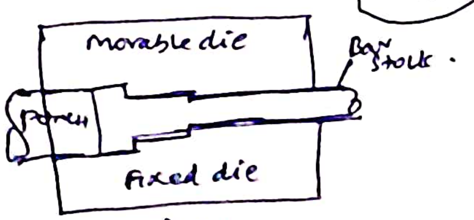
* heated metal squeezed



Roll forging

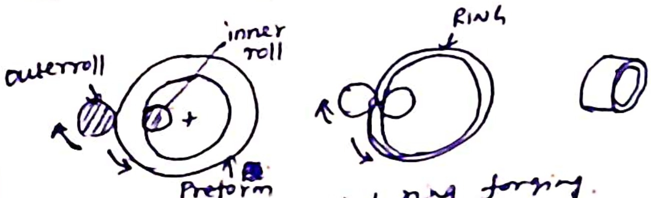


BOLT
NUTS
washers
collers



upset forging

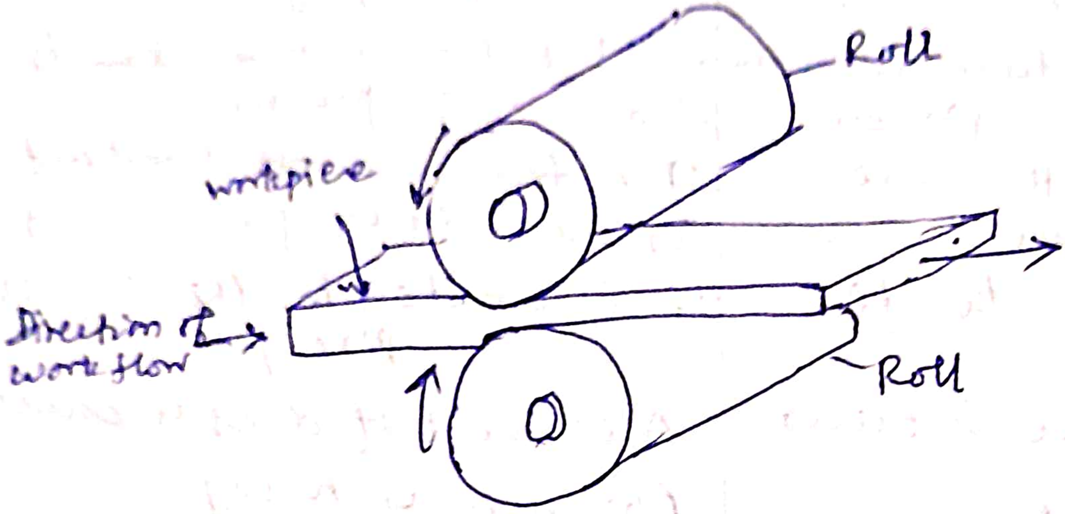
* increase the dia of the workpiece by compressing it's length.
* A machine having die-set.
* Fixed die, movable die, Punch.



Seamless rolled ring forging.

Punching a hole in a thick, round piece of metal (like donut) & then rolling and squeezing the donut shape to thin ring.

Rolling of metals

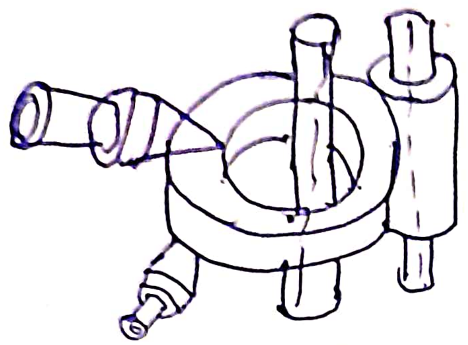


The process of plastically deforming the metal into semi-finished or finished condition passing it b/w two roller.

The cross section of the w/p is reduced by compressive forces exerted by two opposing rolls.

Shape rolling operations

(1) Ring rolling.
(2) Thread rolling
(3) Tube piercing



RING ROLLING

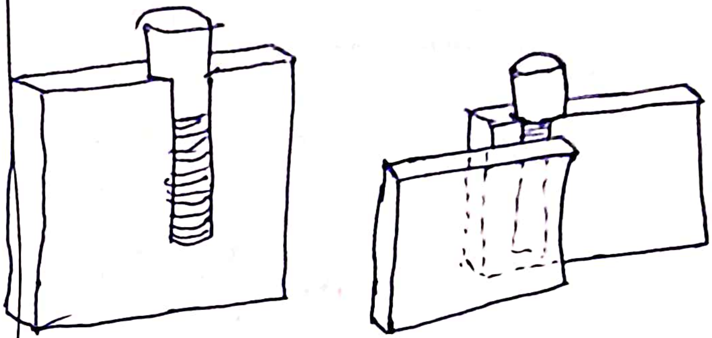
* A thick ring is expanded into a large diameter ring with a reduced cross section.
* First, the ring is placed b/w two rolls in which one of the roll is driven and other is idler.
* A pair of edging rollers is used for maintaining the height constant.
* The ring thickness is reduced by bringing the rolls close together as they rotate.

Adv: Short producing time, high productivity, close tolerance.

App: Large rings for turbines,
roller bearing races.
Flanges & rings for pipes.

Thread rolling

It is a cold forming process by which straight or tapered threads are formed.



(a)

* Threads forming process on round rod or wire with each stroke of pair of flat reciprocating dies.

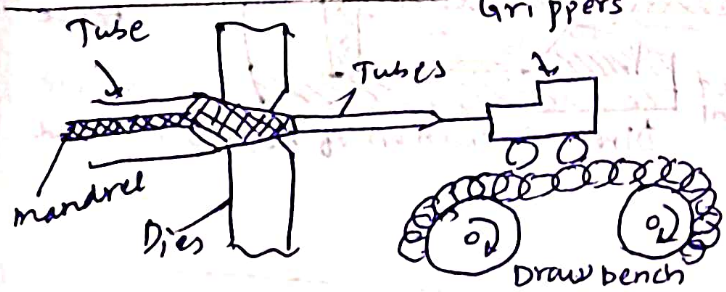
Adv: generating threads without loss of materials.
Surface finish good & fatigue life is high.
Lubrication required.

App:
spur & helical gear can be produced

Principles of rod & wire drawing & Tube drawing

Wire drawing	Rod or bar drawing
1. This process is used to reduce the diameter of a wire by pulling it through a single or series of drawing dies.	It is the process to reduce the size of rods by shrinking it from a large diameter rod by drawing through a die
2- Drawn wires are wound as a coil using power reel.	Bars or rods that are drawn cannot be coiled therefore straight -pull draw benches are used.
3- Wire is a drawn product having less than 5mm.	~~Bra~~ Bar is a drawn product having more than 5mm.
4. For wire drawing, smaller dia bar stock is used.	For rod drawing, larger dia bar stock is used.
5. Wire drawing is a continuous process usually done in multiple steps.	Bar drawing is done in single step.
6. Wire drawing is used for producing wires eg. Electrical wires, cables, strings, welding electrodes, fencing, etc	Drawn rods are metal raw material for making bolts, nails, screw, rivets, & springs.

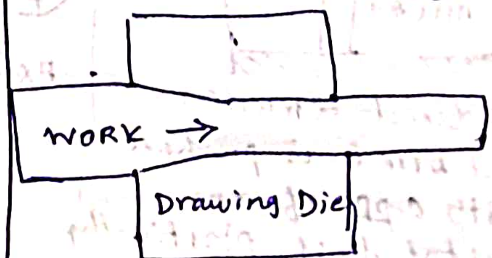
* Similar to bar drawing except the beginning stock is a tube.



Tube drawing.

* cylinders & tubes which are made by extrusion process ~~are~~
& decrease the dia, reduce wall thickness, improve surface finish.
* C/S may be circular, square, hexagonal or in any shapes.

of ~~Redue~~ to be drawn should be straight and maxi. lengths depend on carriage movement



Rod drawing

Draw speed = 1500 mm/s.
maxi reduction in C/S area per pass is to be 45%.

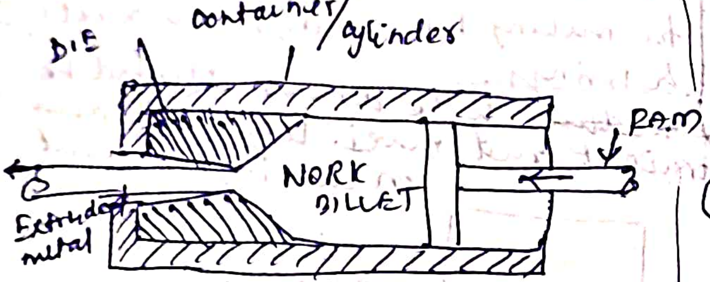
* Tensile stress may increase & surface finish may become poor

Principle of Extrusion, Types

Extrusion
- Hot extrusion
  - Forward (or) Direct extrusion
  - Backward or Indirect extrusion
- Cold extrusion (or) Impact extrusion

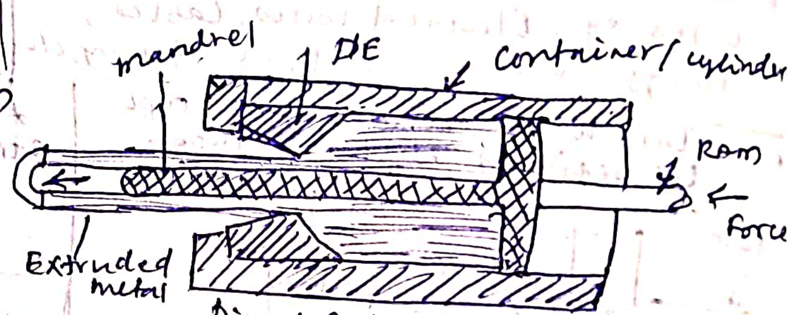
* commonly performed in Horizontal Hydraulic presses.
* Heated billet metal is placed in a press which is operated by the ram and a cylinder

* Hollow sections such as tubes can be extruded by direct method using hollow billet & a mandrel attached to the dummy block.

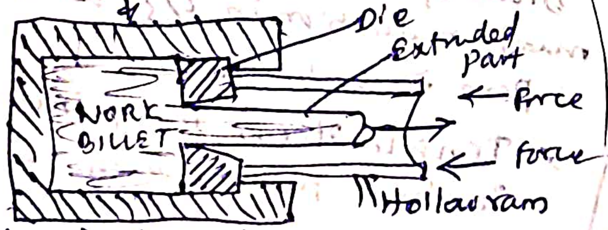


Forward or direct extrusion.

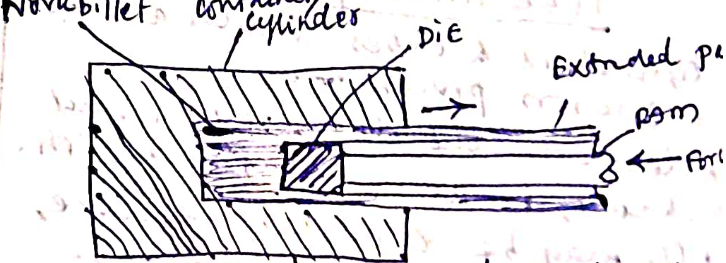
Heated metal billet is pushed by ram & with app. of ram pressure the metal first plastically fills the die.



Direct Extrusion of hollow sections



Indirect or backward extrusion.



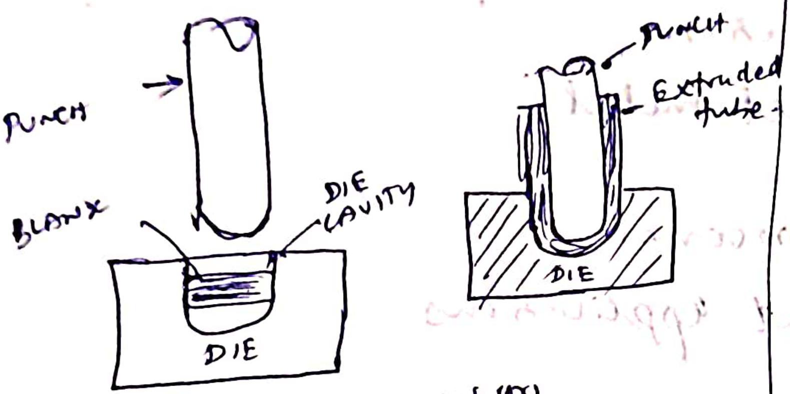
Indirect or backward extrusion.

* Extruded part is forced through the hollow ram.
* Hollow ram containing the die is kept stationary & the container with the billet is caused to move.
* The heated metal is placed in the die & the force is applied by the power operated hollow ram.

In hollow extrusion, the material gets forced through the annular space b/w solid punch and container.

Hot & Cold Extrusion & Shaping Operation.

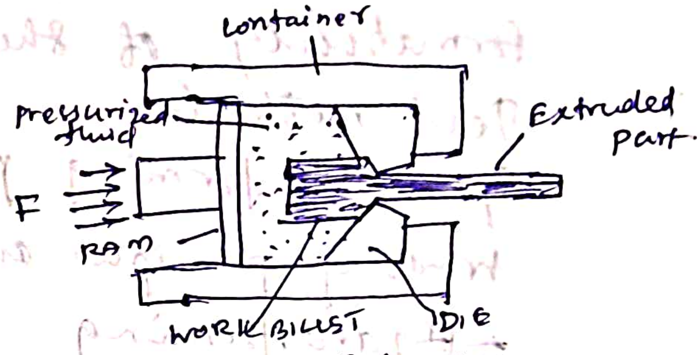
- Extrusion at room temp (or) slightly elevated temp.
- Indirect extrusion.



COLD EXTRUSION (or) Impact Extrusion.

- Work material is placed b/w die & ram.
- The punch is connected with the ram.
- When sudden impact is given to the ram, the metal flows plastically in the upward direction.

- The container is filled with a fluid.
- Extrusion pressure is transmitted through the fluid to the billet.



Hydrostatic Extrusion.

The whole area of the billet is completely circumscribed by a pressurized liquid, except the area of billet which is in contact with die.

- Al or Cu wires - especially for reducing their diameters.

Adv
No friction, minimize force req.
Allows high reduction ratio
uniform flow of material.

Dis
under the effect of high ps liq.
More handling time req.

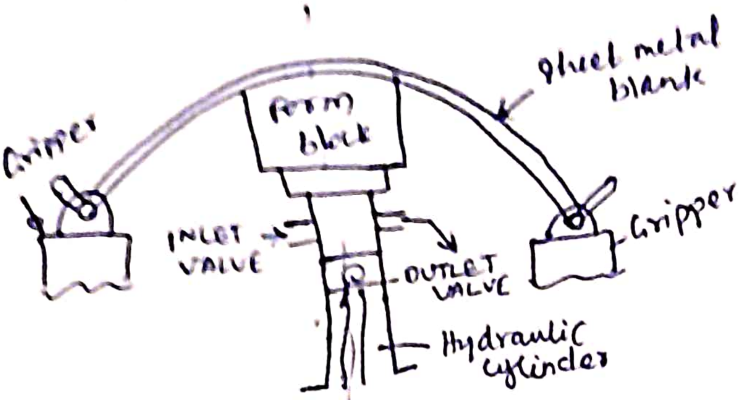
Unit-IV

SHEET METAL PROCESSES.

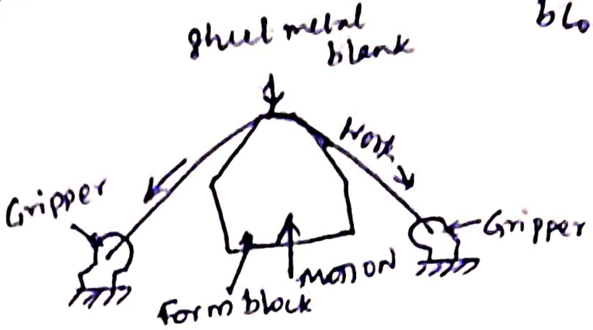
Sheet metal characteristics
Typical shearing, bending & drawing operations.
Stretch forming operations.
Formability of sheet metal
Test methods.
Special forming processes.
Working principle and applications
- Hydro forming
  Rubber pad forming
  Metal spinning
Introduction to Explosive forming.
Magnetic pulse forming.
Peen forming
Super plastic forming
Micro forming
Incremental forming.

Stretch forming (Operation)

* for producing large accurately contoured sheets.
* stressing the work blank beyond the elastic limit by means of form block



Form block method

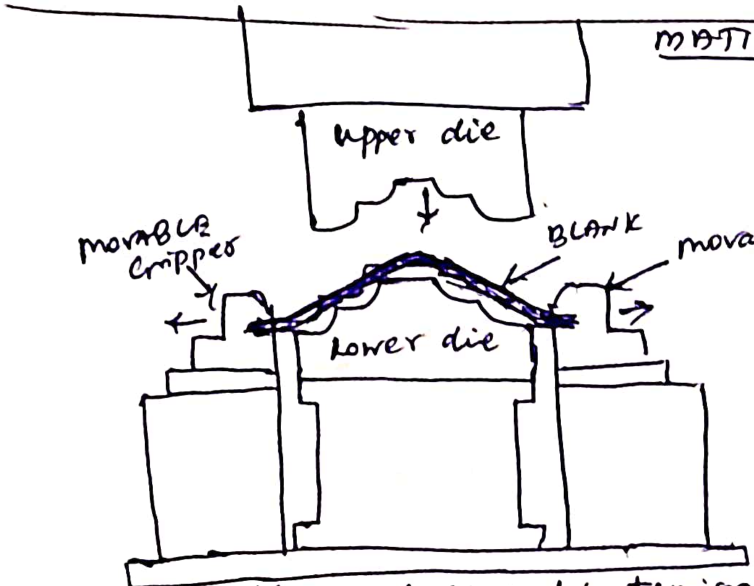


Stretch forming.

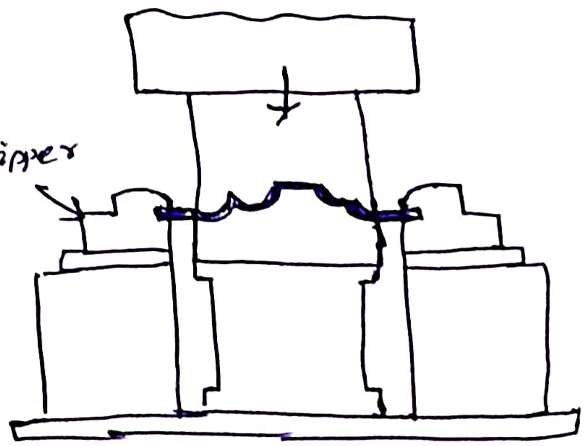
Types
(1) Form block method
(2) Mating Die method

* Two ends of the blank or sheet metal are tightly held by adjustable grippers.
* These grippers are fixed but adjustables.
* Then the form block is moved towards the blank to make the required shape.
* Form tool operated by hydraulic cylinder.
* Fixing the Form block stationary, and moving the grippers towards the form.

MATING DIE METHOD



(a) work blank subjected to tension



(b) Movement of upper die towards blank.

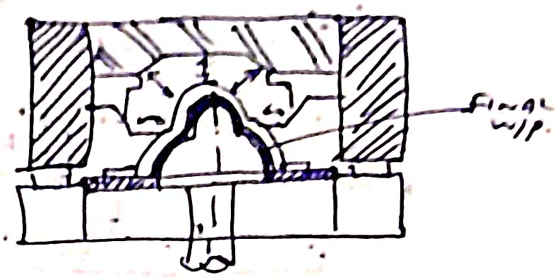
* The blank is held in movable grippers.
* The blank is placed b/w lower & upper dies.
* lower die kept stationary & upper is movable which is operated by hydraulic or pneumatic cylinders.
* Due to continuous stretching of the blank by the upper die, the plastic flow of sheet metal takes place b/w lower & upper dies.
* When the upper die edges reaches the top surface of the blank, the stretching process is completed.

Hydroforming

↳ (1) Hydro mechanical forming
↳ (2) Electro hydraulic forming.

Hydromechanical Forming

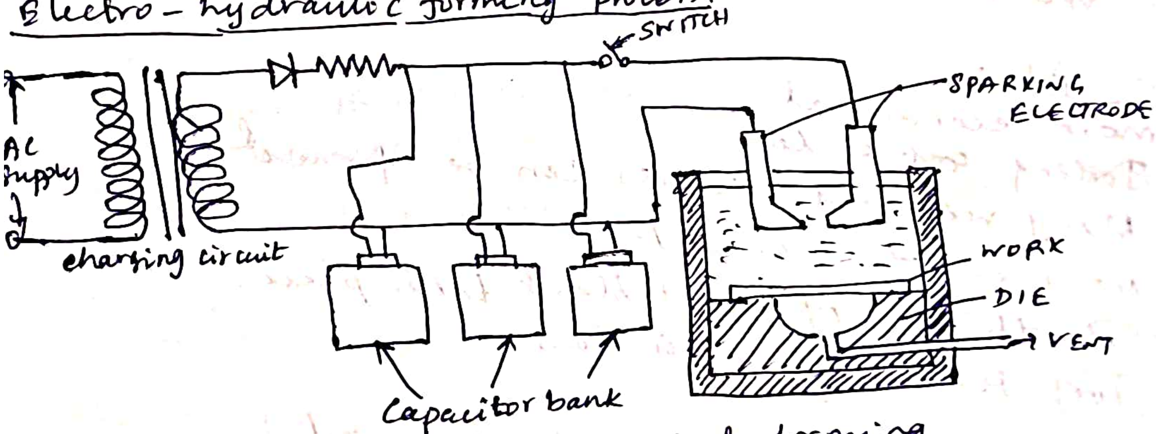
In this type of forming process, the punch is connected to the lower die.



The required shape of inner configuration is made on the punch.
A rubber diaphragm or seal is used for making perfect sealing b/w dies.
This seal is placed across the bottom of the pressure forming chamber.
The pressure-forming chamber is filled with hydraulic fluid.

Adv.
Thinning of metal, spot stresses & spring back are drastically reduced or completely eliminated.
mass production b/c
complicated contours can also be made.
sharp corners are also possible.
Tolerance of upto 0.005 mm/mm is practically possible.

Electro-hydraulic forming process

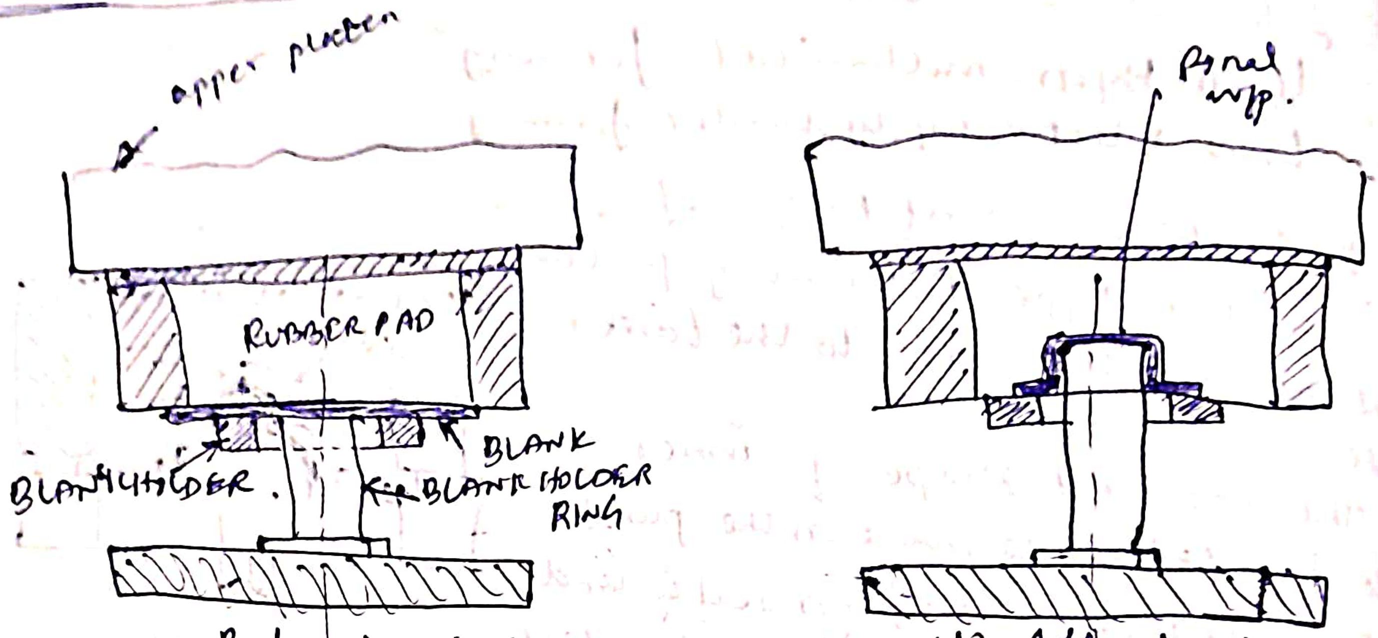


* similar to hydro-mechanical forming.
* But the applied pressure over the blank differs b/c the pressure inside the pressure forming chamber is produced by electrical.
* when the supply is given to electrical circuit, a high energy is discharged through a bank of capacitor to the hydraulic fluid contained in the chamber.
* Discharge energy in the chamber is in the form of shock waves & pressure.
* mechanical energy is used for metal forming operations in the same manner as mentioned in hydro-mechanical forming operation.

Adv-
Pressure inside the chamber is high due to confined shock wave & fluid pressure
Time require is low.

Disadv.
Energy loss occur.
Due to shockwaves both drag & lift created.

Rubber pad forming (MARFORM process)



(a) Before forming (b) After forming

* This process is mainly used for bending & stretching or drawing operations.
* For facilitating, the different form blocks called punches are arranged at regular intervals along the pressing bed called rubberpad.

DIE: is a set of flexible material such as rubber or polyurethane membrane. (placed on a ram of a press.

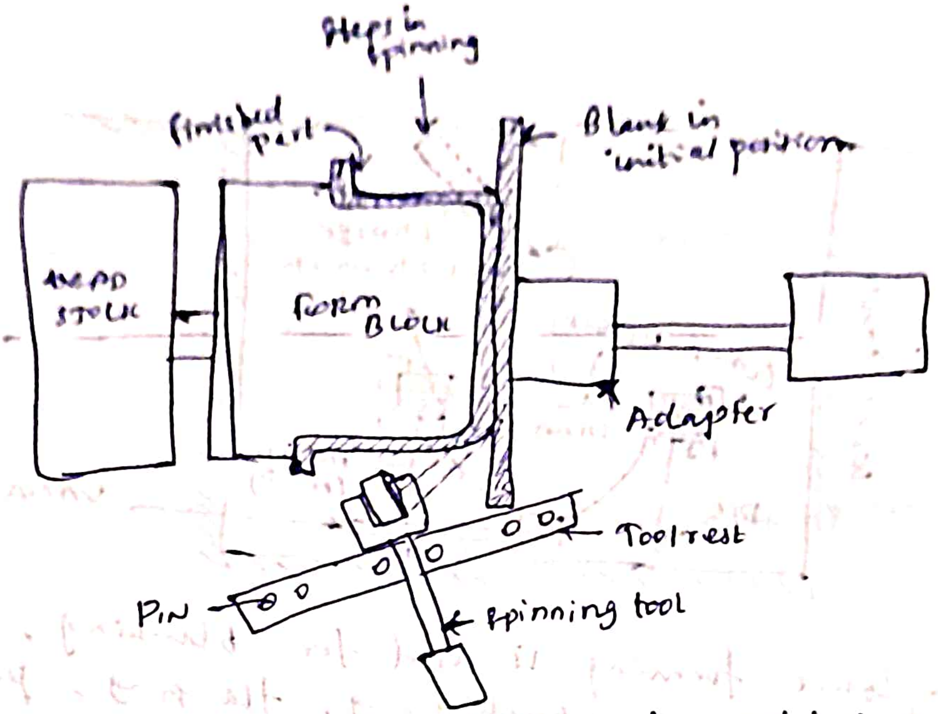
Process Adv.

↳ more economical
↳ Tooling cost is less
↳ Many required shapes can be formed
↳ No need of lubricant
↳ No thinning of metal blank takes place.
↳ Tool setting time is less.

Limi

↳ Rubber pad will rapidly wear out.
↳ Sharp corners cannot be accurately made.

Metal Spinning



Metal spinning, is a metal working process by which a disc or tube of metal is rotated at high speed and formed into an axially symmetric part.

This process produces hollow parts that are typically circular in c/s.

Process

(1) Circular sheet metal blank is centered on a lathe which is placed against a form block.

(2) The form block is mounted on the headstock of the spinning lathe.

(3) The blank is tightly held b/w form block and tailstock spindle.

(4) The required contour surface is made on the form block.

(5) The pressure is applied by the roller type forming tool, which is placed on the toolpost of the spinning lathe.

App

1) Al, cu, brass & stainless steel can also be spun.

2) Suitable for conical shape parts & for low volume production

Adv.

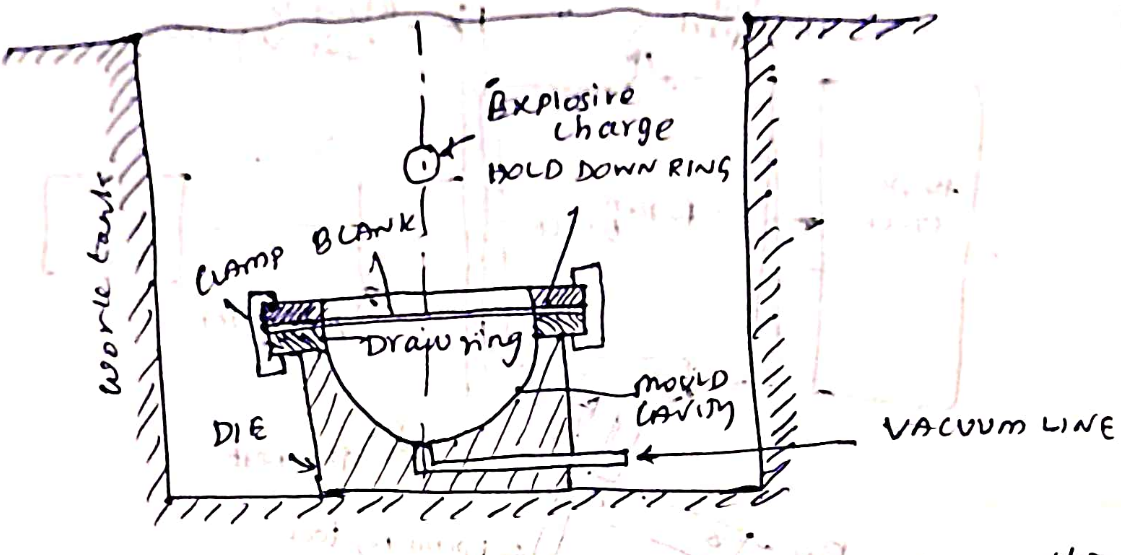
- Heat generated due to friction is used to retain the sheet metal in the plastic state.
- More economical for low volume production.

Dis.

more complex shapes requires segmental chucks. leads to increase the cost

Skilled labour required for better Accuracy & Quality of finish.

Explosive forming



* Explosive forming is used for blanking, cutting expanding, coining, embossing, flanging, powder compacting, drawing, sizing operations etc.

* Explosives used can be high energy chemicals such as TNT, RDX & DYNAMITE or gaseous mixtures or propellants.

* These chemicals are used in various forms such as rod, sheets, liquid, stick etc

Process

1. Explosive charge is located at some distance away from the blank and it's energy is transmitted through some fluid medium such as water.
2. Water ⇒ acts ⇒ as energy transfer medium.

Adv.

1. Capital investment is less & presses are not required
2. Only one die is enough to form the sheet metal
3. Components formed in one stroke.

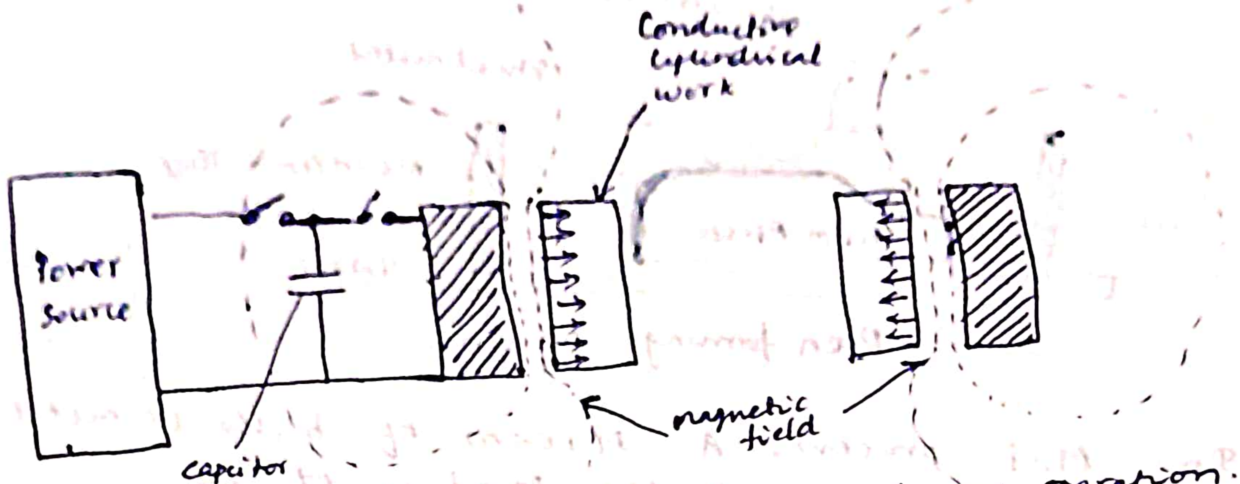
Disadv.

1. High trained operator req. & noisy operation handle with care ⇒ Explosive.

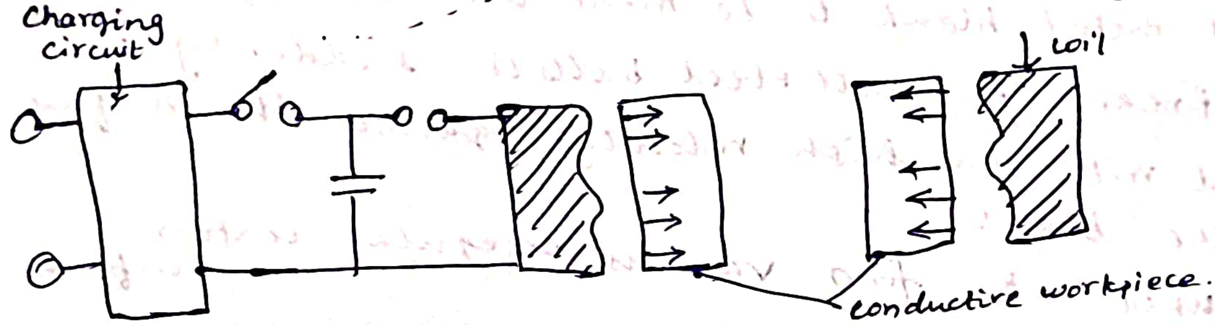
Appli.

Aerospace comp.

Magnetic pulse forming



Magnetic pulse forming operation.



Magnetic pulse forming process

The required shape of the sheet metal is obtained by specially designed magnetic coil.

Principle :

The required shape is obtained by discharging a capacitor through coil over a period of microseconds in the blank.

During this discharges, the magnetic flux densities of the order of hundreds of kilogausses can be produced.

Adv.

1. Carried out with uniform rate of forming.
2. better process for certain materials.
3. Surface finish excellent & less time required

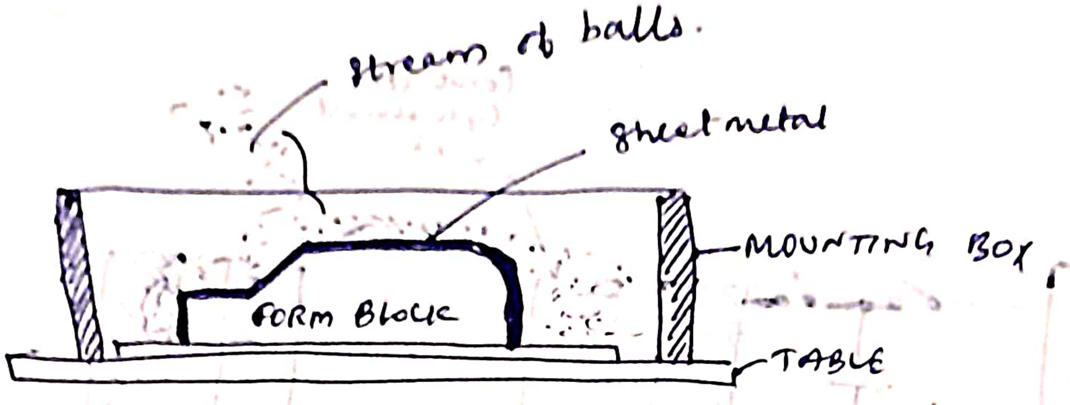
Disadv.

1. Non-conducting materials are not processed.

App.

1. Both compression & expansion of circular bar can be carried out.
2. Instrument gear assembly, embossing, sizing of cups. etc.

Peen forming



Peen forming process.

In this process, a stream of balls or metal shots is blasted against the surface of the sheet metal blank to be made into the required shape.

Stream of small steel balls is suddenly forced with very high velocity against the surface of the blank.

Used to form various irregular contour surfaces of sheets and plates.

Adv

1. complex contours can be easily produced.
2. Peening is also used as salvage operations for correcting bent or distorted parts.

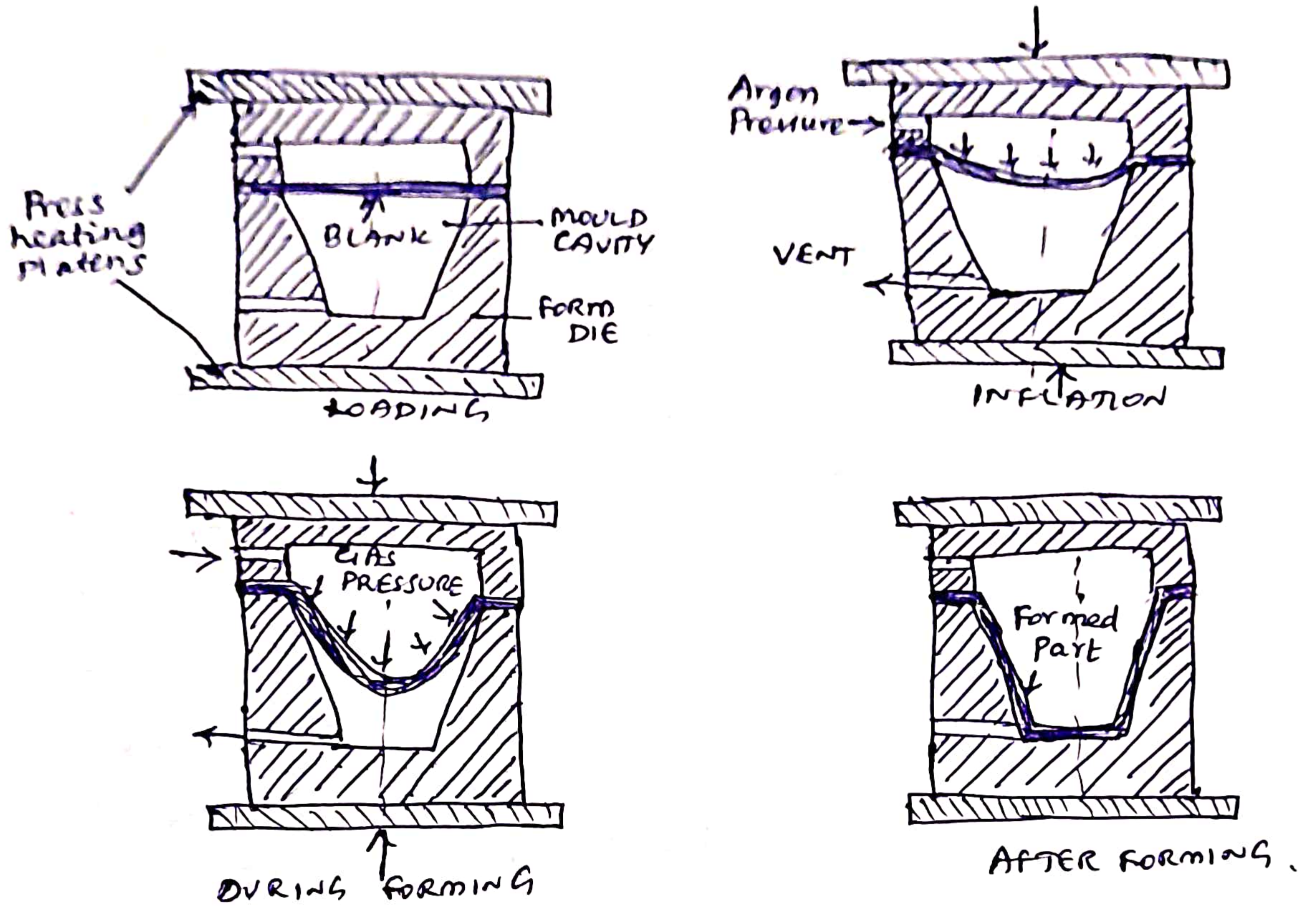
Dis

1. requires longer time for forming the required shape
2. requires additional devices for forcing out metal shots

App

1. Producing Honey comb Panels such as aircraft wings and larger tubular shapes.
2. Smoothing the complex curvature of aircraft wings.

Super plastic forming



* process of sheet metal clamped b/w a die cavity and a plate which are kept at the suitable temp.
* Gas pressure is applied to deform the sheet by forcing it against the walls of the die cavity, under suitable stress and deformation rate.

SUPER PLASTICITY

↳ ability of certain materials to undergo extreme elongation at the proper temp. and strain rate.

Materials developed for superplastic forming are

1. Bismuth – tin (200% elongation)
2. Zinc – Aluminium
3. Titanium – (Ti – 6AL-V)
4. Al – (2004, 2419, 7475)

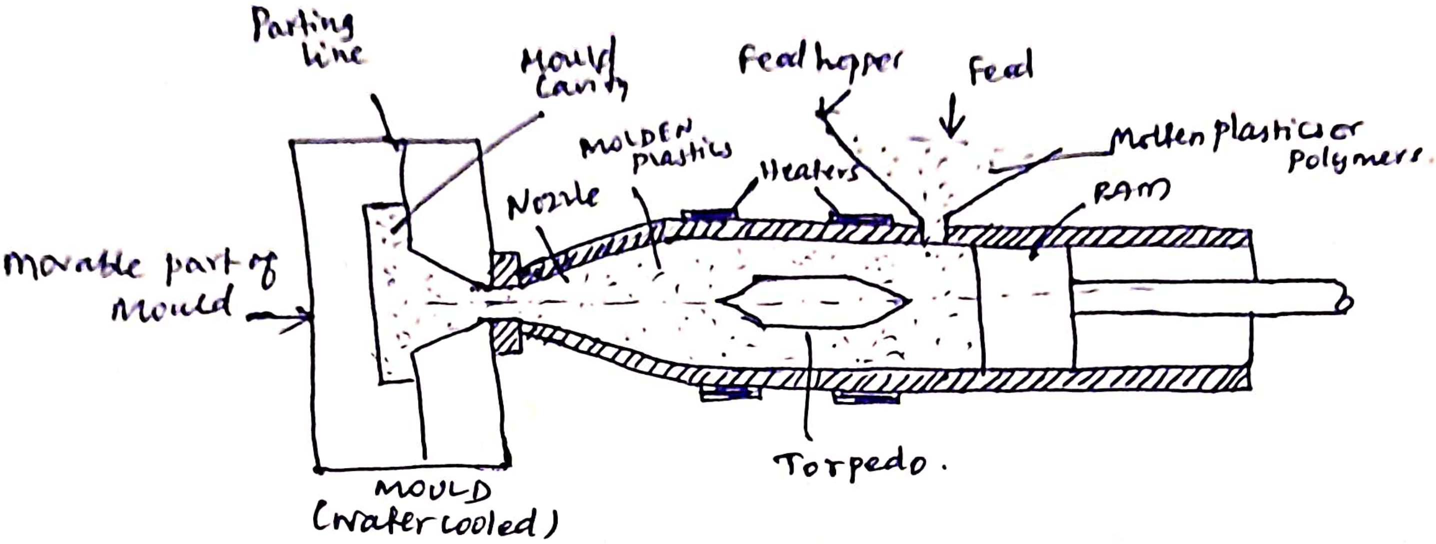
Process
Hot forming – up to 1000°C – superplastic Alloy
Innert gas pressure – up to 50 bar.
Material ~~can~~ can allow 500% elongation.

Unit - V

Manufacturing of plastic components

Types & characteristics of plastics
Molding of Thermo & Thermo setting polymers.
Working principles & Typical Applications.
Injection moulding
Plunger & screw M/c
Compression moulding
Transfer moulding
Typical Industrial Applications.
Introduction to blow moulding
Rotational moulding
Film blowing
Extrusion
Thermoforming
Bonding of Thermoplastics.
Duff moulding.

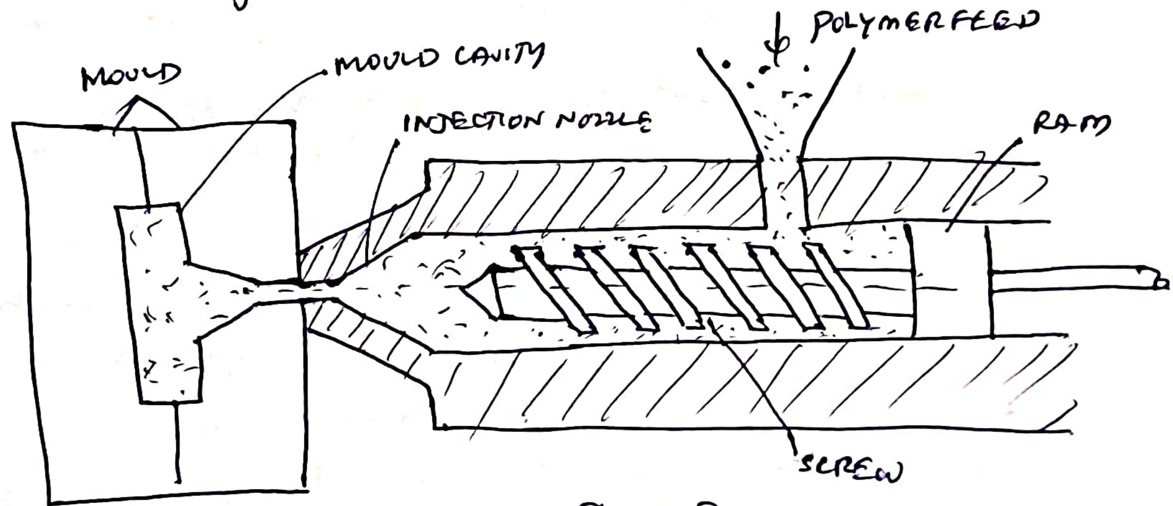
Injection moulding



Ram or Plunger type injection moulding machine.

* used to achieve high speed moulding of thermoplastics.

Working Principle } => ~~melting~~ ~~not moulding~~ molten thermoplastic is injected into a mould under high pressure.



Type-I

Ram or plunger type.

* loaded in hopper
* At heating section temp 150° to 370°C.
* melted plastic => liquid state
* Force by injection ram or plunger.

TORPEDO

spread the polymer melt into thin film in close contact with the heated cylinder walls.

* heated material is injected by ram

Type-II

screw type injection moulding.

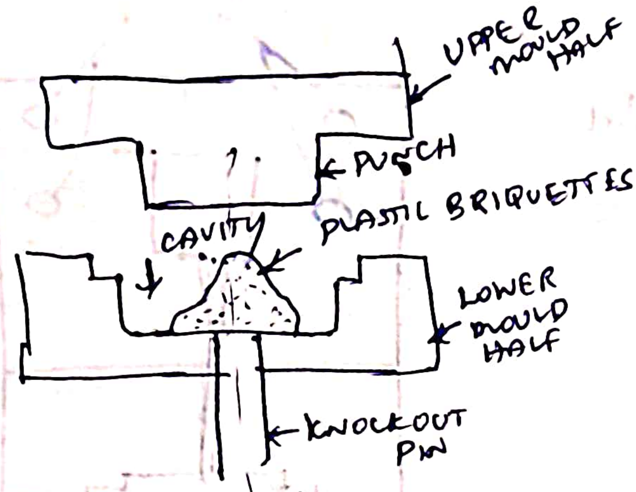
↳ 2 units
  ↳(1) Injection unit
  ↳(2) Moulding unit

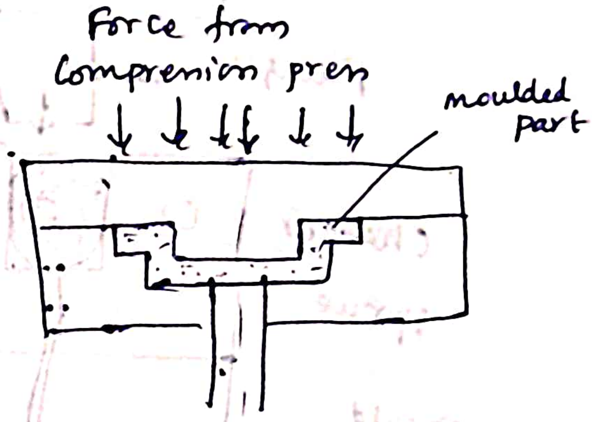
Injection unit

↳ has hopper, screw & heating section.

↳ pellets are initially ~~fed~~ fed inside hopper.

Compression moulding.





It is widely used for thermosetting polymers and it is also used for thermosetting polymers.

A pre-measured quantity of plastic in the form of particles or briquettes is placed in a heated mould and compressed at suitable pressure & temp.

The charge is placed in the heated mould cavity and mould is closed.

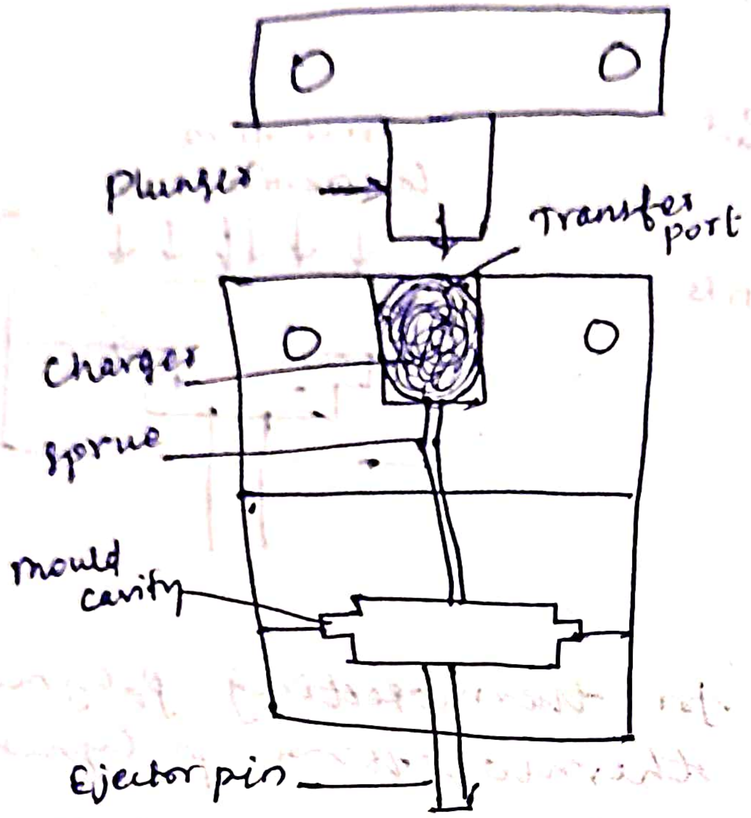
The desired compression is given by compression press thereby resulting an immediate contact of the polymer charge with all parts of the mould.

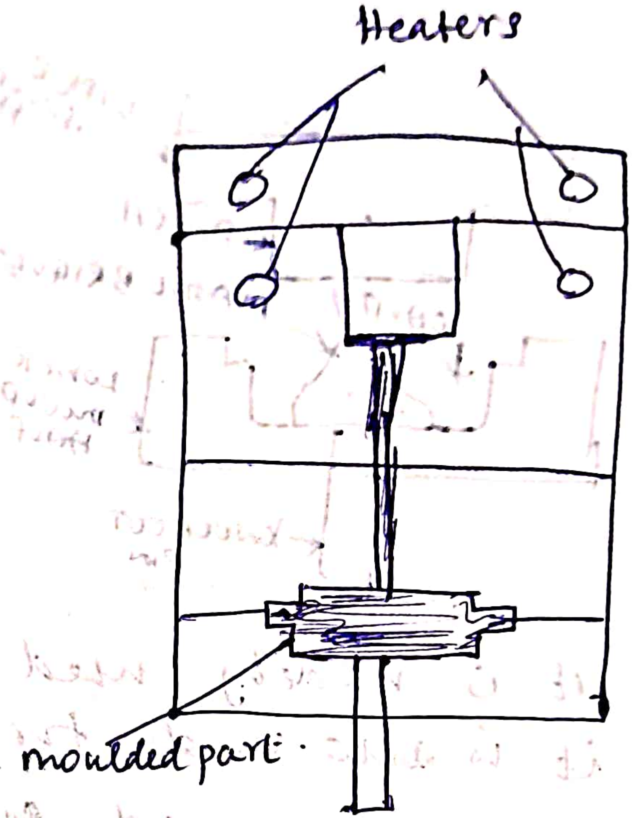
Both pressure and heat ensure the flow of resin, filling of all parts and corners of the cavity.

For thermosetting systems, the pressure is maintained till the intermolecular linking is obtained to an optimum level.

Finally, the mould is opened and ejected from the cavity.

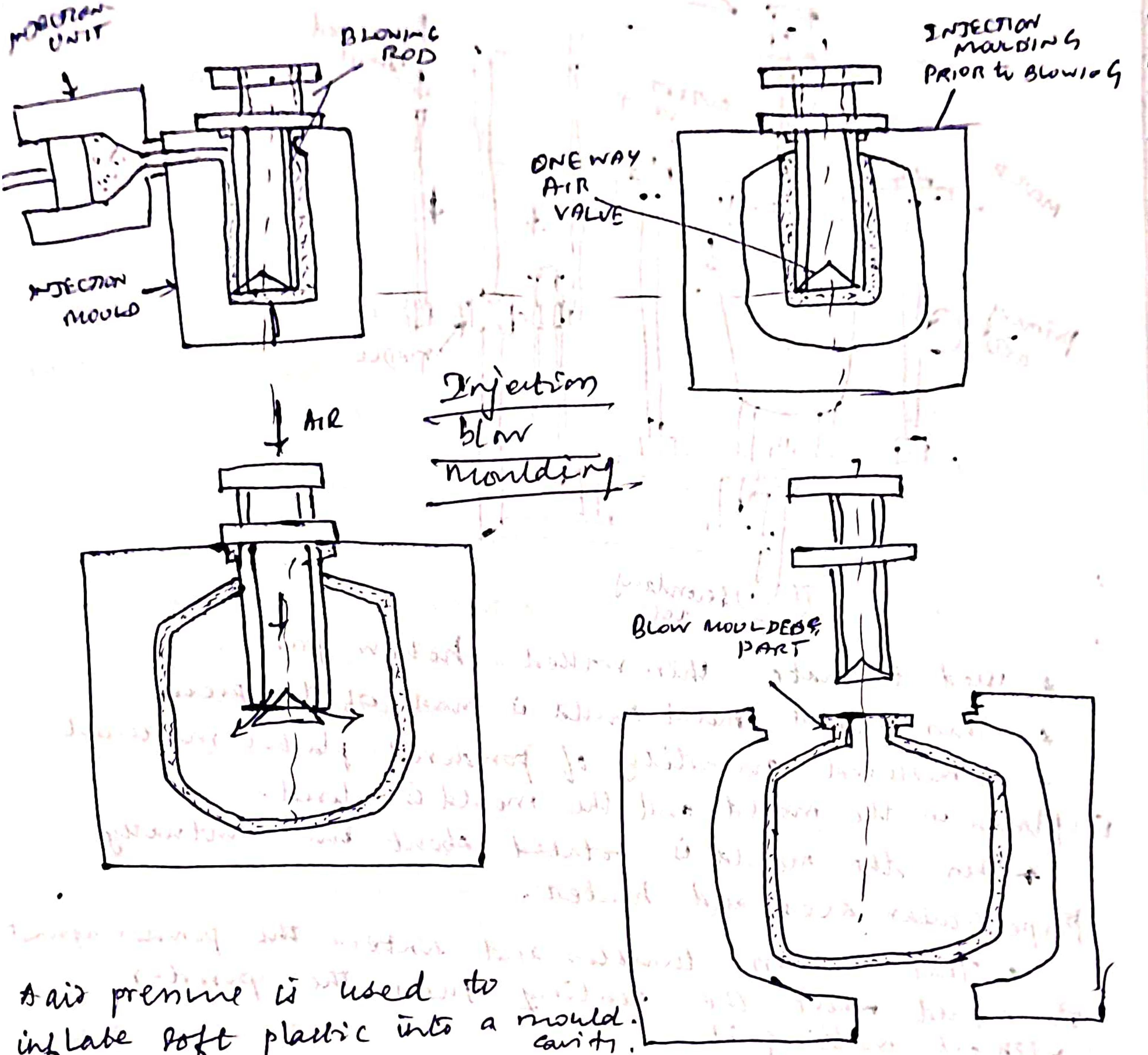
Transfer moulding — Industrial Applications.





It is a modification of compression moulding. In this process, the amount of material is measured and inserted in a separate chamber called transfer pot before the moulding process takeplu

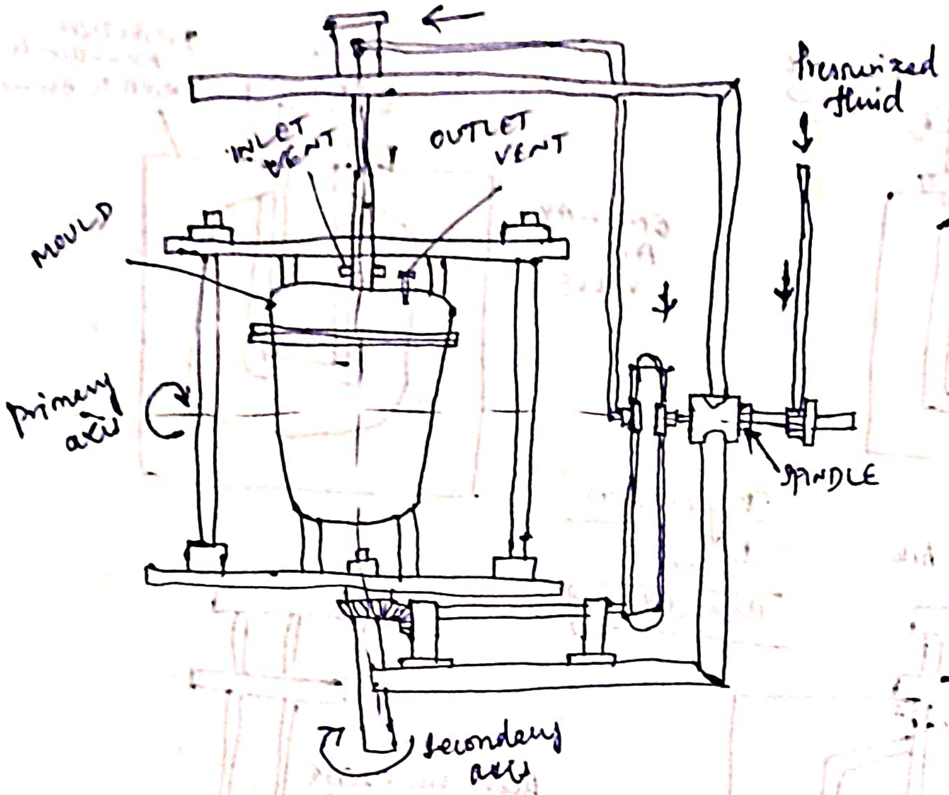
Blow moulding
↳ Injection blow moulding.
↳ Extrusion blow moulding.



* air pressure is used to inflate soft plastic into a mould cavity.

* Process:
a hot extruded tube of plastic called PARISON is placed b/w two parts of open moulds.
* Bottom end of the PARISON is sealed.
* Compressed air is used to blow the molten plastic into the mould and the tubes gets pinched off and also welded at the bottom by closing the mould.
* Air pressure 0.7 to 10 kg/cm²
* Air pressure will force the tube against the walls of the mould.
* Finally the component is cooled & the mould is open & release

Rotational moulding



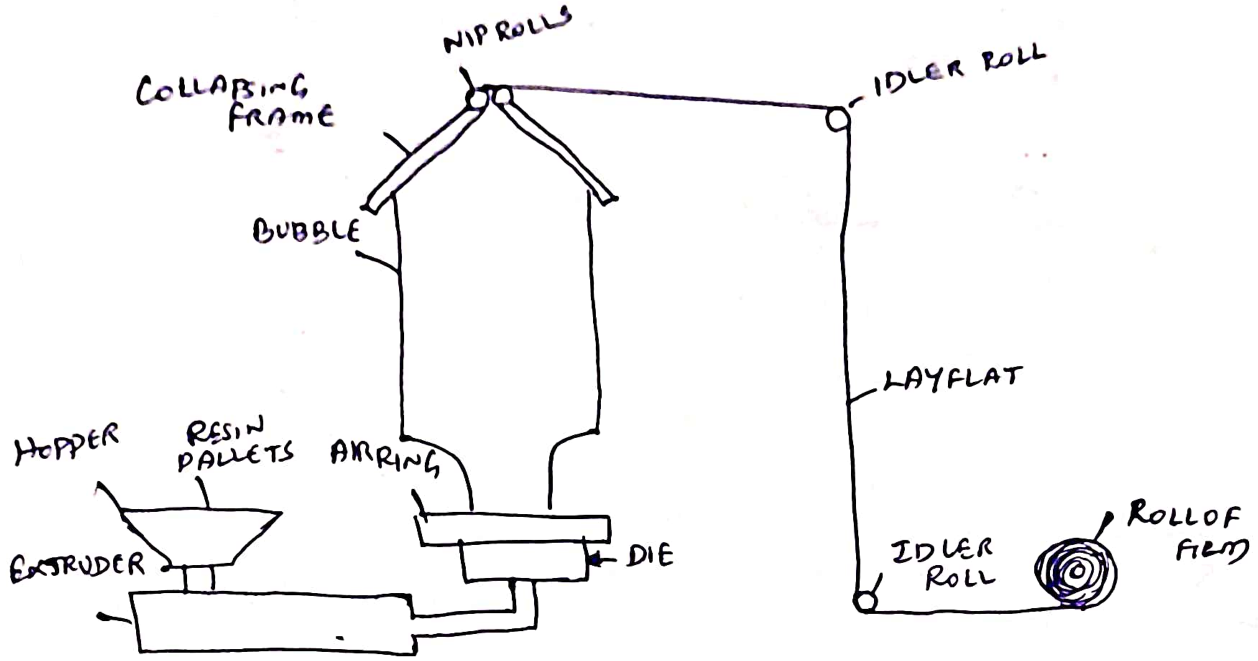
* used to make thin walled hollow parts.
* Thin walled metal mould is made of two pieces.
* A measured quantity of powdered plastic material is placed in the mould and the mould is closed.
* Then the mould is rotated, about two mutually perpendicular axes and heated.
* This action tumbles and sinters the powder again the mould where the heating fuses the powder without melting it.

Application.

- Produce toys using PVC
- used to make large containers of polyethylene
- make petrol tanks for motorcars from polyethylene & nylon.
- buckets, housings, boat hulls are made by this process.

* Parts with complex hollow shapes with wall thickness 0.4mm minimum.
* Large size parts as 1.8m x 1.8m x 3.6m can also be formed.
* Surface finish similar to walls.
* Temp — Time relationship during the oven cycle very important.

Flim ~~Moulding~~ blowing



* Most widely used method of film forming.
* Crystalline sharpe melting polymers such as nylon or poly ethylene terephthalate (PET) are very much suited for the film production.

Process

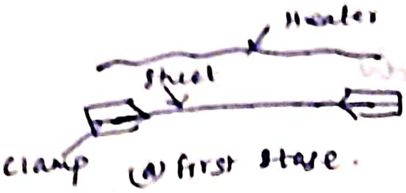
* Initially heated plastic powder is extruded by using extrude machines called extruder.
* Air is introduced through a hole at the center of the die to blow up the tube similar to a balloon.
* The die is similer to die used for making pipes or tubings.
* A high speed air ring mounted at the top of the die used for making pipes or tubings.

Application

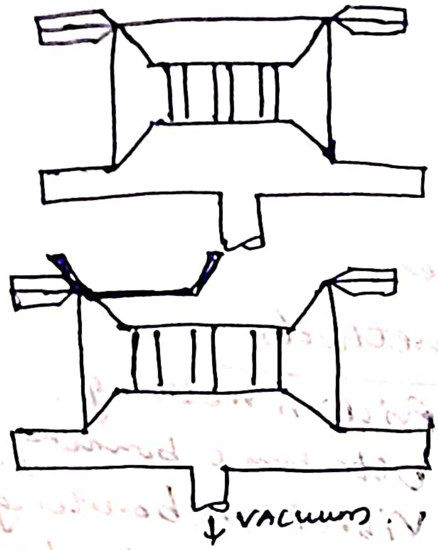
* Most plastic bags and wrapping films are made.
* Industry packaging (eg. shrink film, stretch film, bag films or container liners).
* Consumer packaging (eg. food wrap film, packaging bags)
* Laminating film
* Barrier film. → EVOH ⇒ Ethylene - Vinyl Alcohol Copolymer

Thermoforming

- ↳ Vacuum forming
- ↳ Pressure forming
- ↳ Matched die forming



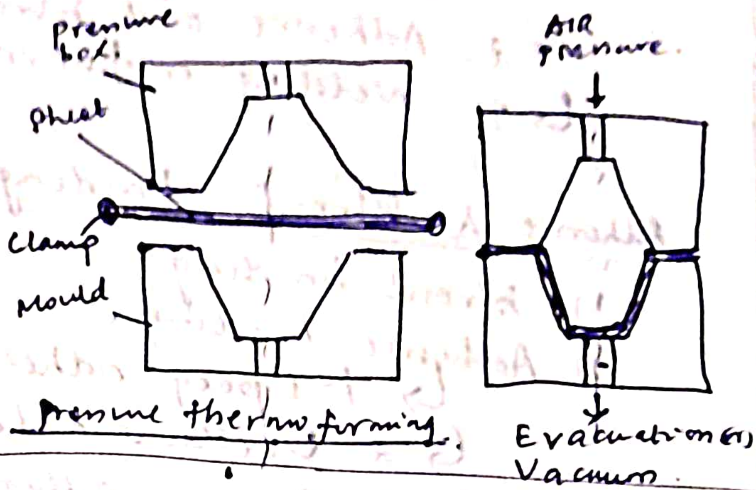
(a) first stage.



Vacuum thermoforming

* Vacuum pressure is used to form the heated thermoplastic sheet into the desired shape.
* Plastic sheet is heated in a heater and the sheet is fixed in a clamp.
* Air b/w sheet & mould is removed
* Surface of the mould when it cools and solidifies.

* Similar to vacuum forming pr...
* Air pressure required is much higher as compared to the vacuum forming.

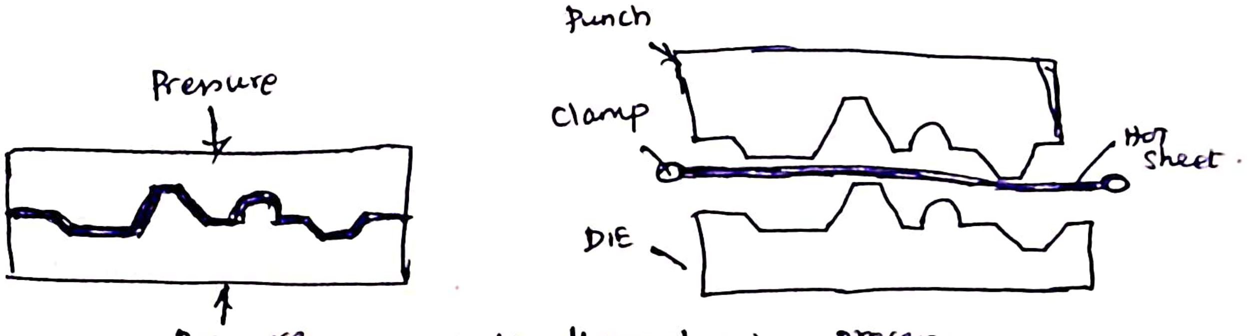


Pressure thermoforming.

* Thermoforming is the process in which the thermoplastic sheets are formed with the application of heat and pressure in mould.
* Thin sheet (upto 1.5mm) and thick sheet (about 3mm) can be formed easily.

Matched die forming:

* Also called mechanical forming.
* Mould consists of Die & punch.
* Thermo plastic sheet is heated with application of heat until it softens.
* Pre heated sheet is placed into the die surface and through punch pressure is applied on the hot sheet.



matched die thermoforming process.

Adv

* Useful for rapid prototype development
* Setup cost, production cost low.
* Dimensional stability is good.

Dis Adv

* Surface finish is poor.
* Non uniform wall thickness produced.

Application

* Trays, drink cups,
* Refrigeration door lines.
* Panel for advertising signs.
* Disposable cups, containers, etc.

Bonding of Thermoplastics

- 1. Mechanical fastening
- 2. Adhesive and solvent bonding
- 3. Welding or fusion bonding.

Adhesive & Solvent bonding methods.

1) Solvent bonding.

2) Adhesive bonding

- 1. Epoxy adhesives
- 2. Urethane adhesives
- 3. Acrylic adhesives.
- 4. Anaerobic adhesives
- 5. Cyanoacrylate adhesives.

Welding or fusion bonding methods.

- 1. Friction bonding (or) Friction welding.
- 2. Ultrasonic welding (or) Ultrasonic bonding.
- 3. Vibration welding (or) Vibration bonding.
- 4. Induction welding (or) Induction bonding
- 5. Hot gas welding.
- 6. Hot Tool welding.